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### ► To cite this version:

Sophie Mense, Karina Höveler, Pauline A. Blohm, Lisa C. Willemsen. Designing a tool for authoring digital problem-solving tasks in an app – an integrative learning design study. Thirteenth Congress of the European Society for Research in Mathematics Education (CERME13), Alfréd Rényi Institute of Mathematics; Eötvös Loránd University of Budapest, Jul 2023, Budapest, Hungary. hal-04410875

**HAL Id: hal-04410875**

**<https://hal.science/hal-04410875>**

Submitted on 22 Jan 2024

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# Designing a tool for authoring digital problem-solving tasks in an app – an integrative learning design study

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*Students need possibilities to discover mathematical structures through experiencing variation in sequenced tasks to develop a conceptual understanding of a mathematical content. Therefore, task designers should be able to design appropriate tasks and tasks sequences in digital learning environments like apps and fit them to the learners' needs. This paper illustrates the design and research process of the mathematic educational app 'Kombi'<sup>1</sup> developed in a design research project following the Integrative Learning Design Framework (ILDF). The paper focuses on the research-based development of a tool to digitally author combinatorial problem-solving tasks and reports on its micro-cyclical and multi-methodical development and evaluation. It gives a comprehensive insight into the implementation of the respective design principle and elements in 'Kombi' and presents results of formative testing by mathematics educational researchers, teachers and pre-service teachers.*

*Keywords: Design research, app development, problem-solving tasks, combinatorics.*

## Introduction

To develop a conceptual understanding of a mathematical content, students need possibilities to discover the respective structures and systematics through experiencing variation in the critical aspects of a concept within a sequence (Marton & Pang, 2006). Learning environments with problem-solving tasks should therefore offer consecutive tasks that allow discoveries of conceptual links by varying contexts, task parameters or (process-oriented) assignments to facilitate a conceptual understanding (Bardy et al., 2021). The design of these consecutive tasks should also be possible for digital learning environments (Bokhove, 2017), which requires a tool for authoring digital problem-solving tasks. Following this demand this paper deals with the question how this kind of digital tool might be designed, and which design principles (DPs) and design elements (DEs) should be considered.

Combinatorial problem-solving tasks are compulsory in the German primary school curriculum and learners are asked to solve simple counting problems increasingly systematic and to justify the completeness of their solution. However, analogue combinatorial solution processes are often accompanied by several challenges, for example the need for many manipulatives as well as the time-consuming active creation of all possibilities, both due the combinatorial explosion (Winzen & Höveler, 2020). These challenges complicate the discourse about systematic strategies in classroom, which is essential for the development of a combinatorial understanding (Winzen & Höveler, 2020). Therefore, the topic-specific design research project 'PAZ-digital' aims at designing and researching digital combinatorial learning environments with a subject-appropriate software for solving combinatorial

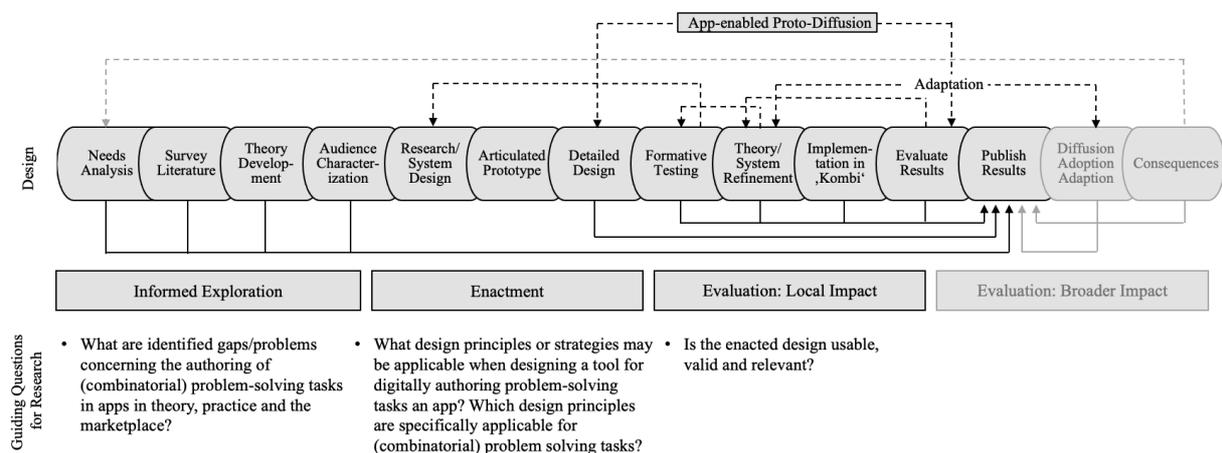
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<sup>1</sup> The German version of the free iOS app for iPads is available at <https://apps.apple.com/de/app/kombi/id1626071081>.

counting problems to overcome these challenges, namely the app ‘Kombi’. ‘Kombi’ includes features like a “free play-mode” that can serve as a flexible, virtual manipulative, an archive of all saved solutions and pre-installed tasks. Previous research on ‘Kombi’ already focused on how the app can be used as a manipulative by learners and how increasingly strategic combinatorial solution processes can be supported by the app (e.g. Winzen & Höveler, 2020). In the process of integrating tasks in the app, the desire came up to also include a flexible tool that allows both teachers and students to author tasks themselves, to enable the design of tasks and learning environments that fit the students’ needs (Bokhove, 2017) and to involve all participants of the teaching and learning processes in the digital task design (Mackrell & Bokhove, 2017). The development and evaluation of this authoring tool will be in the focus of this paper. Due to the lack of comparable apps that could serve as a model for ‘Kombi’, the design process showed early on that the development of the elements of the app required a cyclic design research approach in itself to ensure both a topic-specific didactically appropriate design and a good usability of the app while integrating the needs of both students and teachers. By integrating processes from several fields such as instructional design, object-oriented software development, product development and diffusion of innovations and educational research, the Integrative Learning Design Framework (ILDF) (Bannan, 2013) offered a suitable framework for the four years long multi-cyclical and multi-methodological development process of ‘Kombi’, a collaboration between researchers in mathematics education and two computer scientists. This paper exemplifies the iterative micro-cyclical design and research process of the tool for authoring digital problem-solving tasks in ‘Kombi’ following the ILDF. It presents the respective DP and DEs as well as their evaluation concerning the usability and relevance. To illustrate the impact of the framework for the design research process, the paper will be structured along the phases of the ILDF, and the respective research questions, methods and results will be presented successively.

## Theoretical framework: The Integrative Learning Design Framework

The ILDF is a “meta-methodological” (Bannan, 2013, p. 115) process model for conducting design research that enhances technology-based instruction design with iterative micro-cycles of research to generate research-based knowledge about “design-principles, aspects of learning, cognition, expert and novice perspectives as well as stakeholder positions” (Bannan, 2013, p. 117).



**Figure 1: ILDF and guiding questions for research for each phase in ‘Kombi’ for the development of a tool to digitally author combinatorial problem-solving tasks (based on Bannan, 2013, p. 116)**

The framework consists of the four phases *Informed Exploration*, *Enactment*, *Evaluation: Local Impact* and *Evaluation: Broader Impact* (see Figure 1) and “challenges researchers to provide improved articulation of design research processes by phase and to consider the entire scope of research from initial conceptualization to diffusion and adoption” (Bannan, 2013, p. 116). For the development of ‘Kombi’ the first phase included explorations of literature, related apps, the target audience, and different stakeholder positions that resulted in design implications. In the enactment phase these implications were transformed into a first detailed design that still had to be tested formatively during the following app development process. This highly iterative testing, revising and refining of the app design took place in the local impact evaluation phase that finally resulted in a publishable version of the app. Following the multi-methodological approach of the ILDF, in this paper the guiding questions for research on ‘Kombi’ (see Figure 1) will be answered by a literature review and market analysis, a task and contextual analysis as well as by usability testing and a qualitative expert review.

### **Authoring digital problem-solving tasks – *Informed Exploration* Phase**

The *Informed Exploration* phase deals with the question what kind of gaps and/or problems concerning the authoring of digital problem-solving – in particular combinatorial – tasks in apps can be identified. To answer this question a literature review and a market analysis have been conducted.

Regarding task construction, English and Gainsburg (2016) highlight that the focus of tasks in mathematics classrooms should be on building opportunities for learning or idea generation for all learners, which promote problem-driven conceptual development of core mathematical ideas as well as heuristics simultaneously. This “calls for more cognitively challenging tasks that encourage high-level thinking and reasoning, have multiple points of entry, and enable the use of varied solution approaches” (English & Gainsburg, 2016, p. 326). In the context of combinatorics ‘Find all solution’-tasks which are embedded in substantial learning environments and focus on the discovery of patterns and structures meet these requirements (Höveler, 2019). According to the Variation Theory, four necessary conditions of learning are contrast, separation, generalization and fusion (Marton & Pang, 2006). Therefore, tasks should be designed in a way that learners can focus on certain aspects simultaneously and separately, and experience differences, similarities as well as varying representations, which can be achieved by varying elements of the tasks in a task sequence of a learning environment (Bokhove, 2017). In a heterogeneous mathematics classroom, tasks should be selected, modified and assigned according to their openness, difficulty (complexity), language, help settings and accessibility (Bardy et al., 2021), since the necessary variations to foster learning depends on the context and background of the learner (Bokhove, 2017). For combinatorial learning environments, Höveler (2019) suggests providing several isomorphic combinatorial counting problems that differ in terms of the context, or analogical combinatorial problems that differ in terms of the respective size of the basic set and of the considered elements in a sample, and the combinatorial operation. She also suggests to allow different approaches to the solution process and explicitly including procedural competences like comparing, explaining and predicting.

So far there is “little focus within mathematics education on the design of the technology itself, or on how technology design might facilitate task design” (Mackrell & Bokhove, 2017, p. 55) especially with regard to designing for mathematics learning. Bokhove (2017) emphasizes the importance of

being able to flexibly change and author materials when designing tasks to include variations conducive to learning and states that ideally, technology would cater for authoring. Mackrell and Bokhove (2017) differentiate between two forms of task design environments: on the one hand the environment within which the task is designed and created, and on the other hand the task environment as the environment within which the task is performed. When it comes to an app without any cloud features due to data protection aspects, authoring tasks would normally require coding skills that cannot be expected from primary school teachers. This implies that for ‘Kombi’ a solution needs to be found to design tasks within the local app itself that can be shared easily in a classroom setting.

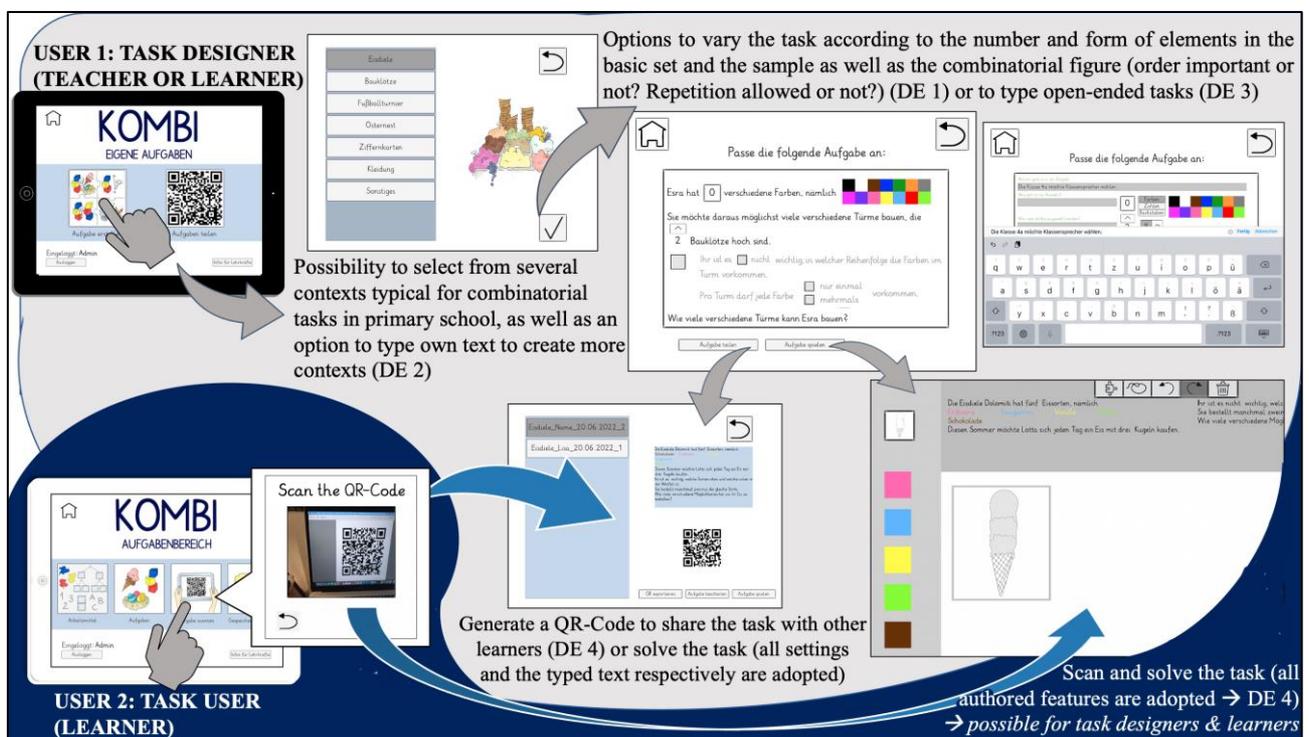
An analysis of the German market of learning apps for primary mathematics shows that only 6% of all apps offer task sequences that include different representations, only 8% include problem-solving and there is no app for combinatorics in primary schools yet (Schwätzer & Walter, 2022). Irrespective of the topic or the age group, no app was found that allowed teachers and/or students to design tasks in the app itself. In conclusion it can be stated that in order to facilitate both conceptual understanding of combinatorics as well as heuristics for the learners, a tool for digitally authoring problem-solving tasks should offer the possibility to design isomorphic and analogical tasks in different contexts that can be adapted according to their complexity as well as their openness. In addition, it should also include process-oriented competencies.

### **Designing an authoring tool in ‘Kombi’ – *Enactment* Phase**

The *Informed Exploration* phase showed the relevance of adapting tasks to the learners’ needs and to allow them to make connections between tasks to promote problem-solving and further process-oriented competencies like explaining and arguing. Therefore, task designers should be able to vary tasks and author their own tasks for learners, not only in analogue settings but also in digital learning environments. The *Enactment* phase of ‘Kombi’ dealt with the question which DP and DEs may be applicable when designing a feature in the app that provides the opportunity to meet these demands, which ones of those are specific for combinatorial tasks and which ones are conferrable to different problem-solving tasks. The potentials of content variation and the inclusion of process-oriented competencies in problem-solving tasks to enable differentiation for heterogeneous learners were presented in the previous section. The informed exploration resulted in the DP ‘Authoring digital problem-solving tasks’ (Mense & Höveler, 2022) that should make it possible to make all these decisions with a digital tool. It is differentiated by the four DEs (Mense & Höveler, 2022) explained below.

A task analysis showed that the tool to digitally author tasks in ‘Kombi’ needs to be specified for combinatorial tasks: The complexity of combinatorial tasks is mainly defined by the combinatorial operation, the nature of elements to be combined and the number of elements in the basic set and the sample (English, 2005). Therefore, the digital tool should include options to vary these factors in particular, which led to the DE 1 ‘Digital adaption of task parameters’. To foster generalizations and the recognition of structural analogies for a conceptual understanding in the spirit of a spiral curriculum, isomorphic tasks should be designed in different contexts (Höveler, 2019). Therefore, the digital tool should offer a selection of contexts as well as the opportunity to generate own contexts, which led to DE 2 ‘Problem-solving tasks in different contexts’. Since the *Informed Exploration* showed the need to integrate process-oriented competencies in the tasks as well, the digital tool should offer the

possibility to pose different assignments. To offer as many options for the fostering of different competencies like describing, explaining, justifying etc., it was decided to integrate DE 3 ‘Digital creation of process-oriented tasks’ to allow the task designer a maximum scope of design. A further context analysis showed that for making the DP ‘Authoring digital problem-solving tasks’ fruitful, it is necessary to be able to digitally distribute the designed tasks with all their authored features without depending on a good internet connection on both the sender and receiver iPads. These considerations led to DE 4 ‘Digital distribution of tasks offline’. Research on technical options for implementation suggested the use of offline QR-Codes since they are easy to scan for primary school learners and might also be embedded in worksheets for combined digital and analogue learning environments. These findings from both the contextual and task analysis led to a detailed design of the DP ‘Authoring digital problem-solving tasks’ with the respective DEs as shown in Figure 2.



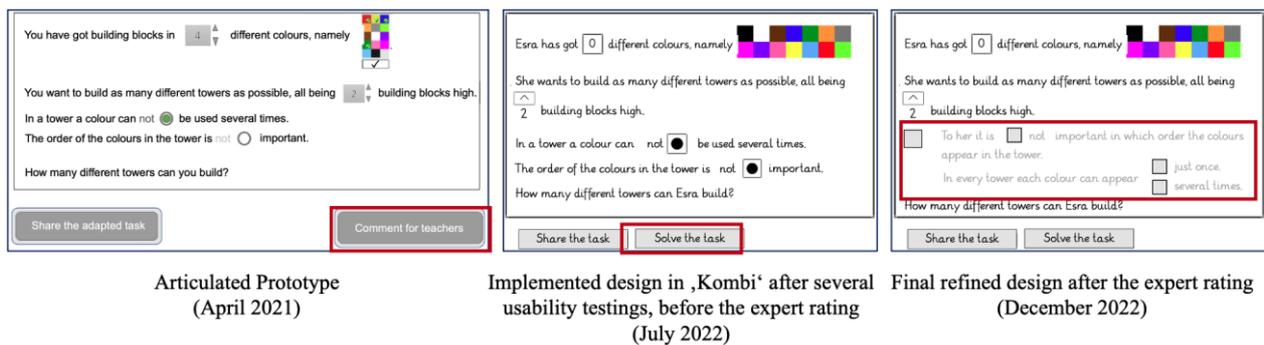
**Figure 2: Detailed design considerations for the DP 'Authoring digital problem-solving tasks' with the respective DEs and their final implementation in 'Kombi'**

A reflection of the DEs suggests that DE 1 ‘Digital adaption of task parameters’ and DE 2 ‘Problem-solving tasks in different contexts’ need to be enacted specifically for combinatorial word problems, DE 3 and 4 however might directly be applied for other word problems. Nevertheless, after further respective task analyses DE 1 and 2 might also be adapted for other problem-solving tasks.

### **Testing, refining and evaluating the tool – Local Impact Evaluation Phase**

The *Evaluation of the Local Impact* stands out by an app-enabled proto-diffusion (see Figure 1). During this phase the researcher continuously deals with the question as to how an enacted design is actually usable when implemented in the app and how it is relevant for different stakeholders. Simultaneously, the enacted design gets constantly improved according to the results of the formative testing. The detailed design considerations from the *Enactment* phase have been implemented in the

app (see Figure 2) using the development platform Unity and the programming language C#. The implementation process took place over several months with bi-weekly micro-cycles of implementation and usability testing by the authors for debugging and improvement of the Graphical User Interface. During these formative tests it was decided to remove the button with background information for teachers and replace the information with an additional manual for the app, since the necessary background knowledge for teachers exceeded the available size of the pop-up-window in the app. Instead, it was decided to integrate the option to solve the designed task oneself (see red highlight in Figure 3, left to middle picture). This change had the intention to allow learners to also design their own tasks and to give them greater autonomy in their learning process.



**Figure 3: Transformation of the authoring tool from Enactment to Local Impact Evaluation Phase**

In summer 2022, the by the time current version of the app was made available for an expert rating by ten teachers, pre-service-teachers (PST), and mathematics education researchers (MER) to get expert feedback before preparing the final version for the release in the AppStore. These groups of stakeholders were chosen because teachers are the future users of the features, the PST were trained explicitly in the topic of combinatorics and the use of digital media in school and MER contribute their high level of didactic expertise. The beta-version of the app was provided via TestFlight and the experts answered a written questionnaire with open and closed questions concerning (amongst others) the DEs presented above. Their answers were analysed and categorised with a qualitative content analysis (Mayring, 2014) and grouped into ‘usable elements’ and ‘elements to be improved’ (see Table 1).

While all groups of stakeholders praised the user friendliness, the guidance in the task design and the variety of adaption options, the teachers and MER also suggested ideas for improvement concerning the wording and length of the pre-installed task contexts, which they reported to be partly too complicated. In consequence, many tasks have been shortened and changed to be easier understandable. In addition, from the next update on, the second half of the task that defines the combinatorial operation will be an optional selection (see red highlight in Figure 3, right picture). This way it will be possible to design shorter tasks that also leave starting points for discussions whether the learners considered repetitive or swapped elements or not, without any of these decisions being right or wrong.

	Categories	Suggestion for improvement	Selected exemplary comments	Comments by
Usable elements	<ul style="list-style-type: none"> <li>• user friendliness</li> <li>• guidance in task design</li> <li>• variety of adaptation options</li> </ul>		<p>“I found the authoring of own tasks very felicitous. The pre-installed tasks make it very easy for the task designer and yet they have enough freedom to make own decisions.” (PST1)</p> <p>“I think that it makes sense that the counter of the elements of the basic set adapts automatically to the number of selected colours.” (MER2)</p>	Teachers, PST, MER
Elements to be improved	<ul style="list-style-type: none"> <li>• wording</li> <li>• length of the task</li> </ul>	<ul style="list-style-type: none"> <li>• ‘just once’ instead of ‘not several times’</li> <li>• possibility to shorten the task</li> </ul>	<p>“In some parts the wording of the pre-installed task contexts is too complicated, especially for younger learners who are not confident readers yet. It should be possible to shorten the tasks.” (MER4)</p>	Teachers, MER

**Table 1: Results of the qualitative content analysis of the expert reviews**

## Discussion, Conclusion and Outlook

The feedback by the expert raters showed that the implemented DP and DEs were evaluated as very relevant for designing combinatorial tasks for primary school learners in an app. The suggestions for improvement received from the raters allowed further refinements that could in large parts be implemented before releasing ‘Kombi’ in the AppStore. This illustrates the impact of the ILDF for the design and development process of ‘Kombi’: while an extensive phase of *Informed Exploration*, planning of DPs and DEs and an articulated prototype, as well as several micro-cycles of usability testing by mathematics education researchers and computer scientists already led to a product that was perceived as valuable for the intended aim, the inclusion of different stakeholders in the design and research process allowed further improvement. The next step in the design and research process of the ILDF will be the *Evaluation of the Broader Impact*, where ‘Kombi’ will be tested in a classroom setting to evaluate the usability and usefulness of the tool to author digital combinatorial tasks for heterogeneous learners and their individual ways of learning. Afterwards, ‘Kombi’ will be ready to be used in a digital learning environment and the first design cycle of ‘PAZ-Digital’ can be concluded. As discussed in connection with the *Enactment* phase, the DEs ‘Digital creation of process-oriented tasks’ and ‘Digital distribution of tasks offline’ are expected to be applicable not only to combinatorial tasks like in ‘Kombi’ but also to author other problem-solving tasks in an app. The DEs ‘Digital adaptation of task parameters’ and ‘Problem-solving tasks in different contexts’ would need some specifications for other contents than combinatorics but might then also be conferrable. It would therefore be desirable to test their applicability in other didactical app development processes.

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