



HAL
open science

The yoyo-bird: A transformative digital game for meaning making on trigonometric functions

Myrto Karavakou, Chronis Kynigos

► To cite this version:

Myrto Karavakou, Chronis Kynigos. The yoyo-bird: A transformative digital game for meaning making on trigonometric functions. Thirteenth Congress of the European Society for Research in Mathematics Education (CERME13), Alfréd Rényi Institute of Mathematics; Eötvös Loránd University of Budapest, Jul 2023, Budapest, Hungary. hal-04410858

HAL Id: hal-04410858

<https://hal.science/hal-04410858>

Submitted on 22 Jan 2024

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

The yoyo-bird: A transformative digital game for meaning making on trigonometric functions

Myrto Karavakou¹ and Chronis Kynigos¹

¹Educational Technology Lab, National Kapodistrian University of Athens, Greece;

karavak@eds.uoa.gr

In this paper, we investigate students' mathematical meanings developed while they played a digital educational game on trigonometric functions. It is a two-player competitive game, called 'the yo-yo bird game' and it was designed in MaLT2; a programmable Turtle Geometry software, affording dynamic manipulation of variable values through the use of sliders. MaLT2 provides a space for graphically and dynamically representing and manipulating models of mathematical concepts and relations. Four students of the 11th grade played this game and their activity and discourse were recorded. The UDGS model was used in order to frame and analyse the evolution of their meaning making processes regarding trigonometric functions. Students' continuously reconsidered game strategies were connected to a cycle of using, discriminating, synthesizing and gradually generalizing meanings on the mathematics embedded in the game.

Keywords: Educational games, educational technology, trigonometry, meaning-making.

Introduction

Digital educational games have gained increasing attention from the field of mathematics education (Byun & Joung, 2018; Totac et al., 2019). This raise of awareness falls into the general wave of educational transformation brought by the emergence of digital technology. In order for education to be genially transformed, pedagogical and epistemological considerations should be taken into account. However, Byun and Joung (2018) in their review claimed that this is not the case for digital games in mathematics education. On the contrary, they highlighted the lack of theoretical perspective, pedagogical orientation and mathematical-background expertise in existing studies. They also reported that most studies used drill-and-practice games, which is a kind of gamified version of traditional paper-and-pencil-based exercises. This type of games does not support the digital transformation of mathematics learning by exploiting affordances of digital tools, such as visualization of concepts, interconnected representations and dynamic manipulation of mathematical relations (Kaput, 1994; Kynigos, 2019; Roschelle et al., 2012). For this study, which was part of two EU projects (see Acknowledgements), we designed a transformative game on trigonometric functions and used it in educational settings. The game was designed in an online software called 'MaLT2', which offers rich opportunities for transformative, creative and explorative mathematical experiences (Grizioti & Kynigos, 2021; Kynigos & Karavakou, 2022). In this paper, we discuss the pedagogical affordances of the game framed within an educational setting, through the lens of students' meaning-making on trigonometric functions. The setting also involved the use of GeoGebra graphic calculator as means for supporting the meaning making process evolved through game-playing.

Theoretical Framing

Transformative digital games in mathematics education

We use the term ‘transformative’ as opposed to the term ‘enhancer’, according to the SAMR dichotomy (Kynigos, 2019; Puentedura, 2006). The SAMR model categorizes digital resources into two broad cases; 1) the ones that substitute or augment traditional educational practices through functional and technical improvement, e.g., in drill-and-practice type of games (enhancement), and 2) the ones that redefine and enable the creation of new types of tasks and practices, previously inconceivable (transformation). Transformative learning situations have been designed and investigated from different perspectives promoting their empowering potential on mathematics learning. We extend the SAMR model to digital games for mathematics, as an appropriate way to discern games in which technology plays a transformative role and imports added pedagogical values to learning practices. In transformative games, mathematics is integrated into the gaming idea, instead of obtaining the gaming ‘camouflage’ superficially in order to ‘sweeten’ mathematical engagement (Kafai & Burke, 2015). For this study, we designed a game as an attempt to incorporate the idea of transformation into digital games for mathematics education, by exploiting the potentials of digital resources in connecting mathematics with authentic experience (Kaput, 1994; Papert, 1980).

UDGS: a theoretical tool to frame mathematical meaning making

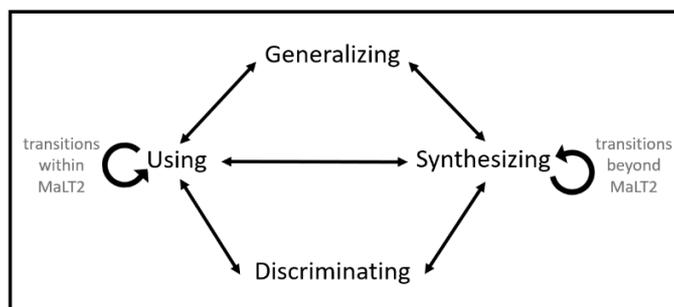


Figure 1: A modified version of the UDGS model

We adopted a modified version of the UDGS (Using, Discriminating, Generalizing, Synthesizing) model, introduced by Hoyles and Noss (1987) and revisited by Kynigos and Karavakou (2022), to frame and analyze students’ mathematical meaning-making process while playing the game. According to this model (Figure 1), this process can be mapped to progressively moving among all phases of 1. *using* (mathematical concepts are used intuitively or reflectively as tools for functional purposes while transitioning among the different representations within the digital resource, in order to achieve particular goals), 2. *discriminating* (locally distinguishing and identifying a mathematical concept or relation initially interwoven within a specific part of the digital tools used), 3. *synthesizing* (connecting the mathematical concepts used/represented in one digital resource with different contexts outside these digital representations, e.g., algebraic expression in paper-and-pencil, gaming ideas or representations of different digital resource) and 4. *generalizing* (extending and expressing mathematical relations and being able to recognize, use, and exploit them through the digital tools). In this study, we adopted this model for conceptualizing how and in what extent a game with transformative potential can affect students’ evolution of meanings on trigonometric functions.

Research Design

The yo-yo-bird game

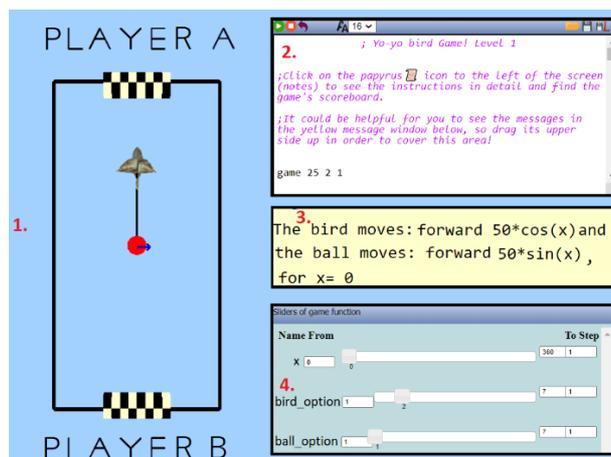


Figure 2: The four elements of the yo-yo bird game (screenshot when the game starts).

The yo-yo bird game¹ is a 2-player competitive game. It was designed in MaLT2, an online programming tool for tinkering figural dynamic models. It combines Logo procedural language with dynamic manipulation of procedure variable values (Grizioti & Kynigos, 2021; Kynigos & Karavakou, 2022). The game's technical part in MaLT2 consists of a 'game scene' (Figure 2.1.) constructed by a series of coded (sub-)procedures. They were written in the editor (Figure 2.2.) and composed the main 'game' procedure, which has three input variables called 'x', 'bird option' and 'ball option'. Each variable activated the function of a corresponding slider (Figure 2.4.). These 3 sliders cause changes on the scene (Figure 2.1.) and the message window (Figure 2.3.) in the following ways: a) the variable 'bird_option' takes values whole numbers from 1 to 7, each one corresponding to a trigonometric function (as shown in the 2nd column of the table in Figure 3) and controls the function $f(x)$ according to which the bird moves forward (its direction is shown by the bird's face); b) the variable 'ball_option' takes values from 1 to 7, each one corresponding to a trigonometric function (as shown in the 3rd column of the table in Figure 3) and controls the function $g(x)$ according to which the red ball moves forward (its direction is perpendicular to the bird's one as shown by the arrow); and c) the variable 'x' takes values from 0 to 360 and corresponds to the input value of both $f(x)$ and $g(x)$. The values of each slider are synchronously displayed in the message window (Figure 2.3.). For example, if $x=0$, $bird_options=2$ and $ball_option=1$ (Figure 2), the bird moves forward $f(0)=50 \cdot \cos(0)=50$ steps, while the red ball moves forward $g(0)=50 \cdot \sin(0)=0$ steps. The game is accompanied by a printed scoreboard² for students to fill in at each round. It consists of 8 rounds, at each of which each player successively: 1) notes down the option of function (1-7) for the bird's movement; 2) notes down a value (0-360) for the variable x ; 3) moves the slider of $bird_option$ to the value noted down; 4) slowly moves the slider of x from its previously left value to

¹ The yo-yo bird game can be found online here: <http://etl.ppp.uoa.gr/malt2/?yoyobirdgame>.

² The yo-yo bird scoreboard can be found here: <https://drive.google.com/file/d/1iY-Q2fKM5og5ySqSoShM-Z3ODwJ7M4Dx/view>.

the one noted down while calculating the earned points (if any) for them and/or their opponent; 5) notes down each player's points; 6) writes the option of function (1-7) for the ball's movement and moves the slider *ball_option* to that value for the opponent to play with.

| Option (slider value) | Bird's movement 🐦 (*) | Red ball's movement 🟠 (**) |
|-----------------------|-----------------------|----------------------------|
| 1 | $f(x)=50\sin(x)$ | $g(x)=50\sin(x)$ |
| 2 | $f(x)=50\cos(x)$ | $g(x)=100\cos(x)$ |
| 3 | $f(x)=100\sin(x)$ | $g(x)=50\sin(2x)$ |
| 4 | $f(x)=100\cos(x)$ | $g(x)=50\cos(2x)$ |
| 5 | $f(x)=100\sin(2x)$ | $g(x)=30\sin(3x)$ |
| 6 | $f(x)=100\cos(2x)$ | $g(x)=30\cos(3x)$ |
| 7 | $f(x)=50\tan(x)$ | $g(x)=30\tan(3x)$ |

Figure 3: Table of functions corresponding to *bird_option* and *ball_option* sliders

Regarding score calculation; while dragging the *x*-slider, if the bird touches the lower finish-line (*forward -100*), player B gets +1 point (Figure 4.1.); if the bird touches the upper finish-line (*forward 100*), player A gets +1 points (Figure 4.3.); if the bird touches the red ball, the round ends and player's gained points are lost (Figure 4.2.). The winning player is the one with the highest total score³.

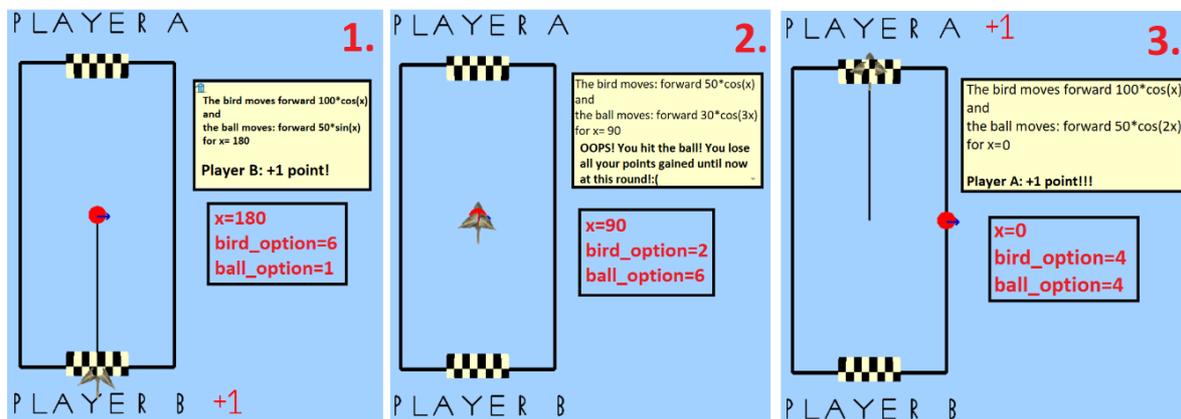


Figure 4: Three screenshots from the scene and message window for different sliders' values

Method

The data analyzed in this paper was taken from its implementation at a computer lab of a Greek public school, where four students from the 11th grade participated. The data consisted of their discourse, written notes and actions with the digital tools, captured through voice and screen recordings. The analysis was based on a coding procedure, initially made by the first author and checked and slightly rearranged by the second one. Students' actions and sayings were corresponded to one out of the four phases of the UDGS model and categorised according to the conceptual aspect of the meaning *used*, *discriminated*, *generalised* or *synthesised*. The implementation lasted for ninety minutes and was divided in two phases. For the 1st phase, the four students (G1: Nikos & Anna and G2: Lukas & Kostas) played the game in pairs (1 vs. 1) and for the 2nd phase, each group competed the other (G1 vs. G2). The 1st phase was defined as a 'trial game' and the 2nd one as 'the main challenge', aiming at provoking discussion and argumentation between them. The GeoGebra graphic calculator was

³ The instructions of the yoyo-bird are also briefly shown in this video: <https://drive.google.com/file/d/1pmqAMzrMEy6ow845dRNAiECMI6MRHLHz/view?usp=sharing>.

available from their mobile phones in case they wanted to visualize the graphical representation of trigonometric functions, through simply typing their formula. The participating students had already been introduced to trigonometric functions in their classroom and had some experience with traditional textbook exercises. According to their teacher, all of them had quite low performance to a relevant math test. The participating researcher (the first author) facilitated the implementation by encouraging students to express their strategies out loud, especially during the 1st phase.

Results

Each group of students developed different strategies while playing, but followed a similar flow of transitions among the phases of the UDGS model. At the 1st phase, they developed meanings on trigonometric functions such as relations on their input and output values, phase difference between sine and cosine functions, symmetry and periodicity. During the first rounds, their gaming actions started from an empirical, intuitive level, as their choices of formulas of trigonometric functions and x input values were random and instinctive. Both groups *discriminated* and *used* properties on the set of each function output values, regarding maximum and minimum values and symmetry. For example, at the 4th round, Lukas (player A of G2) chose the function $f(x)=50\sin(x)$ for the control of the bird's motion. He disappointingly realized that the bird did not go far enough in order to reach the upper finish-line and get +1 point, even though the x slider moved from 0 to 360.

Researcher: What do you think happened and you got no points?

Lukas: It must have been because of the front number. It is only 50, while the other choices are 100. Same thing happened with number 2. It only went halfway. But when I chose number 6 before, I made it. It's ok, I won't choose it again.

Kostas: It goes halfway to the bottom line too. (...) It goes down as much as it goes up, so yes. Just erase these two first options from here, they will not give us any points.

Lukas: Should we do the same for the ball? We could erase the first two choices, in order to stay closer to the center and have more chances of hitting their bird.

Kostas: Yes, if the front number is 100, the ball will go too far on both sides and the bird will have much time to escape.

Kostas *used* the property of symmetry around the x-axis for trigonometric functions of the forms $asin(\beta x)$ and $acos(\beta x)$ in order to adjust his game strategy. G2 also empirically *discriminated* the role of the parameter α , as only affecting the extreme points of the functions and *used* it for fixing strategies for the main challenge. They further *discriminated*, *synthesized* and *generalized* these properties during the 2nd phase, after making transitions among MaLT2 and GeoGebra representations and the gaming context. During the first rounds of the 2nd phase, both groups tended to use the plotting tool in GeoGebra either for understanding the reason of failure of their strategies or for formulating new ones by predicting the bird or ball's motion. For instance, in their first round, G2 (player B) adopted the aforementioned strategy by choosing the function $g(x)=50\sin x$ for the ball's motion, aiming at making it more difficult for G1 to avoid it.

Lukas: It did not work. Even though the ball did not go as far as before, it reached the centre at the same time. When $x=180$.

Kostas: I don't get it. If it goes further, how does it get to the centre at the same time?

Lukas: The speed doesn't have to do with this number 50. Go put it in GeoGebra. (...) We can compare the two graphs if we design both of them.

Kostas created graphs in GeoGebra for the functions $f(x)=100\sin(x)$ and $g(x)=50\sin(x)$ (Figure 5.1.). They compared them and *discriminated* properties for both the output values and their period:

Lukas: See, it's just the height that changes. The $f(x)$ goes up to 100 and to -100, while $g(x)$ to 50 and to -50. It is exactly this front number! $\sin(x)$ and $\cos(x)$ go from -1 to 1 and the front number is multiplied. But they (the graphs) go through x -axis at the same point. So, it doesn't matter. Try out $\sin(3x)$ and $\sin(2x)$ (Figure 5.2.).

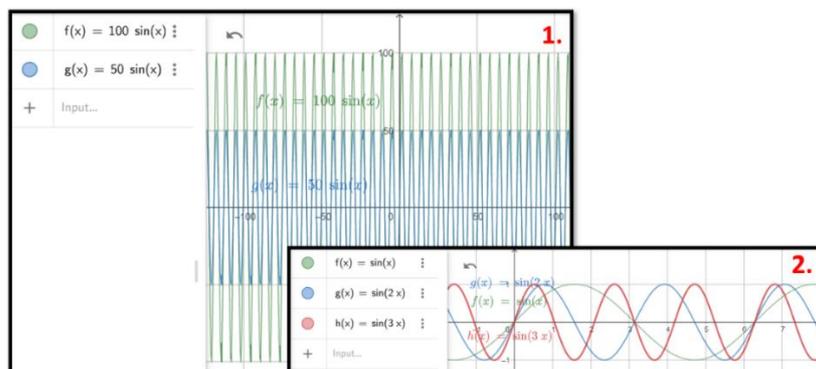


Figure 5: Two screenshots from G2's exploration on graphs in GeoGebra

Lukas *used* the graphs represented in GeoGebra and *synthesized* his meanings on output values. He transitioned from the graphical representation in GeoGebra to the dynamic one in MaLT2 and to the gaming context and vice versa. He *generalized* the case of extreme points of the functions $a\sin(x)$ to be a and $-a$ and from that point on, G2 *used* it to its strategies. Their next exploration involved meanings on the period of the sine function, in a quest to find a better strategy for making the ball “faster”. After creating and comparing the graphs of the functions $\sin(x)$, $\sin(2x)$ and $\sin(3x)$, G2 *discriminated* the role of β in the function of the form $a\sin(\beta x)$. G2 then *generalized* meanings on its period and made the same type of transitions as before. They *used* these *generalized* meanings for strengthening their strategy of ball movement to hit the opponent bird in an offensive-play context.

Lukas: Wow, look how the red goes up and down. (...) It is much faster, it reaches the x -axis more times than the others. When green cuts it three times, red cuts it seven.
 Kostas: Yes, so choose $30\sin(3x)$ for the ball. The bigger this number is, the faster it'll go.

G1 generated and evolved meanings on the period of trigonometric functions by developing a different game strategy, based on a defensive rationale. The following dialog took place during the 3rd round of the 1st phase, when Anna chose the function 4. $f(x)=100\cos(x)$ for the bird's movement.

Researcher: Why did you choose number 4?
 Anna: Well, just because in the previous round the ball was again at 1, so it was $50\sin(x)$, and I selected 5 for the bird, which was $100\sin(2x)$, and the ball hit me! So now that the ball is again at 1, I'm thinking of trying the cosine. I chose $100\cos(x)$.
 Researcher: What makes you think that cosine would save you?
 Anna: Just a feeling. Before, at Nikos's turn, the ball was moving with number 2, so $g(x)$ was $100\cos(x)$, and he chose number 3 for the bird, so $f(x)$ was $100\sin(x)$. He had better luck, maybe because one was moving with sine and the other with cosine.

Anna's answers indicate that she intuitively *used* the relationship of phase difference between sine and cosine in order for the bird to avoid touching the red ball, whose movement was set by player A to the function $g(x)=50\sin(x)$. She expressed that it was an intuitive decision based on her observations on her opponent's gameplay, formulated through transitioning among the

representations in MaLT2 and the gaming context. While playing that turn, she managed to avoid the ball, but sent the bird to player A's finish-line – so she gave +1 point to Nikos. Then she reflected on her move and gradually *discriminated* this mathematical relation, as designated by her next sayings:

- Anna: Yes, now I think I get it. See, when the bird goes at the highest point, the ball is exactly in the middle, and vice versa. It's like they move with some delay. (...) If I move x here we would see... it is 90 exactly. When $x=0$, the bird is at the starting middle point and the ball is at its farthest point. When $x=90$, then the ball is at that middle point. Is it because of the 90 degrees? A right angle?
- Nikos: Yes, I think it does. You can think of the circle. At 90 degrees one is 0 and the other is 1. They never become 0 at the same time. We can avoid all the balls!
- Anna: Cosine of 90 degrees is 0. I remember it because I had it wrong at the test.

G1 *synthesized* the relation between the functions $\sin(x)$ and $\cos(x)$ within the algebraic context of the trigonometric circle with which they were familiar. At the 2nd phase, they *discriminated* the role of the parameter β in the functions $\sin(\beta x)$ and $\cos(\beta x)$, regarding their period and affecting their phase difference. When it was G1's turn, in the 3rd round of the 'main challenge', they *used* the above relation in order to prevent the bird from touching the ball. G2 had set the ball's movement to $g(x)=50\sin(2x)$ and left the x slider to $x=0$. Nikos confidently noted down ' $f(x)=100\cos(x)$ ' for the bird's movement and ' $x=100$ ' at the scoreboard. When Anna *used* the sliders in MaLT2 to implement these moves, she immediately gained +1 point since the bird's initial position was at the upper finish-line, as $f(0)=100$, but then lost it as the ball hit the bird at $x=90$, when $f(90)=g(90)=0$. This unexpected result led G1 to further investigation using plotting tools in GeoGebra. At the first rounds, both groups used GeoGebra to plot the graph of each function for the bird's motion together with the one (chosen by the opponent team) for the ball's motion. They intentionally avoided the two graphs crossing the x -axis at the same point, as they *synthesized* this case in GeoGebra to be a meeting point for the ball and the bird in MaLT2. They further *generalized* meanings on each function's period and *used* them to strengthen their strategy. During the last three rounds, both groups had optimised their strategies based on their *generalised* meanings, without turning to GeoGebra for confirmation.

Conclusions

Students from both groups encountered a similar flow of transitions among the phases of the UDGS model, mapped to the development of different types of strategies, either defensive or offensive. However, all students' gameplay started from an intuitive, empirical mode and gradually turned to a reflective, intentional and attentive one. In the 2nd phase, they increased their transitions among different representations of MaLT2, GeoGebra and the game context, developed stronger strategies and *generalized* concepts and relations in *use*. *Synthesizing* between the graph representations in GeoGebra and the dynamic representations of the bird or the ball's movement in MaLT2 played a decisive role for the evolution of their strategies. They gradually transitioned from *using* the context of the graph to *discriminating* and *generalizing* its practical aspect in the game. Apart from meanings regarding symmetry, periodicity, input and output values of trigonometric functions presented above, they also developed meanings on their intervals of monotonicity, the function of tangent and the relation between degrees and radians. This educational setting that framed the yoyo-bird game provided rich opportunities for authentic mathematical activity through argumentation, problem posing and conjecture development and testing, where mathematical meanings naturally evolved.

Acknowledgements

This study is co-funded by the Erasmus+ project ‘GAMMA’ (Game-based learning in mathematics) (project no: 2020-1-HR01-KA201-077794) and the HORIZON-WIDERA project of the European Research Executive Agency (REA) ‘TransEET’ (Transforming Education with Emerging Technologies) (project no: 101078875). The European Union’s support for the production of this paper does not constitute an endorsement of the contents, which reflect the views only of the authors.

References

- Byun, J., & Joung, E. (2018). Digital game-based learning for K–12 mathematics education: A meta-analysis. *School Science and Mathematics*, 118(3-4), 113–126. <https://doi.org/10.1111/ssm.12271>
- Grizioti, M., & Kynigos, C. (2021). Code the mime: A 3D programmable charades game for computational thinking in MaLT2. *British Journal of Educational Technology*, 52(3), 1004–1023. <https://doi.org/10.1111/bjet.13085>
- Hoyles, C., & Noss, R. (1987). Seeing what matters: Developing an understanding of the concept of parallelogram through a logo microworld. In J. C. Bergeron (Ed.), *Proceedings of the 11th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 2, pp. 17–24). IGPME.
- Kafai, Y. B., & Burke, Q. (2015). Constructionist gaming: Understanding the benefits of making games for learning. *Educational Psychologist*, 50(4), 313–334. <https://doi.org/10.1080/00461520.2015.1124022>
- Kaput, J. J. (1994). The representational roles of technology in connecting mathematics with authentic experience. In R. Biehler, R. W. Scholz, R. Strässer, & B. Winkelmann (Eds.), *Didactics of mathematics as a scientific discipline* (pp. 379–397). Kluwer.
- Kynigos, C., & Karavakou, M. (2022). Coding dancing figural animations: mathematical meaning-making through transitions within and beyond a digital resource. *Digital Experiences in Mathematics Education*, 9, 1–32. <https://doi.org/10.1007/s40751-022-00118-x>
- Kynigos, C. (2019). Adaptive Learning in Mathematics: Situating Multi Smart Øving in the Landscape of Digital Technologies for Mathematics Education. *SLATE Research Report 2019-3*, Centre for the Science of Learning & Technology (SLATE).
- Papert, S. (1980). *Mindstorms. Children, computers and powerful ideas*. Basic books.
- Puentedura, R. (2006). Transformation, Technology, and Education. Presentation given on August 18, 2006 as part of the *Strengthening Your District Through Technology* workshops, coordinated by the Maine School Superintendents Association. <http://hippasus.com/resources/tte/>
- Roschelle, J., Kaput, J. J., & Stroup, W. (2012). SimCalc: Accelerating students’ engagement with the mathematics of change. In M. J. Jacobsen & R. B. Kozma, *Innovations in science and mathematics education* (pp. 60–88). Erlbaum.
- Tokac, U., Novak, E., & Thompson, C. G. (2019). Effects of game-based learning on students’ mathematics achievement: A meta-analysis. *Journal of Computer Assisted Learning*, 35(3), 407–420. <https://doi.org/10.1111/jcal.12347>