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# Follow the actors – Mathematical learning in digital settings

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*Mathematical learning and understanding at primary level are closely connected to handling and exploring manipulatives or other didactic materials. Recently, digital tools have found their way into mathematic classrooms alongside traditional analogue didactic material. In this paper, we focus on learning processes in digital settings with tablet apps. We explore the question: What role does the tablet play in the process of negotiating and understanding? We take a sociological perspective and accept not only humans but also objects as actors. Working in the interactionist paradigm, we follow the actors turn-by-turn in order to reconstruct specific learning conditions.*

*Keywords: Interaction analysis, mathematics education, objects, tablet computers.*

## Introduction and focus of interest

Mathematical ideas and concepts are abstract in nature and accessible to the senses only to a limited extent. Consequently, an understanding of mathematical objects and relations is possible only indirectly. Mental ideas are developed in processes of interaction with different concrete representations. The challenge of mathematical learning is to discover similarities in all sorts of situations. With reference to Skemp (1986), we understand this process of identifying a common ground as an interactive process of abstracting. Accordingly, we cannot even think of mathematical learning without active operating and working with all sorts of things. Blackboard and whiteboard do play an important role in mathematical learning processes as well as cubes, rods or rulers. However, in the context of the pandemic digitalisation was pushed and the portfolio of common materials in mathematics education has expanded. Today, many children work with digital media in mathematic classrooms. Among others, tablet apps are used in the context of mathematical learning situations. Working with applets may well train calculating skills and abilities. But, together with many others (e.g. Scharlau & Walter, 2023), we are rather interested in aspects that enable mathematical understanding and process-related competencies like communication.

In this paper, we focus on learning processes in digital settings with tablet apps and reconstruct specific interactional structures and learning conditions. As a starting point, we concentrate on the app “Klötzchen” (“blocks” or “cubes”) by Etzold (2020), which is well-known in Germany. From a didactic point of view, it is basically a suitable app for mathematical understanding (Bönig & Thöne, 2018). Specifically, we analyse interaction processes that emerged during the work of two fourth graders with the exemplary applet “Klötzchen”. We explore the questions: How does the mathematical interactional process emerge? What role does the tablet play in the process of negotiating and understanding? In doing so, we adopt a consistently interactionist perspective by also accepting objects as actors, drawing on Latour (2005). This approach is rather new in German research on mathematics education but increasingly discussed internationally (de Freitas & Sinclair, 2014). However, many sociologists share an understanding that takes objects as participants in the emergence of social reality (Schildermans et. al., 2023).

In the following, we first present our theoretical framework and the methodological approach. Afterwards, we outline the digitally based setting that serves as the basis of our empirical study. We then present excerpts of the empirical analyses of interaction and empirical findings and conclude with a discussion and an outlook.

### **Theoretical framework: Sociology of objects**

In our research, we refer to the perspective of a sociology of objects as the theoretical framework. This implies two specific theoretical aspects: On the one hand, we understand mathematical teaching and learning as social processes. On the other hand, we accept not only humans but also objects as actors that contribute to the interactional process of learning (see also de Freitas & Sinclair, 2014; Schildermans et. al., 2023). In the following, we explain our theoretical framework.

Mathematical learning takes place in mathematics classes. We understand these mathematical learning situations as social processes in which learners and teachers interact (see also Cobb & Bauersfeld, 1995; Krummheuer & Fetzer, 2005). Referring to Latour (2005) and his actor-network-theory, we go beyond the traditional boundaries of interaction. We extend the understanding of interaction and consider not only humans as actors in those processes. Instead, we also accept objects as actors that are involved in the creation of social classroom reality. “Objects too have agency” (Latour, 2005, p. 63) and contribute to the progress of interaction in which learning takes place. Following Latour and the perspective of the sociology of objects, all kinds of actors, human or not, are participants in the process of negotiating and learning mathematics. This theoretical framework proves empirically true (Fetzer, 2022; Fetzer & Tiedemann, 2018). Tracing associations and processes of networking reveals: Object-actors can contribute on a structural level to processes of learning in (at least) two ways (Fetzer, 2022; Goffman, 1981). On the one hand, object actors can act in assemblance with human actors. Then both actors perform in a “collective action” (Latour, 2005, p. 74) and fulfil an interactive turn together. Their action can only be understood in the assemblance. On the other hand, object actors may fulfil turns on their own and thus act in a solo-action (Fetzer, 2022).

Referring to human and non-human actors and their actions, Latour points out that the “modes of action” (2005, p. 74) differ. These different modes of action appear to influence learning conditions dependent on two aspects:

- Structure of participation: For analogue object actors different conditions for mathematical learning could be reconstructed depending on the way objects are structurally involved. Collective actions provide other learning conditions than solo-actions (for more detailed information see Fetzer, 2022).
- Digital and analogue object actors: Digital object actors contribute in a different way than analogue object actors (Fetzer, 2022). As a consequence, we distinguish analogue object actors (AOA) like traditional manipulatives from digital object actors (DOA) like tablets when we try to identify certain learning conditions.

## **Methods**

Our videos are transcribed. The analyses of the transcripts are conducted in a reconstructive manner applying analyses of interaction (Cobb & Bauersfeld, 1995). This method refers to the interactional theory of learning and is based on the ethnomethodological conversation analysis (Sacks, 1996). This micro-analysis is a method to trace the social, to focus the thematic development and thus to reconstruct the emergence of mathematical learning processes. First, the individual actions and utterances of the actors are analysed extensively in sequence. This includes all modes of actions. We ask: How can a certain human action be understood? How can the contribution of an object actor be interpreted? In a second step, the individual utterances and actions are analysed turn-by-turn, actions are considered as turns on previous actions. This is the particular analytical step of tracing associations and processes of networking on a micro-level. It helps to “follow the actors” (Latour, 2005, p. 156) and to capture their contribution to the interactional process on the basis of empirical analyses. In this way processes of networking and interacting can be reconstructed. Eventually, a summarizing interpretation is carried out. For details on empirical analyses in the empirical framework of a sociology of objects compare Fetzer (2015).

### **Empirical digitally based setting**

For our survey we exemplarily chose the “Klötzchen” app, which involves creating cube buildings that are shown simultaneously in two different representations: A 3D image of the cube building on a grid that can be rotated in any direction, and a building plan showing the number of cubes on the specific square of the grid. An additional feature is the possibility to turn on one or two side view(s) of the 3D building in the form of a shadow on a wall. Based on this feature, we asked the students how many cubes can be removed from certain buildings without changing their shadow(s).

The survey took place with two fourth graders from an elementary school near Frankfurt am Main, Germany. To introduce the mathematical content and to activate the students' prior knowledge about cube buildings, free building with wooden cubes on grid paper that exactly matches the wooden cubes took place first. The two students then replicated building plans and finally had a brief conversation about the side view of (wooden) cube buildings with the teacher. Following on from this, the app was introduced with the task described: How many cubes can be removed without changing the building's shadow(s)?

Without a time limit, the two students were able to immerse themselves in the app and the mathematical process depending on their contributions. The intended role of the teacher present was to stay in the background as much as possible, observing the students' interaction and asking impulse questions to keep their thought process, conversation and interaction going. The situation was filmed from two angles and the screen of the tablet the students were working with together was recorded.

### **Analyses of interaction and empirical findings**

As a starting point, we gathered first impressions on the empirical data. The entire working process of the two children is characterised by an extremely high mathematical density. They exchange mathematical discoveries and negotiate mathematical content with great intensity.

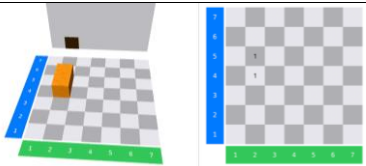
Analysing the data, a striking regularity in the interactional structure could be reconstructed.

1. First, a collective action is carried out by one of the children and the tablet.  
*(1) collective action between digital object actor (DOA) & child*
2. Afterwards, we find a solo-action of the DOA.  
*(2) solo-action digital object actor (DOA)*
3. This is followed by a verbal recontextualization of the object offer by one of the children.  
*(3) solo verbal action child*
4. This repeated process ends in a pause.  
*(4) pause*

The summarizing interpretation of a first sequence may illustrate this recurring structure that dominates the entire working process. In transcript 1 the children try to answer the question of why only the shadow of one block can be seen on the tablet, even though two blocks have been placed.

Some remarks on the transcripts: B and R are the students, T is the tablet, K are the wooden blocks. Regular font is spoken language, stressed verbal expressions are in **bold** and actions are expressed in *italics*. 3D is the left side of the tablet (see Table 1, <3>) showing the blocks in 3D, building plan is the right side of the tablet (see Table 1, <3>).

**Table 1: Transcript 1<sup>1</sup> – Two blocks behind each other**



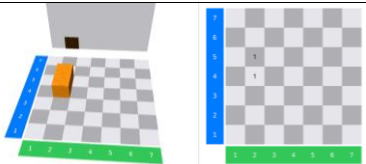
1	B	Because	
2	B, T	<i>Points two fingers at 3D and looks at it</i> That	(1) collective action between digital object actor (DOA) & child
3	T		(2) solo-action digital object actor (DOA)
4	B	<i>Takes hand under the table</i> Is <b>behind</b> each other	(3) solo verbal action child
5	R	<i>Looks B in the face</i>	(4) pause

“Because” <1> indicates that an explanation will follow. B points with two fingers to the two blocks that can be seen in the 3D image <2–3>. In doing so, he establishes a connection between an analogue world in which it is possible to physically touch two blocks and the digital representation. He transfers the two-dimensional digital representation into a spatial representation by using two fingers to illustrate and indicate the two blocks lying behind each other. Here, the boy and the tablet act in tandem. You get the impression that B supports the reading of the tablet as a spatial representation in the plane. The collective action serves as the enacted establishment of a link between the analogue world with its real-world experiences and the digital plane representation. It demands an interpretation of both worlds (collective action DOA-B). In <4> B takes his hands out of the scenery and thus enables the interactive focus on the tablet. In a solo-action it may now unfold the

<sup>1</sup> Transcript 1 was translated from German into English by the authors.

representation and “speak for itself” (solo-action DOA). Finally, B recontextualizes the DOA’s turn through the verbalization “are behind each other” (solo verbal action B). With R looking straight into B’s face one may understand this very moment as a taken as shared interpretation: The DOA tablet shows a spatial situation. Although we cannot touch the blocks, they are represented on the plane of the tablet (pause). Immediately afterwards, the four-step of (1) combined action, (2) solo-action digital object actor, (3) solo verbal action child and (4) a pause is repeated.

**Table 2: Transcript 2<sup>2</sup> – You can also only see one**

6	B, K	And <u>if</u> you now <i>Looks at the wooden blocks</i>		
7	B, T	From the front <i>Bends to the edge of the T as if he were peering over the T exactly in line with the two blocks in 3D</i>		(1) collective action between digital object actor (DOA) & child
8	B, T	Look <b>there</b> <i>Moves 2x with thumb and index finger of the right hand above the T from front to back</i>		
9	R	<i>Looks at the T</i>		
10	T			(2) solo-action digital object actor (DOA)
11	B	<i>Sits up straight again and puts his hand under the table</i>		(3) solo verbal action child
12	B	[Then] You can <b>also</b> only see one		
13	R	<i>Looks at T 3D, nods</i>		(4) pause
14	B, R	<i>Both children look at 3D</i>		

In the collective actions of lines <7–9>, B and the DOA tablet enact and establish the link between real world experiences and plane tablet-representation, again. With B peeping over the edge of the table he invites the observer into a real-world situation. He imitates a real experience that would actually be meaningless in the digital representation. But, in this way a sustainable reading of the tablet is illustrated and established. It becomes physically comprehensible that one could see a shadow of a block from this position. The digital representation is linked to a primary experience and thus to a viable mathematical interpretation. Again, the combined action strengthening the link between analogue and digital situations is followed by a solo action of the tablet <10>. After that a verbal recontextualization is put forward: “you can also only see one” <12>. This four-step sequence again concludes in the mode of a taken as shared meaning and interpretation on the mathematical

<sup>2</sup> Transcript 2 was translated from German into English by the authors.

level: R nods <13>, B has both hands under the table and both children look at the tablet. This very interactional moment of shared meaning emerges and is interactionally realized as a pause.

Scene 2 also illustrates another empirical finding that can be reconstructed again and again in the entire work process: If-then-constructions. The collective action of B and T, which establishes the connection between the analogue and digital world interactively and makes it possible to experience, is framed by an if-then-formulation. If-then-constructions are typical for mathematical learning processes. They describe the systematic nature of changes, mathematical relations and conditions. Thus, if-then-formulations are a central element of mathematical learning, especially in the process of mathematical abstraction. Exploring operational changes and playing with systematic changes is an intense process of learning on relations between numbers, operations and geometric relations. In the given data, such if-then-formulations occur frequently. In most cases, they frame a collective action of tablet and human actor, as in scene 2: B opens up with “And if you” <6>. This is followed by the interactive realisation of the connection between real world experience and digital representation. In <12>, B finishes the sentence with “[Then] you can also only see one”.

The formulation of if-then-relations to describe connections frames a collective action. The verbalization itself is carried out as a solo-action.

In some recorded cases the pause in step four appears to be longer than we as observers would expect. However, the interactional process does not stop at these points, but is carried on without intervention of the teacher or any other external input. Different interpretations seem possible: Working their way through exploring the shadow-problem, the children go step by step. Again and again, they seem to reassure each other of a taken as shared understanding of the offer the DOA makes. Once they appear to agree they work their way further through systematic variations: If ... then. At the same time, these pause moments are moments of taking turns with the DOA. The children sit at the table, hands off the table and look closely at the depicted representation of the tablet. In these moments of interaction, they appear to “follow the actor” (Latour, 2005, p. 156) and try to make sense of its interactive offer. However, obviously the DOA *has* something to share and tell on the mathematical level. The working process carries on.

## **Discussion**

Eventually, we sum up our findings on the research questions of this paper. How does the mathematical dense interactional process emerge? What role does the tablet play in the process of negotiating and understanding?

As analyses show, there is a repeated structure in the interactional process. Four steps are repeated frequently. In (1) collective actions, when human and non-human actors act in assemblance, the physical experience and the digital representations are related to each other. The linkage between both contexts is enacted interactively. However, this might be the potential of collective actions of learners and DOA. In assemblance, different contexts of experience may get interactively connected by collective actions. This might be one key to mathematical understanding: The similarities in different contexts, in the analogue und the digital world, have to be identified. This happens in a process of interaction and exchange. With actors contributing collectively, the conditions for the social process of abstracting appear to be good. The learning conditions are favourable. In step (2)

the DOA takes its turn and makes an interactive offer. Thus, it contributes to the emergence of the social learning process. Its offer is recontextualized in a solo verbal action by a learning child (3). This is another moment of intense learning. Mathematical ideas are expressed in different ways; represented by the tablet (DOA) and verbally by the learner. Eventually, this four-step-sequence culminates in a pause (4). This may be understood as a taken as shared moment. All participants agree on the (mathematical) interpretation of the interactive situation.

If-then-formulations are verbal representations of mathematical variation and relations. Those if-then-sentences occur predominantly as solo-actions. The data shows the interactional conditions of emergence of such mathematical dense and demanding verbal utterances: They frame collective actions of human and non-human actors. In this way, the sequential character of systematic variation is enacted and realized on an interactional level and thus gets accessible by senses.

Eventually, analyses give hints at the interactive conditions for emerging pauses that are interpreted as interactive moments of mathematical agreement and reflection. They are situations, in which the digital object actor comes into play in a solo-action. A silent interaction between DOA and participating human actors possibly emerges. If we accept digital objects as actors in interactive learning processes and follow them as contributing actors, their contribution to the emergence of mathematical meaning might strengthen learners' mathematical understanding.

Our next research steps focus on better understanding the interaction structure and emerging learning conditions. Methodologically, we work with comparative analyses that include scenes

- with particularly long acoustic pauses of more than 5 seconds.
- in which AOA (analogue object actors), as opposed to DOA, are involved in the learning process.
- where there are less dense mathematical interactions.

Thus, we follow the actors with a special focus on digital and analogue object actors in order to better understand the sociology of objects in mathematical learning situations. Building on these reconstructive analyses, we attempt to formulate constructive points of contact for mathematical learning. We try to identify interaction conditions that are specific to the emergence of dense mathematical learning situations and to distinguish them from less fruitful interaction conditions for mathematical learning and understanding.

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