



Teachers' challenges in integrating technology in mathematics teaching through the lens of the instrumental distance concept

Mariam Haspekian, Carolyn Kieran

► To cite this version:

Mariam Haspekian, Carolyn Kieran. Teachers' challenges in integrating technology in mathematics teaching through the lens of the instrumental distance concept. Thirteenth Congress of the European Society for Research in Mathematics Education (CERME13), Alfréd Rényi Institute of Mathematics; Eötvös Loránd University of Budapest, Jul 2023, Budapest, Hungary. hal-04410780

HAL Id: hal-04410780

<https://hal.science/hal-04410780>

Submitted on 22 Jan 2024

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

Teachers' challenges in integrating technology in mathematics teaching through the lens of the instrumental distance concept

Mariam Haspekian¹ and Carolyn Kieran²

¹Université Paris Cité, EDA, F-75006 Paris, France; mariam.haspekian@u-paris.fr

²Université du Québec à Montréal, Canada

Addressing the issue of technological integration in mathematics teaching, this contribution presents a theoretical construct, the instrumental distance, that both sheds light on deep obstacles to integration and, consequently, provides analyses that could be useful for improving the integration efforts. After presenting the theoretical frame, the paper gives two examples of its application.

Keywords: Instrumental distance, mathematics teaching practices, technology integration.

Introduction

Research in mathematics education on the use of digital technologies has since the mid-1990s moved its focus from learners to teaching practices (Sinclair et al., 2023). This evolution has been partially driven by the observation that these technologies are not well integrated by teachers, whereas their use was supported by institutions, research and societal developments. The official programmes and prescriptions, combined with the growing accessibility of resources for teachers and the educational potential of technology, as demonstrated by research, make the lack of widespread adoption of these tools in classrooms a puzzling paradox. Researchers have therefore attempted to uncover the reasons and various hypotheses have been proposed with respect to the so-called *teacher barrier* (Ruthven, 2007). Using theoretical concepts derived from research on mathematics education in general or specifically designed to account for the role of technology, multiple resistance factors have been outlined, such as lack of training opportunities, difficulties in accessing materials, lack of time, teachers' beliefs, attitudes or perceptions about technology and their own self-competence/expertise, as well as their pedagogical and didactic knowledge of technology (Hoyle et al., 2020).

Within the hypotheses, several researchers evoke an idea of “distance” using different terms, such as distancing, gap, non-transparency (Bretscher, 2022) or different degrees of fidelity, either relative to the mathematics that technology favours, or that affect teachers' practices with digital technologies. For example, Bretscher (2022) applies Adler's (1999) transparency dilemma to digital tools and stresses the non-neutrality of technology on the mathematical concepts that can be taught. She argues that paper-and-pencil figures are relatively flexible in that they do not need to obey fixed rules regarding the definition and measurement of angles, unlike figures in GeoGebra with their rigid construction rules. She uses Adler's analogy of the window, an object that is both visible and transparent, to define the dilemma of transparency: in order to teach mathematical knowledge, technology must be “simultaneously both visible, so that it can be noticed and used in the practice of teaching school mathematics, and invisible (...), so that attention is focused on the subject matter (...) allowing teachers to make mathematical connections across technological contexts” (*ibid.*, p. 7). In her case study, the non-transparency of GeoGebra leads the teacher to reject or strongly restrict its use. Kasten and Sinclair (2009), for their part, highlight a notion of *fidelity*. In their study on teachers' use of dynamic geometry with Geometer's Sketchpad, their analysis of the reasons why teachers choose certain technological tasks over others reveals that “the strongest criterion for task selection

was very high textbook fidelity” (p. 133). This idea of matching with curricula is also emphasised by other researchers. For Hoyles et al. (2020), a major challenge in incorporating technology into math education in schools is that numerous teachers fail to perceive the alignment of technology with the prescribed curriculum: “many teachers (...) don’t see how technology matches with official curricula” (*ibid.*, p. 87). Hoyles et al.’s notion of fidelity relates not only to official curricula but also to mathematics itself. Despite the fact that spreadsheets have great potential for teaching algebra, Hoyles et al. (2020, p. 80) use the notion of “mathematical fidelity” to express the idea of distance between spreadsheets and paper-and-pencil environments with respect to some algebraic objects: “the *distance* [our emphasis] between the syntax of Spreadsheet formulas and algebraic syntax may be a hallmark of weak mathematical fidelity; in the former case, formulas allow for representation and manipulation of generalisations, but they cannot be transformed with internal rules from that system of signs; (...) continuity through the different educational levels is absent in the Spreadsheet environment.” Hoyles et al. argue further (p. 81) that “the spreadsheet method for solving word problems (...) is far from the Cartesian method of solving problems (...), finding the didactic connection between the versions of notions and methods used in the digital technologies and the 'paper and pencil' versions used in school mathematics is still a significant challenge for teachers.”

In the early 2000s, also based on a study on the use of spreadsheets in teaching mathematics, a concept of *instrumental distance* (Haspekian, 2005) was developed which we believe effectively captures all these ideas. It precisely addresses the non-transparency of the tool and the lack of mathematical and curricular fidelity by examining the impact of technology on mathematical contents and its consequences for mathematics teachers. To address technological integration issues, this theoretical construct seems a helpful tool to consider with other non-integration factors identified by research. Below, we first present the theoretical frame in which this construct is rooted. We next provide examples of its application to illustrate how it explores the phenomena of non-transparency. Lastly, we discuss the implications that can be driven regarding teaching mathematics with technologies.

The concept of instrumental distance¹ and its theoretical frame

The concept of instrumental distance has developed with the aim of addressing the consequences of these phenomena (non-transparency, “far away from”, mathematical/curricular fidelity, etc.), on the teaching of mathematics with new technologies. It derives from the Instrumental Approach in Didactics, a frame based on Verillon and Rabardel’s (1995) work in cognitive ergonomics.

Instrumental approach in didactics

As a subject repeatedly uses a tool to perform similar tasks, he/she gradually develops schemes of use. According to Rabardel (2002), this process, referred to as *instrumental genesis*, is not neutral with respect to the subject's conceptualisations. This emphasis on instrumental geneses provided new and valuable insights into didactic phenomena identified in mathematics education by researchers such as Artigue, Drijvers, Lagrange and Trouche, who were interested at the time in students' use of symbolic calculators and CAS software. This psycho-ergonomic lens unveiled how the implementation of CAS alters traditional algebraic concepts and moves the techniques away from the conventional institutional techniques. For instance, using a graphing calculator leads to schemes that

¹ This section is based on the habilitation dissertation currently being written by one of the authors: Haspekian (to come)

impact on students' conceptualisations of the tangent line (Guin et al., 2005). As new instrumental techniques and objects emerged, integrating digital technologies in education appeared in a new, more complex light, requiring consideration of both institutional and instrumental contexts.

To address this double complexity, the instrumental approach in didactics (Artigue, 2002; Guin et al., 2005) combines Chevallard's (1992) anthropological theory of didactics with this psycho-ergonomic approach: while the former considers the institutional dimension of education, the latter accounts for the non-neutral role of instruments on subjects' conceptualisations.

Mathematical worlds, norms, and values

In the initial research on spreadsheets, instrumental distance addressed the gaps between the algebraic objects/techniques/representations in new versus traditional paper-and-pencil environments. These gaps pose a challenge for teachers if they are too large. For example, in France, algebra teaching targets traditional paper-and-pencil methods. While the official instruction also includes the use of the spreadsheet, it fails to address the changes brought by spreadsheets, treating their integration as neutral and transparent to mathematical knowledge. This explains the weak integration of the spreadsheet in algebra teaching. Spreadsheets impact both local mathematical knowledge (modifying objects, techniques, representations) and global norms/values of conventional algebra teaching. These norms and values are crucial to understanding teaching practices. For example, Hoyles et al. (2020, p. 82) note from their study of Mexican-British classrooms on the integration of digital technologies in mathematics: "When analysing the data collected in the two countries, differences emerged that could not be explained, other than on account of what was emphasised or valued in the mathematics classroom; that is, it could be said that the differences originated in the school mathematics culture."

Therefore, to theorise the notion of "*distance to*", one must consider a global level of coherent mathematical networks and a contextual dimension specific to each educational institution, with its own norms, values and social practices of reference (Saussez, 2009). To address the fact that technology impacts these networks too, Haspekian (to come) has designated them as "mathematical worlds," using Vergnaud's conceptual field and Chevallard's praxeologies (see below). The concept of instrumental distance (ID) refers then to the differences between the mathematical worlds fostered by two different environments: it is defined as the set of praxeological and conceptual differences that emerge in the instrumental genesis of these two environments. From a teaching perspective, note that among mathematical work environments, the paper-and-pencil holds a unique referential role for teachers and institutions. This referential role is one of the causes of teachers' difficulty in integrating new tools. School knowledge institutionally refers to paper-and-pencil. Therefore, the ID considered in the following examples is that to paper-and-pencil. This does not mean it is the only interesting type of ID: for example, Bakos's (2023) study shows the relevance of exploring the ID between two digital tools. Our hypothesis is that integrating new technology becomes harder as its ID to the referential environment increases. Shortly, we illustrate this issue through the example of algebra that presents different teaching approaches: in each, different mathematical worlds can develop according to the specificities of the institutional context (official prescriptions, cultural pedagogic habits, etc.).

The concept of instrumental distance with respect to the referential environment

Instrumental distance typically manifests itself in three ways:

- 1) through distortion: the device modifies or eliminates typical content (vocabulary, objects, tasks, techniques, representations). For instance, the objects are not represented in the same way. The concept of “geometric figure” in dynamic geometry is a good example (Laborde & Capponi, 1994);
- 2) by bringing in instrumental elements that do not exist in traditional mathematical contexts (have no paper-and-pencil equivalent) but interfere with the conceptualisations of the mathematics at stake. For example, the recopy tool in spreadsheets affects the conceptualisation of the notion of formula (these are syntactically different on each line, but semantically the same through the recopy gesture);
- 3) by introducing mathematical elements that are unusual in the specific institution in question, thereby altering the ecology and usual connections between tasks and techniques.

Consequently, the didactic possibilities introduced by the implementation of a new technological environment affect the usual didactic approaches to school mathematics content for a given institution and offer alternatives to the usual way of teaching this content. The concept of ID highlights this gap. Even for identical mathematical tasks, the use of a new environment reveals different content, different coherencies and, ultimately, a different mathematical world.

Table 1. Studying instrumental distance: comparing the characteristics valued in Environment 2 (studied) with those of the mathematical world of Environment 1 (reference)

Main characteristic of the mathematical worlds	Environment 1	Environment 2
Main Fostered Objects (and associated vocabulary and symbolic representations)		
Pragmatic potential/major field of mathematical problems		
Fostered processes for solving tasks, types of techniques		
Other issues (ex: nature of solutions)		

Analysing ID consists in describing all these differences that may occur. Table 1 gives the main directions of study for assessing the ID induced by a new environment. The mathematical world solicited by an environment can be mainly characterised by: the *objects* involved (are the usual objects modified, especially those emblematic of the world studied? does the new environment introduce new objects? do these have an equivalent in paper-and-pencil and how do they interfere with the usual mathematics at stake?); the *vocabulary* (is it the same? if not, how does it relate to the conventional one?); the *representations and symbolisations* (are they congruent between the two environments?); the *type of tasks* and their solving *techniques* (Chevallard, 2006); and the *field of mathematical problems* (Vergnaud, 1990) that the new environment fosters (does the environment open new problems? For example, dynamic geometry creates new construction problems thanks to macros or to customisable menus); are there *other issues specific* to the mathematical world at stake? (as the nature of solutions in numerical processes compared to algebraic ones). This next section illustrates the operationality of ID with two examples. It draws on a recent work (Haspekian et al., in press) carried out with three colleagues, Paul Drijvers, Kajsa Bråting and Michal Tabach, with whom we have analysed the ID of three tools with respect to different algebraic perspectives.

Some examples

Distance between paper-and-pencil and spreadsheet for generalised arithmetic

The connection between cells through the use of formulas identifies spreadsheets as arithmetico-algebraic tools, leading numerous researchers (see Haspekian, 2005) to recognise their potential for teaching algebra within a generalised-arithmetic approach. Applying the elements listed above, we

examined the ID of the Excel spreadsheet in the context of a generalised-arithmetic approach to algebra. Spreadsheets alter the contents and the problem-solving methods of algebra by creating new objects and modes of action. The algebraic world experienced in spreadsheets compared to paper-and-pencil (variable and formulas arise, unknowns and equations disappear, new instrumental knowledge as the copy functionality comes into use...) is quite different. Table 2 summarises the significant distance between the conventional algebraic world in the generalised-arithmetic approach and the algebraic world of spreadsheets, mainly coming from its design purpose (teaching algebra or not) and from the computational transposition of the mathematics in play.

Table 2: Instrumental Distance between the algebraic frameworks offered by paper-and-pencil versus spreadsheets for a generalised-arithmetic approach

Main characteristics	Paper-and-pencil	Excel
Fostered Objects	Symbolisations; Unknowns; Equations; equal sign indicating a relation of equivalence	symbolisations not necessarily visible; variables (cell/ column/line); formulae (with loss of the syntactic invariance); new meaning of the equal sign (indicating a formula) coexisting with the standard one
Pragmatic potential/ major field of problems	tool for solving word problems (sometimes involving proof)	tool for generalisation or optimisation (problems of generalisation/patterns, of optimisation or model)
Fostered processes for solving tasks	structural transformations that maintain top-down equivalence	No literal solving process, arithmetical process of trial-and-refinement
Nature of solutions	exact	approximate

From an institutional point of view, the changes described in Table 2 have different impacts that contribute to explaining why spreadsheets are poorly used by teachers, all the more in the algebra domain. Whereas the referential paper-and-pencil algebraic world in this approach presents algebra as a tool for solving word problems, including unknowns and equations with well-defined algebraic techniques, leading to exact expressions of solutions, spreadsheets offer an experimental tool for numerical or graphical conjectures, including variables and formulas with numeric trial-and-error techniques, leading to approximate solutions.

Distance between paper-and-pencil and Scratch for Early Algebra

In Early Algebra, developing algebraic thinking can be fostered through particular approaches using, for example, word problems and patterning tasks (see Kieran, 2022). Teachers use these problems specifically to give meaning to the objects that are common to algebra and arithmetic (as the equal sign, which needs a shift from focusing on operational to structural aspects), while patterning tasks nourish the generalisation skills associated with algebra. Focusing on two tasks illustrative of these two directions in the Early Algebra approach, we analysed the ID (summarised in Table 3) between the mathematical world carried by the Scratch software and the referential paper-and-pencil one. Unlike spreadsheets, Scratch is an educational tool, yet it is not designed for algebra teaching. The results indicates that the algebraic world it invites is distant from that of paper-and-pencil and requires important adaptations in order to be used with the same usual Early Algebra goals. The use of Scratch for solving a word problem significantly affects the usual algebraic contents referential for this approach (i.e., the meanings of the equal and operational signs). In addition, new algebraic content involving the concept of variable takes center stage. As for the use of Scratch to solve patterning problems, this brings into play some objects belonging to the field of computer science, such as loops and the diverse interpretations of the concept of variable. As a result, creating a comprehensive

understanding of the concepts shared with algebra is not straightforward, and discovering innovative approaches that utilise computer science knowledge is necessary to foster the ability to generalise.

Table 3. Different algebraic frameworks invited by paper-and-pencil versus Scratch in Early Algebra

Main charact.	Paper-and-pencil	Scratch
Fostered objects	equal sign and operational signs (particularly addition and subtraction signs used in arithmetic); and meaning of equality as a relation rather than a sign indicating a process; unknowns; possible use of letters for unknowns	equal sign used in loops for coding condition; unknowns not apparent, instead: introduction of different types of variables; possible use of letters for variables; introduction of new instrumented objects such as loops and blocks and new notions such as “assignment”
Pragmatic potential/ major field of problems	introducing algebraic thinking as a tool for representing and solving word-problems in relation with arithmetical thinking (procedural/ structural aspects of expressions, etc.);	relation with arithmetic differs from that in paper-and-pencil and is not as obvious as it was in paper-and-pencil with such arithmetic word problems;
	introducing algebraic thinking as a tool for representing and solving; numerical and figural patterning tasks in relation with the idea of generalising	computational aspects rather than structural; pattern generalisation can be related to loops, yet, this may amplify the differences for the notion of variable; developing the idea of generalising must be done by finding new ways
Fostered processes for solving tasks	analysis/synthesis process; applying syntactic rules respectful of algebraic properties of the signs	stepwise problem solving; using an intermediary: the programming of a sprite to make the machine carry out the solution

Discussion

We have highlighted how the notion of instrumental distance captures well the ideas of the lack of mathematical fidelity and connections to math curricula, as well as the non-transparency of the tool. We have also illustrated these ideas with examples drawn from specific digital-tool environments. At first sight, one might think that the instrumental distance as shown in the examples above, and the other phenomena related to distance, as reported in research in general, would be a hindrance to the integration of technology by teachers. As mentioned by Kilhamn et al. (2022, p. 1287) in the context of programming in Scratch with early algebra learners: “Just as important as how a Scratch code can enrich learning is to know also when it is not useful or even runs the risk of complicating things for algebra learners.” However, these researchers also point out how teachers can rethink programming tasks in such a way as to bridge the gap between the way that mathematical objects, such as variables, are thought about in Scratch and how they are treated in non-digital mathematical contexts. More generally, Kilhamn and her colleagues use the notion of didactical transposition (Chevallard, 2006) to discuss the way teachers can explicitly transform the theoretical underpinnings and justifications of the knowing-how in a digital environment so as to induce the intended mathematical knowledge. However, developing such a praxeology is not easy for teachers. As Kilhamn et al. (2022, p. 1287) advocate: “If we want students to develop algebraic thinking, tasks need to be justified by an algebra logos [theoretical underpinnings and justifications].”

In that the mathematics of paper-and-pencil environments is highly valued by various stakeholders, teachers wishing to take advantage of the affordances of digital environments for the learning of mathematics have found that they need to spend instructional time in coordinating the mathematics of such environments with the corresponding paper-and-pencil mathematics. In a paper that drew upon the Task-Technique-Theory frame, derived from the instrumental approach to tool use, Kieran and Drijvers (2006) examined the impact of a combined computer algebra (CAS) and paper-and-

pencil environment on students' learning of algebraic thinking and technique. The task-technique combination in CAS and the same combination in paper-and-pencil inevitably led to conflicts and questioning among the students. Such questioning is not to be avoided, as the discussion of these conflicts can be very productive for the students. As Kieran and Drijvers (2006, p. 236) emphasise about the complications in applying CAS techniques: "our experience suggests that they should be considered occasions for learning rather than as obstacles; however, a precondition for these complications to foster learning is their appropriate management in the classroom by the teacher."

This quotation argues for the positive use of ID for addressing issues of integration of new technologies. The ID analyses of the illustrations above shed light on the obstacles to integration and thus offer support in three areas: ID could help in designing new educational tools for mathematics, in providing training and support for classroom implementation, and in developing suitable resources. By taking into account the insights and awareness that ID analyses raise, educators and designers can improve the effectiveness and practicality of technology integration efforts.

References

- Adler, J. (1999). The dilemma of transparency: Seeing and seeing through talk in the mathematics classroom. *Journal for Research in Mathematics Education*, 30(1), 47–64. <https://doi.org/bbp5m9>
- Artigue, M. (2002). Learning mathematics in a CAS environment: The genesis of a reflection about instrumentation and the dialectics between technical and conceptual work. *International Journal of Computers for Mathematical Learning*, 7(3), 245–274. <https://doi.org/cvt5jg>
- Bakos, S. (2023). Mathematics, TouchTimes and the primary school teacher: Generating opportunities for transitions across and beyond. *Digital Experiences in Mathematics Education* 5–30. <https://doi.org/10.1007/s40751-022-00109-y>
- Bretscher, N. (2022). Teachers' knowledge for supporting transitions between dynamic digital technologies and static environments. In J. Hodgen, E. Geraniou, G. Bolondi, & F. Ferretti (Eds.) *Proceedings of the Twelfth Congress of the European Society for Research in Mathematics Education (CERME12)*. Free University of Bozen-Bolzano and ERME. <https://hal.science/hal-0374748>
- Chevallard, Y. (1992). Fundamental concepts in didactics: Perspectives provided by an anthropological approach. *Research in Didactique of Mathematics, Selected papers*, 131–168.
- Chevallard, Y. (2006). Steps towards a new epistemology in mathematics education. In M. Bosch (Ed.), *Proceedings of CERME4*, (pp. 21–30). FUNDEMI IQS-Universitat Ramon Llull.
- Guin, D., Ruthven, K., & Trouche, L. (Eds.). (2005). *The didactical challenge of symbolic calculators. Turning a computational device into a mathematical instrument*. Springer.
- Haspekian, M. (2005). An "instrumental approach" to study the integration of a computer tool into mathematics teaching: the case of spreadsheets. *International Journal of Computers for Mathematical Learning*, 10(2), 109–141. <https://doi.org/dcgdgs>
- Haspekian, M. (to come). *Genèses, distances et repères : les dynamiques instrumentales pour penser les pratiques enseignantes en mathématiques avec les technologies*. Université Paris Cité.
- Haspekian, M., Kieran, C., Drijvers, P., Bråting, K., & Tabach, M. (in press). Teaching and learning of algebra with digital resources. A question of distance? In B. Pepin, G. Gueudet, & J. Choppin

(Eds.), *The international handbook of digital (curriculum) resources in mathematics education*. Springer.

Hoyles, C., Kieran, C., Rojano, T., Sacristan, A. I., & Trigueros, M. (2020). Reflections on digital technologies in mathematics education across cultures. In A. I. Sacristan, J. C. Cortés-Zavala, & P. M. Ruiz-Arias (Eds.), *Proceedings 42nd Meeting of N. A.* (pp. 69–92). PME-NA.

Kasten, S. E., & Sinclair, N. (2009). Using dynamic geometry software in the mathematics classroom: A study of teachers' choices and rationales. *International Journal for Technology in Mathematics Education*, 16(4), 133–143.

Kieran, C. (2022). The multi-dimensionality of early algebraic thinking: Background, overarching dimensions, and new directions. *ZDM – Mathematics Education*, 54(6), 1131–1150. <https://doi.org/10.1007/s11858-022-01435-6>

Kieran, C., & Drijvers, P. (2006). The co-emergence of machine techniques, paper-and-pencil techniques, and theoretical reflection: A study of CAS use in secondary school algebra. *International Journal of Computers for Mathematical Learning*, 11, 205–263. <https://doi.org/10.1007/s10758-006-0006-7>

Kilhamn, C., Bråting, K., Helenius, O., & Mason, J. (2022). Variables in early algebra: Exploring didactic potentials in programming activities. *ZDM – Mathematics Education*, 54(6), 1273–1288. <https://doi.org/10.1007/s11858-022-01384-0>

Laborde, C., & Capponi, B. (1994). Cabri-géomètre constituant d'un milieu pour l'apprentissage de la notion de figure géométrique [Cabri-geometer as a medium for learning the concept of a geometric figure]. *Recherches en Didactique des Mathématiques*, 14(1–2), 165–210. <https://revue-rdm.com/1994/cabri-geometre-constituant-d-un/>

Rabardel, P. (2002). *People and technology*. Université Paris 8. <https://tinyurl.com/4w42mfak>

Ruthven, K. (2007). Teachers, technologies and the structures of schooling. In D. Pitta-Pantazi & G. Philippou (Eds.), *European Research in Mathematics Education V: Proceedings of the Fifth Congress of the European Society for Research in Mathematics Education (CERME 5)* (pp. 52–67). University of Cyprus and ERME.

Saussez, F. (2009). Entre disciplines scolaires et disciplines universitaires, l'affiliation des enseignants de l'enseignement secondaire supérieur en devenir à des cultures disciplinaires [Between school subjects and university subjects, the affiliation of future teachers in upper secondary education to disciplinary cultures]. In E. Richard (Ed.), *L'université peut-elle vraiment former les enseignants?* [Can universities really train teachers?] (pp. 77–92). De Boeck.

Sinclair, N., Haspekian, M., Robutti, O., & Clark-Wilson, A. (2023). Revisiting theories that frame research on teaching mathematics with digital technology. In A. Clark-Wilson, O. Robutti, & N. Sinclair (Eds.), *The mathematics teacher in the digital era. vol.2* (pp. 391–418). Springer.

Vergnaud, G. (1990). La théorie des champs conceptuels [Conceptual field theory]. *Recherches en Didactique des Mathématiques*, 10(2.3), 133–170. <https://tinyurl.com/2cpkr34d>

Verillon, P., & Rabardel, P. (1995). Cognition and artifacts: A contribution to the study of thought in relation to instrumented activity. *European Journal of Psychology of Education*, 10(1), 77–101. <http://www.jstor.org/stable/23420087>