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Upper secondary school teachers' views on CAS from an epistemic and pragmatic point of view

Ragna í Geil and Ingi Heinesen Højsted

University of the Faroe Islands, Faculty of Education, Tórshavn, the Faroe Islands;

ragna.i.geil@gmail.com

This paper reports on Faroese upper secondary school teachers' perspectives on the benefits and challenges of using Computer Algebra Systems (CAS) in their mathematics classroom. The aim is to investigate to what extent teachers view CAS as a pragmatic calculating tool or as a contribution to mathematical learning, furthermore how they see CAS affecting students' representation competency. Eight mathematics teachers from all five upper secondary schools in the Faroe Islands were interviewed using semi-structured interviews focusing on pragmatic value, epistemic value as well as representation competency. Hereafter, thematic analysis of the interviews was conducted. The analysis suggests that the teachers primarily see CAS as an advanced calculator and to a lesser extent a pedagogical tool supporting the students' mathematical understanding. The visual aspects of CAS are seen as advantageous to support the students' development of representation competency.

Keywords: Teachers' views, CAS, representation competency, epistemic and pragmatic value of techniques.

Introduction and context of the study

During the last three decades, the use of Computer Algebra Systems (CAS) has become commonplace in mathematics education in many countries around the globe. In Denmark, there was a reform in 2005, that introduced CAS in upper secondary mathematics curriculum (Undervisningsministeriet, 2013), and in the subsequent years, CAS usage has steadily become an integrated part of secondary school mathematics teaching (Jankvist & Misfeldt, 2015). This process has not been without difficulties, and the National Mathematics Commission report (Grønbæk et al., 2017) states in a section on CAS usage that "There is agreement in the commission that the way in which CAS has been implemented has had a negative effect on the students' development and possession of basic mathematical skills." (Grønbæk et al., 2017, p. 9).

In the Faroe Islands¹, CAS was introduced in the upper secondary school curriculum in 2013 and has since then steadily become an integrated part of the secondary school praxis (NÁM, 2020a; 2020b; 2020c). There has been an ongoing debate in the mathematics education community on the effects of this transition. However, no evaluation or research study has been conducted into the effects of this CAS implementation.

In the vast literature on the use of CAS in mathematics education, there have been differing foci over the past decades: initially, there was an focus on describing software possibilities, oftentimes in an optimistic manner (e.g. Dreyfus, 1994), followed by focus on characterising students' learning processes, and finally on the role of the teacher in facilitating CAS usage (e.g. Guin et al., 2005,

¹ The Faroe Islands is a self-governing country within the Kingdom of Denmark with a population of approximately 54.000 (Hagstovan, 2023).

Gueudet et al., 2012; Tabach et al., 2013). Some studies describe the beneficial effects that CAS use can provide for mathematical learning, such as providing calculation resources that can support students' systematic explorations or providing different representations of mathematical objects (Pierce et al., 2009). Other studies describe difficulties associated with CAS use, such as loss of arithmetic skills or difficulties with concept formation (Artigue, 2010; Jankvist & Misfeldt, 2015). Some effort has been made to encapsulate teachers' perception of the value of CAS usage. For example, Pierce et al. (2009) found that secondary school teachers perceived CAS to be worth the effort overall, although the teachers problematized technical overhead, initial workload, and indicated that while CAS may benefit high achieving students, it could present an obstacle to low achieving students' learning of mathematics.

In this paper, we treat data from a study (Geil, 2022) focusing on Faroese upper secondary school teachers' views on CAS. We ask the following research question: *What are Faroese upper secondary school mathematics teachers' views on CAS in relation to representation competency and from an epistemic and pragmatic point of view?*

We begin by introducing pertinent theoretical notions in the next section followed by a description of the applied methodology. We then present data in combination with ensuing analysis. Finally, we provide a concluding discussion of the results of the study.

Conceptual framework

Our study is framed in the lens of two theoretical constructs - representation competency from the Danish KOM framework (Niss & Højgaard, 2011, 2019), and epistemic and pragmatic values of techniques, from the instrumental approach (Rabardel & Bourmaud, 2003; Rabardel, 1995), which we will briefly introduce in the following sections.

Epistemic and pragmatic value of techniques

In Rabardel's (1995) instrumental approach, a distinction is made between an artefact and an instrument. An artefact is a physical or non-physical object (e.g., a calculator or language) while an instrument is a subject's cognitive development which allows the subject to engage in goal-directed activity with the artefact (Drijvers & Trouche, 2008).

Rabardel and Bourmaud (2003) suggest that a person's activity is mediated by instruments. This mediation serves a pragmatic purpose, as it helps the person achieve their goal and to produce mathematical results. However, the mediation also serves an epistemic purpose, as it helps the person develop a better understanding of the mathematical content. Artigue (2002) highlights this pragmatic/epistemic distinction in her work on the use of CAS in mathematics education. She identifies both pragmatic and epistemic values in the techniques that students develop when using CAS. In this perspective, a technique is considered to be a way of solving a task, and its pragmatic value is related to its potential to produce a successful outcome, such as how effective or versatile it is in various situations. Meanwhile, the epistemic value of a technique is related to the extent to which it enhances the understanding of the mathematical content being studied.

Representation competency in the KOM framework

The Danish KOM framework (Niss & Højgaard, 2019) describes what it means to have mastery of mathematics across different topics and educational levels, based on mathematical ability rather than just subject matter. More specifically, the framework defines mathematical competency as the ability to be ready to act appropriately in a well-informed manner in situations that involve mathematical challenges. It is thereby not sufficient to possess mathematical knowledge one has to be able to actively use it. The KOM framework includes eight mathematical competencies: mathematical thinking competency; mathematical problem handling competency; mathematical modelling competency; mathematical reasoning competency; mathematical representation competency; mathematical symbols and formalism competency; mathematical communication competency; mathematical aids and tools competency (Niss & Højgaard, 2019, pp. 15), which are interrelated and woven together. In this study, we focus on mathematical representation competency, which involves the ability to work with various representations of mathematical concepts, such as verbal, material, symbolic, tabular, graphic, diagrammatic, or visual. It includes interpreting and translating between different forms of representations, as well as the ability to choose appropriate representations for different mathematical tasks or situations. Understanding the strengths and weaknesses of each form of representation is also an important constituent of representation competency, as different representations may not convey the exact same information about a mathematical concept (Niss & Højgaard, 2019).

Methods

To investigate our research question, we conducted semi structured interviews with eight mathematics teachers in upper secondary schools in the Faroe Islands, which accounts for more than 20% of the total population, consisting of 39 upper secondary school mathematics teachers. A maximal variation strategy was used to make the participation group as representative as possible. We chose teachers that teach predominately at different levels (Faroese upper secondary school has three levels in mathematics, A, B and C), and took into account gender, age, seniority and educational backgrounds.

An interview guide was developed to include an operational part, containing the questions for the interviewees, along with a parallel theoretical part pointing to which part of the theory each question refers to. This enabled us to secure a continuous theoretical anchoring during the interview while giving an inkling to what extent the theory could be operationalized. For example, indicators for pragmatic value could be found in expressions of effectiveness, speed, and correctness of answers, and if interviewees were to mention this it would be wise to ask follow-up questions.

After transcribing the interviews thematic analysis was conducted using Braun and Clarke's (2006) six step description. This includes initial familiarisation with the data, development of introductory codes which are then gathered in headlining themes. These were all done by one and the same person.

In the next section, we present data in the form of examples of teacher utterances in each prevalent category.

Presentation of data and ensuing analysis

First, we focus on the categorization of utterances according to the teachers' perceived pragmatic and epistemic value of CAS in the mathematics classroom. A first point to note is that there were two main programmes mentioned by the teachers, Maple and GeoGebra. Since some teachers use both, their references to CAS sometimes include comparing these two programs.

In table 1 and table 2, below, we have collected some of the excerpts that pertain to teacher utterances that can be characterised as pragmatic and epistemic assignments of value, respectively. In addition, we could characterize further subgroups of utterances, each of which is presented in both tables.

Table 1: Teachers' utterances categorized as pragmatic value, exemplified from each subgroup

Pragmatic value	Subgroups pragmatic value
Teacher (1): [...]a sort of enhanced calculator because that's what it is, it's not more than that, it's a calculator.	Computational features
Teacher (2): [...]they could be using more time on the rest of the set, e.g. a statistics assignment it takes at least 20min if they have to draw [the graph] by hand.	Time-and energy saving
Teacher (1): [...] regression exercises could be an annoyance in the way that if you had numbers from millions, thousand and ten thousand, it was a bit difficult to draw [by hand]. There were often errors in units of the like, which spoiled everything. Teacher (6): [...] back then you used single- and double logarithmic paper to draw on and you could [then] draw the best line possible and that line was not always as precise.	Precision
Teacher (5): [...] what speaks for Maple is that it's a text editor where you can solve the mathematical problems while you write, then you solve a problem and then continue writing.	Documentation

The teachers are quite clear on CAS' pragmatic value. Overall, they regard CAS to be a tool with a strong computing power, which saves time and energy and provides the students with more accurate answers whilst also permitting the students to document their work.

One teacher (1) compared CAS to an enhanced calculator, a practical tool saving students both time and energy by outsourcing long calculations, thereby freeing capacity to use on other tasks. In addition, some teachers find that when students use CAS it reduces the number of errors in calculation and produces more accurate graphs than those drawn by hand. All the while the students' work gets documented with text, calculations and figures all gathered in one sequential document. This suggests

that the teachers seem to assign to CAS' some pragmatic value. In contrast to the pragmatic value, the teachers do not so easily identify the epistemic value of CAS. In fact, many comments that we have categorized as pertaining to epistemic value, are comments that describe how CAS cannot replace mathematical understanding.

Table 2: Teachers' utterances categorized as epistemic value, exemplified from each subgroup

Epistemic value	Subgroups epistemic value
<p>Teacher (1): You can't [do] anything with Maple if you don't know mathematics. It's not like Maple can do anything if you can't. Well, except for template problems.</p>	Prerequisite mathematical understanding
<p>Teacher (2): [...] it [is] important that they've learnt everything by hand.</p> <p>Teacher (3): First paper and pencil then the computer. Not directly to the computer because then it's like: what is this? What is the theory behind this?</p>	Paper and pencil first
<p>Teacher (1): [...] the advantage is that you can draw graphs. When you have an exercise, I always ask them to draw the graph. it's a huge advantage.</p> <p>Teacher (5): Students have better apprehension of differentiating due to the quick access to graphical representations, especially with GeoGebra.</p> <p>Teacher (5): What happens if you change the period? And then you can also have scrollbars inside, right? And then you can adjust the amplitude from 0.1 up to 5, and then you can draw parallels to waves. You can take the period, change it, what happens? It looks just like an accordion. I think it's very powerful visually, to show them, well, what happens when you change things?</p> <p>Teacher (2): Visualisation gives better comprehension of a task</p>	Visualisation
<p>Teacher (4): It's hard to experiment by hand, if you make a little change then you have to do computations for 10 minutes before you see the difference in outcome. If you use CAS it is a split second from change of input to new result.</p>	Repetitions
<p>Teacher (1): CAS becomes a black box if the students do not have a mathematical preconception.</p> <p>Teacher (4): [...]chi2 test, it's so hard that the students don't understand it at all, then you use a black box, CAS, to do the computations and practise to say the right things, but they haven't understood it.</p>	Black box

<p>Teacher (6): There are always students that don't understand, and those students still need to learn to think in a structured manner ... in this case it's good to have a program to help them solve equations and compute and such.</p> <p>Teacher (4): CAS became an extra gear that resulted in more [students] not understanding mathematics because they didn't understand the tool. If you don't know anything, you can't enlarge zero, it just becomes zero... Thus, you need to understand some mathematics before you can use CAS.</p> <p>Teacher (L7): You need to understand the basics and the work from there, then you might understand the mathematics much better.</p> <p>Teacher (L4): The advantage of CAS is that you can progress further. It's a good magnifying glass, or a gear, I use it to gear up the mathematics.</p>	Lever
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The teachers seem to agree that a certain mathematical understanding is required before the students should start using CAS. For building such understanding the teachers prefer using paper and pencil or Dynamic Geometry Environments rather than CAS. Some teachers mention some features of CAS that contribute to mathematical understanding such as easy access to illustration and experimentation. Compared to experimenting by hand the students don't have to do all the calculations, only adjusting input and thereby quickly receive a new output. This outsourcing is, according to some teachers, also the reason why a certain mathematical understanding is required, mainly to reduce the risk of CAS becoming a black box. One teacher points out that although using CAS allows for more advanced mathematics, what is taught still needs to be at a level where the student can grasp it. However, if the student has acquired an adequate understanding of some mathematical content, then CAS can support further and deeper understanding of that content. Here CAS can work as a magnifying glass.

Representation competency

Several teachers believe that visual functions of the digital tools are beneficial for the students' understanding of mathematical relationships. For example, when introducing functions, Teacher (5) believes that Geogebra's affordance of quick access to graphical representations of algebraic expressions is beneficial for students' conceptual understanding. This gives an insight into what a function is, what its graphical representation looks like and can thereby create a connection to its algebraic representation. Another teacher (T5) mentions the advantage of being able to experiment with the sliders in Geogebra in connection with oscillations and periodic functions. We argue that the teachers perceive that these functionalities can support students' development of representation competency.

However, it is primarily Geogebra's drawing function, and not the CAS tool that is used for visualisation. This function allows for manipulation of constructed objects, which according to the teachers helps to strengthen the students' understanding of the connection between algebraic and graphical representation.

Concluding discussion

In this paper we presented a synthesis of Faroese secondary school mathematics teachers' perceptions of the role of CAS in the mathematics classroom. We did so through the frame of representation competency and the notions of epistemic and pragmatic value. Our thematic analysis of interview data suggests that the teachers mainly see CAS as an advanced calculator that is beneficial referring to its pragmatic value. The teachers highlight the possible epistemic value CAS may bring to the mathematics classroom, which may support the development of representation competency further. However, the teachers also point out the necessity of prerequisite mathematical understanding for CAS to be of epistemic value. For example, it is important for students that they already understand mathematical concepts and algorithms that can then be outsourced to CAS. Following this argument, we can surmise that high achieving students in particular may benefit from these functionalities, which is in alignment with the findings of Pierce et al. (2009).

We can conclude that Faroese secondary school teachers primarily view CAS as adding pragmatic rather than epistemic value, while the visual aspects of CAS, particularly the connection between algebraic and graphical representation in systems containing CAS, like GeoGebra, are seen as advantageous to support the students' development of representation competency.

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