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How odd are odds? Students' difficulties in converting relative frequencies into odds

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There are many ways in expressing relative frequencies with numerical values: 1) percentages 25%, 2) decimal fractions 0.25, 3) common fractions 1/4, 4) natural frequencies "1 out of 4", 5) notations like "every fourth", or 6) odds "1 to 3". The latter one plays an important role in communicating changes of risk, e.g., in medical research results in scientific papers. Therefore, we investigate medical students' and—as a comparison group—also law students' abilities in converting different representations of relative frequencies into odds. 52 medical students and 51 law students answered three items, which addressed the conversion of a relative frequency into odds. The low solution rates indicate that odds ratios are probably poorly understood because the underlying concept—namely the odds themselves—are frequently misinterpreted. These findings support previous claims for not reporting odds ratios in scientific papers, because these can be misunderstood.

Keywords: Odds, relative frequencies, odds ratios, conversion.

Introduction

This paper presents a study that investigates how well students can deal with odds (e.g., "the odds of winning the game are 2 to 1"). An understanding of odds can provide an important basis for understanding risk changes, for example. There are different ways of statistically expressing the *reduction (or increase) of risks* (or the *reduction or increase of chances*): For instance, with the help of an absolute risk reduction, a relative risk reduction or with the help of odds ratios (Schechtmann, 2002, Monaghan et al., 2021). Considering the medical situation described in Table 1: The risk of suffering a side effect is $\frac{15}{50} = 30\%$ with drug 1 and $\frac{10}{55} \approx 18\%$ with drug 2.

How much better is drug 2 in comparison with drug 1 (c.f. Batanero et al., 1996)? The **absolute risk reduction** is $30\% - 18\% = 12\%$. The **relative risk reduction** considers the ratio of the absolute risk reduction to the proportion of patients in the control group (drug 1) and is calculated by $\frac{12\%}{30\%} = 40\%$.

Odds ratios are even more complex: At first the odds of a side effect have to be calculated as a part-part relationship (instead of a part-whole relationship, see e.g., Singer & Resnick, 1992): "15 to 35" for drug 1 and "10 to 45" for drug 2. Then the odds ratio yields: $\frac{\frac{10}{45}}{\frac{15}{35}} = \frac{10 \cdot 35}{15 \cdot 45} = \frac{350}{675} = \frac{14}{27} \approx 0.52$. The

odds of suffering a side effect with drug 2 is lower with the factor 0.52 compared to the odds with drug 1.

Table 1: Two different drugs, which lead to different risks in suffering a certain side effect

	Drug 1	Drug 2
Side effect	15	10
No side effect	35	45
Sum	50	55

Odds ratios, and also relative risk reductions are not easy to interpret. Therefore, there are a lot of recommendations to not use odds ratios or relative risk reduction for risk communication (Schechtmann, 2002; Gigerenzer, Wegwarth, & Feufel, 2010, Katz, 2006, Persoskie & Ferrer, 2017). At the same time, however, it can be observed that exactly these difficult-to-understand parameters are frequently communicated, e.g., in medical papers (Schwartz, Woloshin, Dvorin, Welch, 2006; Sedrakyan & Shih, 2007).

In this contribution, the basis for understanding odds ratios is examined—the odds themselves. Therefore, we examine the hypothesis that the underlying odds are not well understood by medical and law students. The group of law students serves as a comparison group.

Theoretical background

Similar to the reduction or increase of risks or chances, there are various numerical options for presenting the risks (or chances), i.e., the relative frequency themselves, all of which are shown in Table 2 (also compare Wiesner & Stegmüller, 2022, Krauss et al., 2020).

Table 2: Different ways of expressing relative frequencies

Way of expressing the relative frequency	Example	Graphical representation of basic mental models						
Percentage	25%							
Decimal fraction	0.25	e.g., place value table <table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <td>O</td> <td>T</td> <td>H</td> </tr> <tr> <td>0</td> <td>2</td> <td>5</td> </tr> </table>	O	T	H	0	2	5
O	T	H						
0	2	5						
Common fraction	1/4							
Natural frequencies	1 out of 4 (or, 1 in 4)							
Notation with “every”	Every fourth							
Odds	1 to 3 (or, 1 : 3)	part-part relationship 						

Alternatively, if you want to communicate that 25% of the people in a study were cured, you could also say the proportion of cured people is 0.25 or 1/4. Furthermore, it would be possible to state, that

on average 1 out of 4 people is cured. In newspaper and media, frequently relative frequencies are communicated with a notation that includes the word “every”. In the case of expressing 25% with this notation, one could say: “Every fourth person is cured”. Another possibility to communicate chances or risks is to use odds: 1 to 3. The odds in favor of an event A are defined as the ratio of the probability of event A and the probability of the complement of A (Fulton et al., 2012), or the number of events in favor of A divided by the number of events in favor of the complement of A. Instead of communicating a part-whole relationship, in the case of odds a part-part relationship is communicated. In common language odds are also known, namely from the phrase: The odds are 50 : 50. The odds are $50 : 50 = \frac{50}{50} = 1$ means, that the probabilities of the event A and the complement of A are equal. If the probability of event A is higher than the probability of the counter event, the odds are greater than 1 and if the probability of event A is lower than the probability of the counter event, the odds are lower than 1. Odds (like 50 : 50) are often used in recipes to clarify the ratio of different ingredients, e.g., of sugar and fruits for making jam.

However, the problem in using odds for describing relative frequencies might be a confusion of probabilities and odds (Fulton et al., 2012). Switching between the different representations of risk shown in Table 2 could be particularly fruitful learning and should also be a focus in mathematics teaching in school, especially considering that some of these representations are often communicated in media but are still neglected in the classroom (Krauss et al., 2020).

At first glance, the different representations in Table 2 seem cognitively easy to understand, but how do students really perform when, for example, their mutual conversion is required? The present paper focusses on this question.

Research interest

The present study is dedicated to the question of how well medical students and law students (as a comparison group) succeed in translating relative frequencies from different forms of representation into odds. Since odds are frequently communicated in medical papers, the hypothesis is, that medical students should perform better in comparison to law students.

Method

Participants

In the experimental group of this study, $N=52$ medical students from three different German universities answered different items regarding conversions into odds. Furthermore $N=51$ law students from three different German universities serve as a comparison group. In sum, 72 of the students were female and 31 of the students were male. Participants age ranged from 18 to 35.

Study design

As part of a larger study (TrainBayes, see http://bayesianreasoning.de/en/br_trainbayes_en.html, compare Büchter et al., 2022) some participants answered one closed and two open questions in which a conversion into odds was requested. The three items can be found in Table 3.

The first item addressed a conversion of the representation with the notation “every” into odds, the second item a conversion of natural frequencies into odds and the third item a conversion from a

common fraction into odds. The first item was a single choice item (true vs. false) with a correspondingly high probability of guessing (50%, or 1:1 if this relative frequency is to be presented in odds). In addition, various covariates were collected (e.g., final grade in mathematics in the secondary school leaving examination or gender).

Table 3: Three items addressing the conversion into odds

Item	Correct solution
Item 1: <i>What does “every fifth” mean? The odds are 1 to 5.</i> <input type="checkbox"/> true <input type="checkbox"/> false	False (correct expression in odds: 1 to 4)
Item 2: <i>Fill in the two missing numbers for the gaps: “4 out of 6” means the odds are “_____ to _____”</i>	4 to 2 (or 2 to 1, etc.)
Item 3: <i>Fill in the missing number for the gap: The relative frequency $\frac{1}{6}$ means “1 to _____”</i>	1 to 5

Results

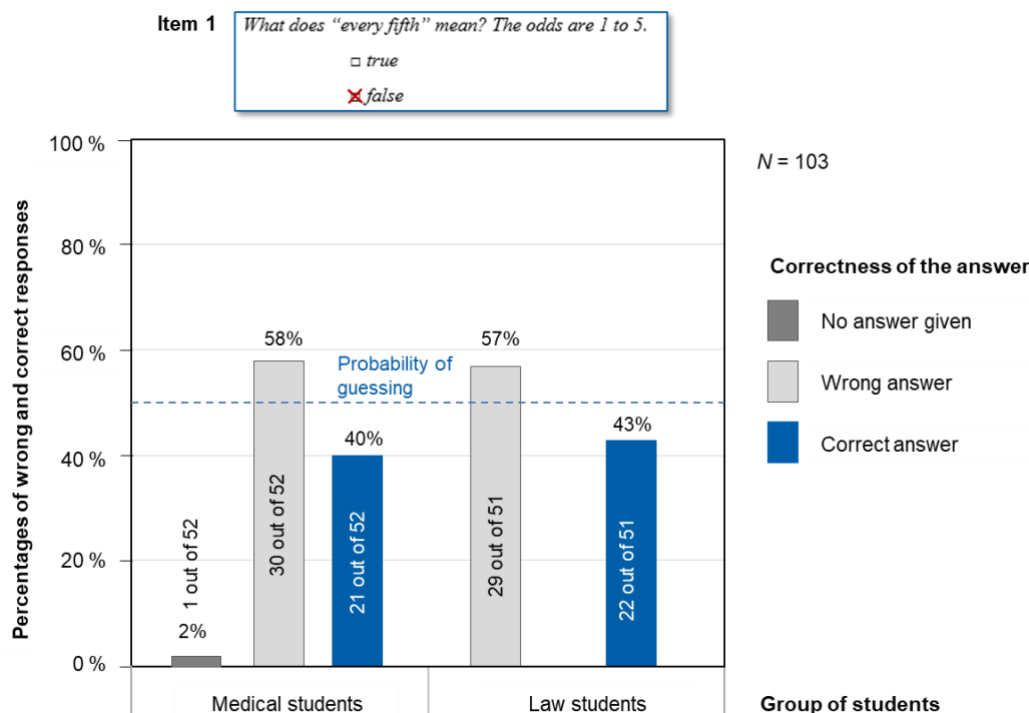


Figure 1: Proportion of correct answers in item 1 (Remark: the probability of guessing is 50%; the solution rate is below this probability)

Participants' performance in item 1 is depicted in Figure 1: Although the probability of guessing is 50% in this single choice task, only 40% of the medical students and 43% of the law students correctly signed "false", because the correct odds corresponding to "every fifth" would be "1 to 4" and not "1 to 5". The medical students did not perform better in item 1 than the law students in our study. More than half of the participants confuses the probability with the odds in this item and assume that "every fifth" should be equal to "1 to 5" (instead of "1 to 4").

Regarding the second item (see Figure 2) it could be stated that medical students performed better than law students in converting natural frequencies into odds. Whereas ten out of 52 medical students correctly filled the gaps with the correct answers "4 to 2" (7 participants) or "2 to 1" (3 participants), only 2 of the 51 law students were able to provide the correct solution.

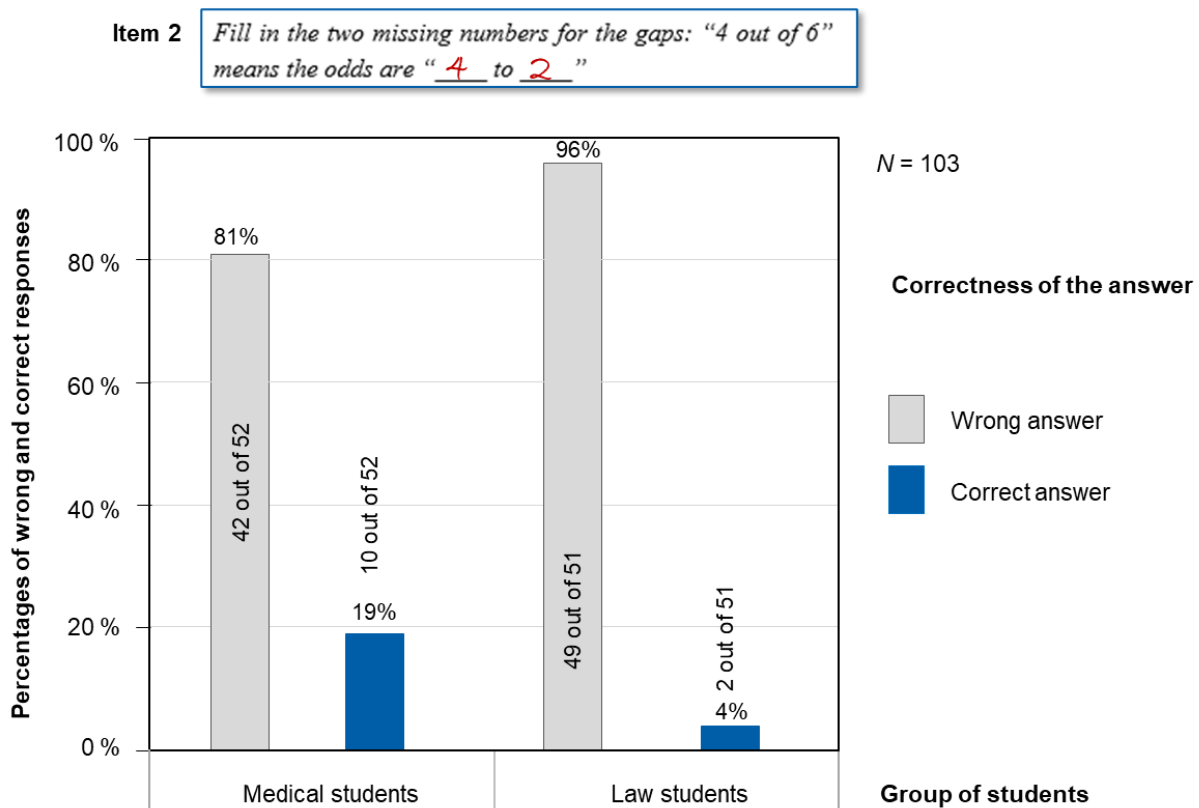


Figure 2: Proportion of correct answers in item 2

Most of the wrong answers in item 2 consisted of simply canceling the given natural frequencies "4 out of 6" into the odds "2 to 3" (29 medical students and 26 law students), which is not correct. Other common responses were also attempts to shorten or expand, like "1 to 1,5" or "8 to 12".

Similarly, in item 3 medical students performed better compared to law students (compare Figure 3). 31% of the medical students, but only 18% of the law students correctly converted the common fraction $\frac{1}{6}$ into the odds "1 to 5". The most common mistake was to confuse the odds with the probability and to not report the ratio from the number of outcomes in favor of A and the number of outcomes not in favor of A, but to report the ratio of the number of outcomes in favor of A and the

total number of outcomes, that means “1 to 6” (32 medical students and 36 law students made this mistake).

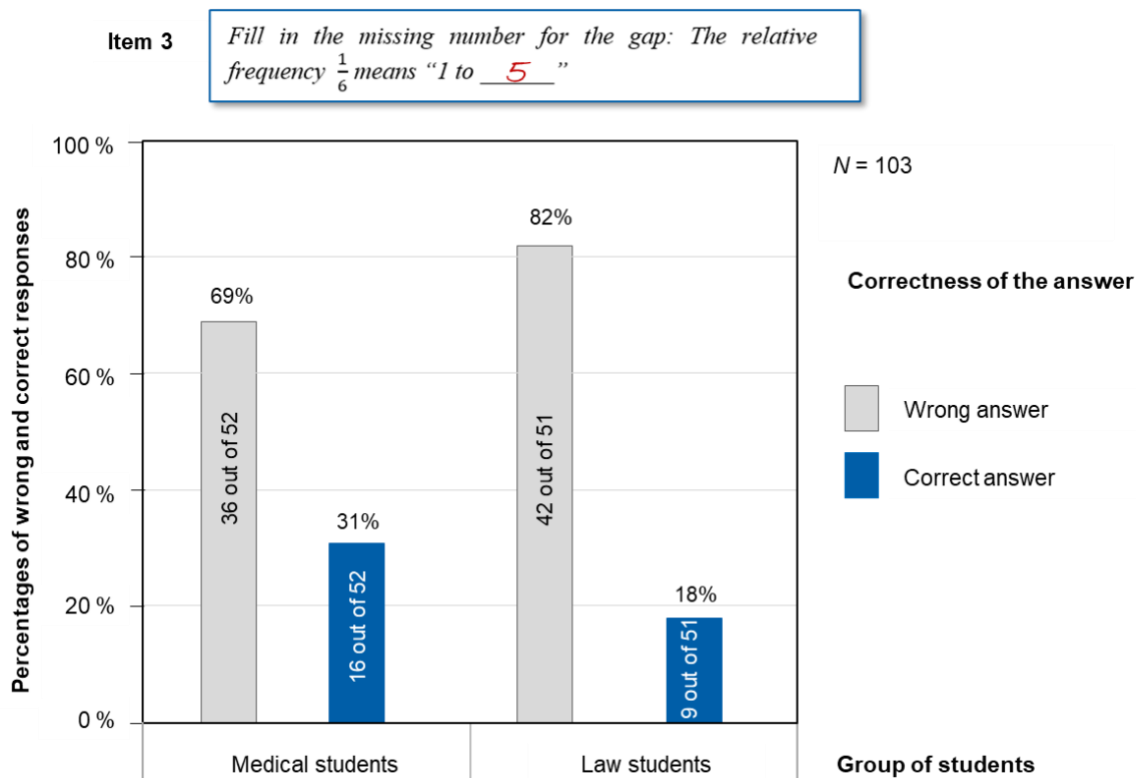


Figure 3: Proportion of correct answers in item 3

Table 4 summarizes the results of the students in all three items. More than half of the students answered all items incorrectly. Only 12% of the medical students and 4% of the law students were able to solve all three items correctly.

Table 4: Proportion of students with no correct answer, one, two or all correct answers

Percentages of students with	Medical students	Law students
no correct answer	52%	57%
one correct answer	17%	25%
two correct answers	19%	14%
all correct	12%	4%

A linear mixed model for predicting the sum of correct answers by subject of the study (law vs. medicine) and the final grade in mathematics in school reveals interesting results, which can be found in Table 5. From an inferential statistical point of view, there is no general effect of the subject or of the final grade. Instead, there is an interaction effect between these two variables: The influence of the final grade in mathematics on participants performance in our items is significant higher for law students compared to medical students. It has to be noted, that final grades in mathematics were significant better in the group of medical students (12,58 points) compared to the law students (10 points).

Table 5: Results from a linear mixed model to predict the sum of correct answers by subject and final grade in mathematics

Results of a linear mixed model to predict the sum of correct answers in the three items	Standardized estimate	Standard Error	t	p-Value
Intercept	0.16	0.13	1.12	0.23
Subject (0: medicine, 1: law)	-0.16	0.11	-1.43	0.16
Final grade in mathematics (higher grades indicate better performances in school)	-0.08	0.13	-0.61	0.54
Subject × Final grade in mathematics	0.33	0.14	2.41	0.02*

Discussion

The present study shows that (medical and law) students have substantial difficulties with the conversion into odds. Descriptively medical students showed better results compared to law students. However, the linear mixed model revealed only a significant interaction effect: For law students (with significant lower mathematics grades) the influence of the final grade in mathematics on the performance in the odds conversion items is higher than for medical students, and in the expected direction: Law students with better grades performed better in the odds conversion items.

In any case, the overall very low performance in the three tasks could explain to some extent why odds ratios are so poorly understood. Of course, to understand odds ratios, one must first understand odds. Otherwise, the misconceptions about the odds are transferred to the calculation of the odds ratios.

A limitation of the current study is that only one direction of conversion (namely the conversion *into* odds) was examined. Of course, the reverse direction of the translation would also be interesting here and we would suspect similar results and misconceptions here.

Fulton et al. (2012) presented five different examples from the media where probabilities and odds were confusing in the communication for the general public. We found similar confusions of probabilities and odds in the current study (see items 2 and 3). The results confirm the demand of many scientists not to use odds ratios in medical journals because they can lead to misunderstandings when read by physicians.

The results of the study suggest that in the classroom, a greater focus should be on odds and that odds should be better differentiated from part-whole relationships. Such a distinction of part-part relationships from part-whole relationships can be made, for example, with the help of 2×2-tables because odds comparisons were identified in Batanero et al. (1996) as an intuitive comparison strategy.

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