A teachers' debate on the design and use of a digital medium for mathematics as a window to their practices

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This paper focuses on a group of Mathematics teachers-colleagues at the same junior high-school, that use a digital medium for teaching probability. They changed it and designed interventions with it; though their discourse, a debate emerged between them. They followed different approaches in the classroom exploitation of this digital medium, they observed the lessons of one another, and they did not share the same view when they reflected on the classroom observations. Seeking a theoretical perspective that was 'sensitive' to the effect of their interaction, we used CHAT (Cultural-Historical Activity Theory) to connect this debate, the diversity of teachers' reflection on design and teaching, and a possible tension between their actions with a description of their practices.

Keywords: Design of digital media, TPD, probability, covariation, CHAT.

Introduction

In this study, which is a part of a broader one, we present our findings around Mathematics' school teachers' practices who adopted and redesigned digital environments for teaching and learning Mathematics, provided by the Ministry of Education in Greece, to exploit them in their teaching in school. These findings became apparent from the analysis of the teachers' interaction and their debate on the possibilities they had using these environments as an offspring of their own designs. One of our study's concerns was to track elements of Mathematics' teachers practices that were related to Teacher Professional Development (TPD). Although the research around the professional development of Mathematics teachers in relation to the use of digital media for teaching offers to researchers a range of perspectives (Clark-Wilson et al., 2022), for the analysis and the reading of the data produced from our intervention we exploited a theoretical point of view not frequently used in this research domain; we adopted cultural-historical activity theory (CHAT) (Engeström, 2001) having in mind to capitalize on the interaction between the elements of the school environment. In this paper, we focus on the diversity around the 'in-classroom' exploitation of a digital medium, by teachers that collaborated to prepare a lesson, seeking for a deeper understanding of their practices, in terms of TPD, through the analysis of their actions and interactions.

Theoretical framework

Based on research results, the study of Mathematics' teachers as learning intervention designers, with the exploitation of digital or non-digital media, has significant contribution in understanding of their practices related to teaching, and their views of the curriculum (Remillard, 2018). Moreover, studies on teachers as designers of digital learning media (collaboratively or on their own), whether they are using them in teaching or not, make the kind of knowledge they enable as they act as professionals, more transparent to the view of the researchers (Laurillard, 2018). In these two aspects of a teacher as designer of learning interventions or media, there is common ground; elements of (TPD) may be

apparent. However, in everyday school practice, teachers may adopt hybrid roles, interacting with the school environment in many ways; as designers (either of lessons or material), as colleagues, as collaborators, as employees, etc. Our purpose was to study these interactions, from a holistic view, searching for elements of TPD. Given that 'in-school' TPD-related activities for teachers include co-design of educational material (Laurillard, 2018), it could be justified to ask if all the 'in-school' interactions and possible diversity in didactical exploitation of the same or similar media -which is something very common- between mathematics' teachers and colleagues, could open us a window to the teachers' professional practices.

In our previous work we have studied a mathematics teacher as co-designer of educational material in collaboration with students (Diamantidis et al., 2019). In that study, the complexity of design as an activity was apparent; the teacher adopted a hybrid role and made decisions related not only to the content, but to social factors as well, like how to make a challenge provocative for the specific students. So, in the present study, our approach was to capture instances of instructional designs and to grasp as much as possible of these complex -social- aspects of teachers' decisions and actions as a unity; apart from their schemes of use for the available resources, we analyzed the results of their interaction with many factors, i.e. their view of the curriculum implementation, the impact of the school environment and rules on their design decisions or their answers to possible exploitation dilemmas, the interaction between them, etc. So, we made use of third generation cultural-historical activity theory (CHAT) (Engeström, 2001), to have a more detailed view of this ecology.

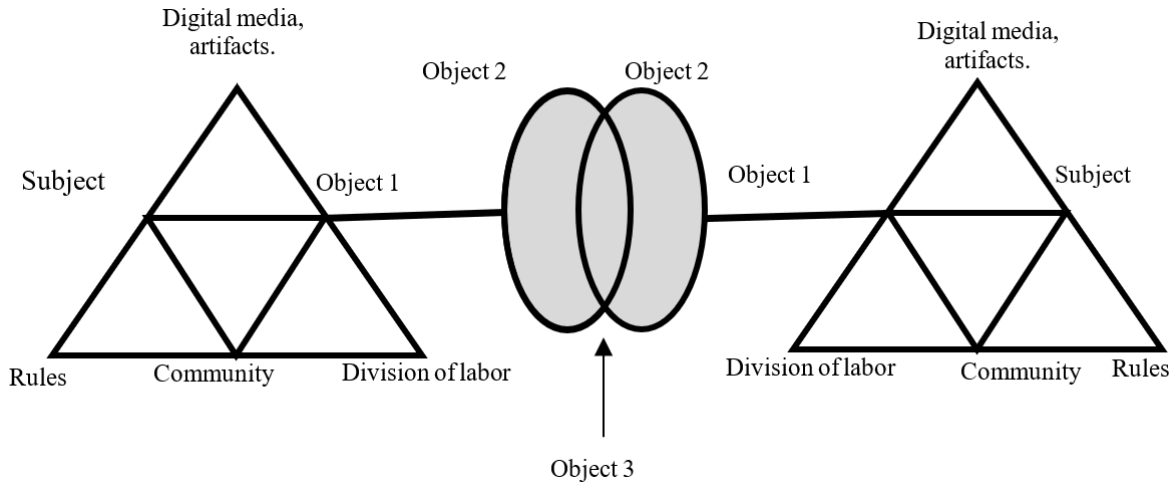

Figure 1: The CHAT 3rd generation activity system.

CHAT in the field of TPD research (figure 1), can be used to interpret teachers' activity having in consideration all the elements that constitute it. In our case the activity is designing interventions with digital media for teaching Mathematics (designing/teaching). The subjects are the teachers, while rules and community refer to the school environment, as well as the division of labor (i.e., the teachers must teach, and the students must do their assignments, etc.). Object 1 is described by the goal of the activity, while object 2 is the result of the activity. If there are different teachers (subjects) engaged in a similar activity (i.e., designing/teaching), then object 2 is not the same for every teacher, since there might be different motives (i.e., different teaching goals), etc. So, object 3 is the shared result, a consensus around objects 2 for each teacher.

From the CHAT point of view, a change in the activity is welcome when tensions in the activity are apparent, that may lead to contradiction. These tensions may emerge in the activity system (the triangle of figure 1), or in the objects 1, 2 and 3 as well. In this study, we were searching for evidence in the activity 'designing/teaching' of mathematics' teachers that, through tensions, may lead to change in their activity and practices in the future, according to the approach of CHAT.

Method

For this study, three Mathematics teachers of a junior high-school in Athens had chosen digital learning objects for Mathematics -'micro-experiments'- from a Greek Ministry infrastructure called 'Photodentro' https://photodentro.edu.gr/lor/. Micro-experiment refers to a digital artefact that simulates the layout of an experiment addressing a challenge and provokes the student to use it and to be involved in inquiry (Kynigos & Grizioti, 2018).

The medium that the teachers of the study used, was a simulator of a fair coin tossing (figure 2). They designed lessons for teaching probability, which in terms of learning outcomes -according to the curriculum- means that students should use theoretical probability to solve problems and to compare it with the results of the corresponding random experiments. This was one of the eight design cycles, that teachers were involved during the broader study. In each cycle teachers chose a micro-experiment from 'Photodentro', and they discussed on how they could use it in teaching. So, they redesigned it, producing one or more different versions and they used these versions to their lessons. In each lesson, one of them had the role of the teacher, while the others participated and observed the lesson, keeping notes, along with another teacher at the same school, who was one of the researchers of the study. After each lesson they made a group discussion to reflect on their observations. For the broader study, we -as researchers- adopted design-based research methods (Bakker, 2018) making modifications after each cycle, while we refined our design principles we started with. In this paper, we refer to the 7th of these cycles. The main design principle in the beginning of this cycle was that 'the interactions between these teachers, while they designed and used digital media for teaching, would help us elaborate our understanding of their professional practices'. So, our question was, which way the study of these interactions and the analysis of the discourse between the teachers, would open us a window to their professional practices?

The corpus of data produced and analysed was voice recordings of teacher's dialogs during the design, recordings from the classroom during teaching, the researcher's field notes and the versions of the digital resources that teachers adopted, redesigned, and used in classroom. We analysed the transcribed dialogs, using 'meaningful segments', as unit of analysis (Strijbos, et al., 2006) and we categorized our findings to describe what we observed (Kelle, 2005) in terms of CHAT. In the results section, we present extracts of our findings, which are indicative of the categories that came up and of the diversity between teachers' approaches on using the micro-experiment (figure 2).

Results

During the 7th iteration of the study, the participant teachers decided to use the micro-experiment that is shown on figure 2. The graph on the upper cartesian plane was a teachers' idea (non part of the original micro-experiment) and represented the trace of a dynamically manipulated point (in black)

that its y-coordinate was equal to the frequency of heads $\left(\frac{number\ of\ heads}{number\ of\ tosses}\right)$, while its x-coordinate is equal to the number of coins tossing. The horizontal line $y = 0.5$ that is also drawn, represented the theoretical value of the "head" event's probability.

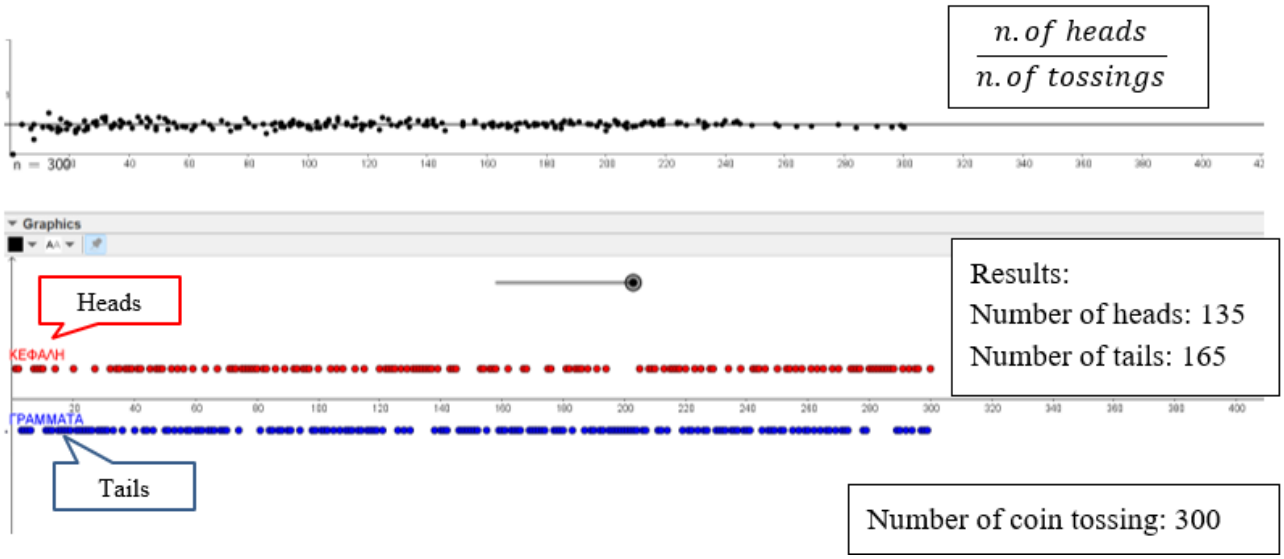

Figure 2: The micro-experiment in its initial version (lower part), and the first teachers' redesign of it, adding the upper's part (black) graph.

A debate around the suggested, by the teaching guidelines, use of the micro-experiment.

The point's position could be manipulated through dragging the slider (in the lower graph) which changed the number of coins tossing. The original version of the micro-experiment did not include the upper graph. It was an element that teachers added to the design of the medium:

096 T1: We should add a point that shows the frequency of heads, so that students, through dragging the slider, can see the variance of the results around the theoretical value of probability.

097 T2: Then, we should represent this value… […]

107 T2: We could use a horizontal line, $y = 0.5$ and then have this trace of the point. Students could realize that there is a natural variability around 0.5.

108 T3: Yes, but this is not the spirit of the teaching guidelines we have! They should learn about probability, so I am thinking of asking them to investigate the relation of frequency with probability. I think I prefer two points. One red for the frequency of heads, and one black for the frequency of tails.

109 T1: Ok, but if we had that straight line representing the theoretical probability of 'heads', students could investigate also what happens when the number of coins tossing increases. Something like the law of large numbers. We could ask them these questions.

110 T3: Too ambitious, I think, but ok, let's try it. However, I would like another version, like the one I said before. I am thinking of asking them to describe the way that these ratios change, as the number of coins tossing increases. I will be very happy if they tell me something about their difference, and how it changes.

This small debate on the micro-experiment's usage and learning exploitation led teachers to make another version of it, based on what T3 suggested about the teaching guidelines (line 108). Instead of the upper graph in figure 2, there was another graph (figure 3). However, both versions were available for them to use.

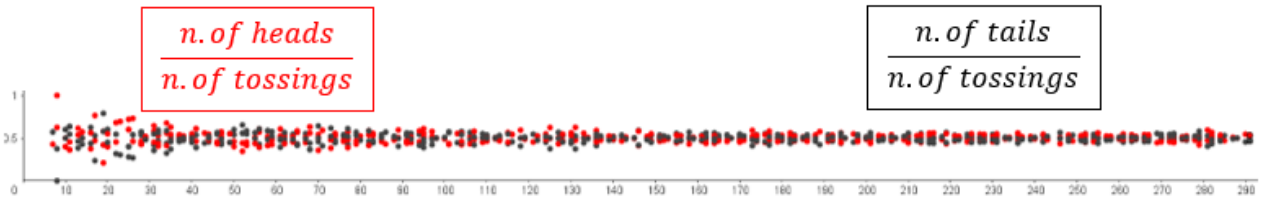

Figure 3: The micro-experiment, and the way teachers redesigned the upper graph, after the suggestion of T3. It has two points, one red for the frequency of heads, and one black for the frequency of tails.

Teachers' diverse understanding of the teaching guidelines became apparent.

T3 used the version with two points, red and black (figure 3), in her lesson, which T1 and T2 participated as participant observers. Students worked in pairs, in the school PC lab. Each pair had access to a PC with the micro-experiment on it. The main challenge that T3 addressed to them was to investigate 'when the two ratios came close to 0.5'. After the lesson a discussion took place around the learning outcomes that students achieved, from their point of view. We refer to students using the capital letters corresponding to the initials of their encrypted names.

351 T1: What about M and J, they seemed to be the ones that made a clear conclusion.
352 T3: Yes, they said that there was a symmetry between red and black point. And that these points came closer to each other and to 0.5 when the number of coins tossing increased.
353 T2: This was a good one. I think that they described the law of large numbers, somehow. And answered fair enough to your question.
354 T3: Or they just realised that the difference of two ratios decrease, as the number of coins tossing increases.
355 T2: So, from this point of view, they almost answered to your task!
356 T3: Almost! At least, they realised that when the number of coins tossing increases, the ratios are mostly near 0.5. But they did not relate this conclusion with the theoretical value of probability.

T1 used the first modification of the micro-experiment (figure 2) for his teaching. The orchestration of the classroom was like the one of T3, while T2 and T3 had the role of participant observers. T1 asked his students to investigate what happened when the number of coins tossing increased. Since the lesson of T1 followed the lesson of T3, the discussion about T1's lesson was somehow comparative.

509 T3: I think that your students made it better. For example, K and I said that when the number of coins tossing increases, the point stays close to the horizontal line, which is the theoretical value of "heads" probability.
510 T1: Yes, that was a good answer. But also, C and D made a very good conclusion.
511 T2: I totally agree! Listen to what they said: "if you move the slider to the right, then you know that there is a big number of coins tossing and a small range of change for the point around the horizontal line. But when you move it to the left, you have less coins tossing and big range of change around this horizontal line". What else do you want them to say?
512 T3: Yes, very good observation, but is it what we want them to realise? It seemed to be more related to the graph, than the concept of probability.
513 T2: It doesn't matter. The slider helped them conceptualize the connection between the number of coins tossing and the variability of the heads' frequency.
514 T1: So, it seems that we could use this micro-experiment in more than teaching probability.

In the extracts above, it seems that the three teachers do not share a common view, neither about the learning goals of probability in 9th grade, nor about the way students addressed the two tasks. In the next paragraph we will try to interpret these findings using the analysis of T1's and T3's designing/teaching activity.

Discussion

On the one hand, we captured two different uses of the micro-experiment for teaching around the concept of probability, corresponding to two different versions of it (figure 2-upper part and figure 3). Through the lens of CHAT, both teachers' activity systems' elements were common. The digital medium was mostly the micro-experiment, or other digital artifacts to simulate coin tossing. The subject was T1 and T3 in each case, while all teachers along with the students belonged to the community. The curriculum could be construed as a text that describes some rules, along with the duration of the school hours, etc. The division of labor corresponds to the role of each teacher in the classroom, i.e., during the first lesson T1 and T2 were participant observers, where T3 was the teacher. The object of each activity is described in the table 1.

Subject	T3	T1
Object 1	To design/teach with the micro-experiment.	
Digital medium	The micro-experiment, other related simulations.	
Community	T1, T2, students.	T2, T3, students.
Rules	The curriculum, teaching guidelines.	
Division of labour	T3 is the teacher, the other teachers are participant observers. Students are addressing a challenge, which should focus on the concept of theoretical probability (her view of the curriculum).	T1 is the teacher, the other teachers are participant observers. Students are addressing a challenge, which focuses on (theoretical) probability, but there are more concepts related, that they are welcome to conceptualize through investigation (his view of the curriculum).
Motive	Students to be engaged in an investigation around the theoretical value of probability.	Students to be engaged in an investigation around the concept of probability.

Table 1: Designing/teaching activity system of T3 and T1

The last row of table 1 also refers to the motive of each activity, which is crucial since an activity can be described by its motive. It seems that T3 designed her teaching intervention having in mind the 'restrictions' of the curriculum and the teaching guidelines, according to her (line 108), while T1 seemed to think that it was legitimate to leave an open door for students to investigate probability in a broader field (lines 096, 109). T3's opinion about students' answers in both lessons (lines 356, 509, 512) showed that she expected them to focus on theoretical probability, while T1's view was more

tolerant -if not receptive- to answers related to preliminary meanings about probability, the law or large numbers, or the natural variability of frequency around the value of theoretical probability.

Moreover, in line 510 T1 said that C and D made a "very good conclusion". These two students coordinated two quantities through the movement of the slider. The first quantity was the number of coins tossing, while the second was the variability ('the range of change') of heads' frequency around the horizontal line. This coordination could be interpreted as a conceptualization of covariation (Thompson & Carlson, 2017); students used the slider as a representation of a multiplicative object which coordinated the two quantities that varied and they referred to the slider as an operative image to represent the covariation between them. Both quantities were coupled by the students and each of their pair of values corresponded to a position of the slider (right or left). T1 seemed to realize and welcome that students' actions and observations extended the possible use of the micro-experiment (line 514).

Trying to describe object 2, for T1 and T3, a diversity became apparent; for T1 it was the designing/teaching for probability and other concepts related to the natural variability and randomness, while for T3 was design/teaching for students to be engaged to the investigation of probability in relation with the frequency of "heads" and "tails". This diversity was crucial for their teaching, as it became apparent through their reflection on it. It might be related to their different approaches to the teaching guidelines (the curriculum), which were evident to us through the categorization of the data and was presented -indicatively- to the results' section. So, from the description of object 2 for each teacher, a tension between their actions was apparent, which, according to CHAT, could be a starting point for the change of their activity as designers and teachers with the use of this digital medium.

Conclusions

In this study, we captured the effect of actions and interaction-in-design between teachers that worked in the same school and collaborated with each other, using CHAT to elaborate our understanding of the diverse approaches around the design and didactical use of a digital medium for Mathematics. We interpreted diverse and somehow conflicting approaches around the design and use of a digital medium, as tensions between diverse professional actions of the teachers. Our better understanding constituted of a) a description of a tension between T1 and T3 teaching actions, as it was evident through their reflection on teaching observation, b) a possible reason of this tension, which was the teachers' different approaches to the teaching guidelines.

Moreover, tensions in teachers' actions, like in this study, may end up to contradictions inside the activity system, according to CHAT (Engeström & Sannino, 2010). A contradiction is the starting point of a novel practice, a new activity. So, in case of teachers' professional actions, these tensions may lead to the change/development in professional level, to a novel practice for these teachers. As a conclusion, we can use (a) to reformulate the design principle of cycle 7th: 'While teachers interact with the motivation to design and use a digital medium for teaching Mathematics, CHAT may help us draw connections between teachers' interactions and actions with their practices'. More research using this theoretical approach would be illuminating on this conclusion.

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References

Bakker, A. (2018). *Design research in education: A practical guide for early career researchers*. Routledge.

Clark-Wilson, A., Robutti, O., & Sinclair, N. (Eds.). (2022). *The mathematics teacher in the digital era: International research on professional learning and practice* (Vol. 16). Springer International Publishing. https://doi.org/10.1007/978-3-031-05254-5

Diamantidis, D., Kynigos, C., & Papadopoulos, I. (2019). The co-design of a c-book by students and teachers as a process of meaning generation. In U. T. Jankvist, M. van den Heuvel-Panhuizen, & M. Veldhuis (Eds.) (2020). *Proceedings of the Eleventh Congress of the European Society for Research in Mathematics Education*. Freudenthal Group & Freudenthal Institute, Utrecht University and ERME.. https://ikee.lib.auth.gr/record/316373

Engeström, Y., & Sannino, A. (2010). Studies of expansive learning: Foundations, findings and future challenges. *Educational Research Review*, *5*(1), 1–24. https://doi.org/10.1016/j.edurev.2009.12.002

Kynigos, C., & Grizioti, M. (2018). Programming approaches to computational thinking: Integrating turtle geometry, dynamic manipulation and 3D space. *Informatics in Education*, *17*(2), 321–340.

Remillard, J. T. (2018). Examining teachers' interactions with curriculum resource to uncover pedagogical design capacity. In L. Fan, C. Qi, S. Rezat, L. Trouche, & J. Visnovska (Eds.), *Research on mathematics textbooks and teachers' resources: Advances and issues* (1st ed. 2018, pp. 69–88). Springer International Publishing. https://doi.org/10.1007/978-3-319-73253-4

Thompson, P. W., & Carlson, M. P. (2017). Variation, covariation and functions: Foundational ways of thinking mathematically. In J. Cai (Ed.), *Compendium for research in mathematics education* (pp. 421–456). National Council of Teachers of Mathematics.