



HAL
open science

A prospective mathematics teacher's instrumental orchestration: Instrumentation of 2D and 3D screens to teach cylinder

Gülay Bozkurt, Melike Yiğit Koyunkaya

► To cite this version:

Gülay Bozkurt, Melike Yiğit Koyunkaya. A prospective mathematics teacher's instrumental orchestration: Instrumentation of 2D and 3D screens to teach cylinder. Thirteenth Congress of the European Society for Research in Mathematics Education (CERME13), Alfréd Rényi Institute of Mathematics; Eötvös Loránd University of Budapest, Jul 2023, Budapest, Hungary. hal-04410674

HAL Id: hal-04410674

<https://hal.science/hal-04410674v1>

Submitted on 22 Jan 2024

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

A prospective mathematics teacher's instrumental orchestration: Instrumentation of 2D and 3D screens to teach cylinder

Gülay Bozkurt¹ and Melike Yiğit Koyunkaya²

¹İzmir Demokrasi University, Faculty of Education, İzmir, Türkiye; gulay.bozkurt@idu.edu.tr

²Dokuz Eylül University, Buca Faculty of Education, İzmir, Türkiye

This case study examines a prospective mathematics teacher's planning and micro-teaching of a technology-based task focusing on the concept of cylinder. This study was designed in the context of a professional development (PD) arrangement organised within a university-based course in its mathematics teacher education programme in Türkiye. Instrumental orchestration framework was used as the theoretical lens of the study. The data included a prospective mathematics teacher's technology-based task plan and video records of her micro-teaching, individual interview, and group discussion. The data analysis showed that coordinated use of 2D and 3D screens has a pivotal role in teaching solids. The findings indicated that prospective teachers' use of planned questions and prompts focusing on the relationships between the objects in 2D and 3D screens are of crucial importance in teaching the concept of cylinder with technology.

Keywords: Prospective mathematics teacher, micro-teaching, cylinder, instrumental orchestration.

Introduction

Teacher education programmes, especially practicum courses, are of crucial importance in developing prospective teachers' professional knowledge of technology integration. In this context, mathematics teacher educators should consider what to do to promote and support professional development of prospective teachers. The work presented in this paper is a follow up work of our previous participatory action research aiming to design a professional development (PD) arrangement for prospective mathematics teachers (PMTs) using the tools of dynamic mathematics software in secondary education (Bozkurt & Yigit Koyunkaya, 2022). The main objective of our previous work was to develop PMTs' planning and teaching with technology with the guidance of Dynamic Geometry Task Analysis (DGTA) and Instrumental Orchestration frameworks. The DGTA framework (Trocki & Hollebrands, 2018) was used to examine and improve the content of technology-based mathematical tasks particularly focusing on the components of mathematical depth and technological action. The IO framework (Drijvers et al., 2010; Trouche, 2004) was employed to develop and illuminate PMTs' classroom practices with the use of digital technologies. The results of our previous work indicated that while planning technology-based lessons PMTs overlooked the exploitation modes of their instrumental orchestrations. They tend to focus on the global view of their teaching but not focusing on detailing their technology use nor coordinated use of technology or prompts. Hence in our follow-up work, we mainly focused on developing PMTs' ability to proactively plan for exploitation modes. In this study, we examined one prospective mathematics teacher's planning and micro-teaching of a technology-based cylinder task in the context of a practicum course.

Theoretical framework

The Instrumental Orchestration (IO) model introduced by Trouche (2004) and developed by Drijvers et al. (2010) is derived from the instrumental approach to didactics that offers a model to describe the process of learning mathematics when using technological tools (Artigue, 2002). IO points out the necessity for teachers to guide their students' instrumental genesis by systematic organisation of their tasks and use of the various artefacts available (Drijvers et al., 2010). When introducing this framework, Trouche (2004) used two concepts: a didactical configuration and exploitation mode. A didactical configuration is mainly about arranging the teaching setting including the technological tools in the environment. An exploitation mode is concerned with teacher's decisions about how they plan to exploit their didactical configuration in order to achieve their lesson aim. These decisions can be related to how they introduce and perform a task and/or how technological tools are used by the teacher and students. Later, when using and developing the framework, Drijvers et al. (2010) added a third concept, a didactical performance involving the *ad hoc* decisions while performing teaching in the chosen didactic configuration and exploitation mode. As an example of these concepts, Drijvers et al. (2014) described didactical configuration including practical elements (whether there is a projection device, enough computers, black board etc.) and global outline of the lesson (mathematical explanation, technical demo, students work on tasks etc.). Similarly, they exemplified exploitation mode by stating the question of how to elaborate elements of the global outline of didactical configuration in detail. These parts of IO are related to decisions, which teachers make before teaching in advance. On the other hand, didactical performance is concerned while carrying out the lesson during which teachers should consider the questions of whether they managed to employ their original global plan and/or their detailed plans worked out as expected (Drijvers et al., 2014).

Following this idea, in our study, while preparing the lesson plans, PMTs were encouraged to consider the possible orchestration types relating to particular didactical configurations and exploitation modes in which they could achieve their planned objectives. The concept of didactical performance, in our study, was concerned with the parts when PMTs performed their micro-teaching and subsequently, their classroom teaching within their school placements, during which unexpected issues may occur in response to using the available technological tools. Based on their didactical performance, PMTs were expected to revise their instrumental orchestrations in particular detailing the exploitation modes of their orchestrations. This was our particular focus since our previous study (Bozkurt & Yigit Koyunkaya, 2022) showed that PMTs mainly focused on the global outline of the lesson but not a detailed exploitation mode of their orchestration, which resulted in a failure in reaching the intended mathematical depth and using coordinated technological actions during their teaching.

Methods

This case study focused on an in-depth investigation of one prospective mathematics teacher's exploitation mode of the area formula of the cylinder with the use of dynamic mathematics software, GeoGebra in her planning of the task and micro-teaching of it. The unit of analysis was the prospective mathematics teacher, and the case was the PD arrangement which was organised within a practicum course in its mathematics teacher education programme (Yin, 2018).

Context, participants and procedure

The context of this study was based on a PD arrangement designed in the context of two-semester compulsory practicum course named as school experience and teaching experience in a four-year mathematics teacher education programme at a Turkish University. In this study, we reported the first part of the PD activities taken place during the first semester in the school experience course. The participants were four PMTs, who were in their final year of the programme. These four volunteer PMTs were selected considering their interests and existing knowledge of using dynamic mathematics software. In this course, the PMTs were required to observe the teaching of the assigned mentor teacher and classroom environments in a school placement for four hours in a week. Additionally, they were required to attend a class at the university to discuss their observations in the school placements for one hour in a week. As a continuation of this course, in the following semester, the PMTs took the teaching experience course.

The contents of the courses do not include the technology integration as a compulsory part, but our aim was to encourage the PMTs to prepare tasks including technology. Based on our experiences and inferences in a previously completed study (Bozkurt & Yigit Koyunkaya, 2022), we planned and conducted the content of the PD arrangement in this study focusing on developing the PMTs' technology integration into their teaching. As a result, the PD arrangement in this study involved seminars, planning and teaching technology-based tasks, group discussions and individual interviews. Initially, we gave seminars about designing tasks that aimed to generalise mathematical concepts, processes, and relationships by using the dynamic features of GeoGebra. Then, we assigned the selected four PMTs the objectives related to the 3D geometry - recognizes, determines, constructs, and draws the parts of *the right prism, pyramid or circular cylinder* and construct the area and volume formulas of the right circular cylinder. Our decision regarding these objectives mainly stemmed from PMTs' tendencies to choose or develop the tasks related to 2D geometry rather than 3D geometry in the software.

In the planning process, we asked the PMTs to plan a technology-based task with a focus on detailing exploitation modes of their use of technological actions and prompts. Then, they conducted a micro-teaching in a room at the university, in which one prospective teacher was teaching and the other fellow PMTs acting as the students. After the micro-teaching, we conducted group discussions to examine each PMT's teaching and exchanged ideas. In addition, we conducted interviews with the PMTs individually to enable them to critique their teaching and collect feedback from us. In this paper, we focused on one PMT's (pseudonym Mira, female) planning and micro-teaching of the concept of cylinder since she provided evidence regarding her encountered difficulties through her planning and micro-teaching. Mira designed her task considering the following objectives: (a) Determines, constructs, and draws the parts of a right circular cylinder; (b) Constructs the area formula of the right circular cylinder and solves related problems.

Data collection and analysis

During the PD arrangement, we collected different types of data using different data collection techniques. In this paper, the data consisted of Mira's planning of a technology-based task, in which she planned to teach the parts and the area of the cylinder to the students aged 13-14, and the video

recording of micro-teaching of this task. The video records of the individual interview and group discussion related to all the PMTs' micro-teaching experiences were also used as the supportive data in the study. She taught her tasks in 30 minutes. We completed the individual interview in 60 minutes and the group discussions in 75 minutes (the discussion regarding her task was completed in 15 minutes). We triangulated the data from video records of the group discussion and individual interview with the data from video records of the micro-teaching by focusing on how Mira herself and her fellow PMTs as a group evaluated the micro-teaching to improve her teaching with GeoGebra. The triangulation allowed us to validate and verify the data.

In her task (see Figure 1), she used four different sliders (representing the height of the cylinder, opening the bases of the cylinder, black line segment on the cylinder, unwrapping the rectangle (netting) around the cylinder, which is also related to the line segment IP on 2D screen, as seen inside the red rectangle in Figure 1). She planned to use 2D and 3D screens collaboratively to guide her students through reaching a generalisation of the area formula of the cylinder. (See the task: <https://www.geogebra.org/classic/nkzbnwut>)

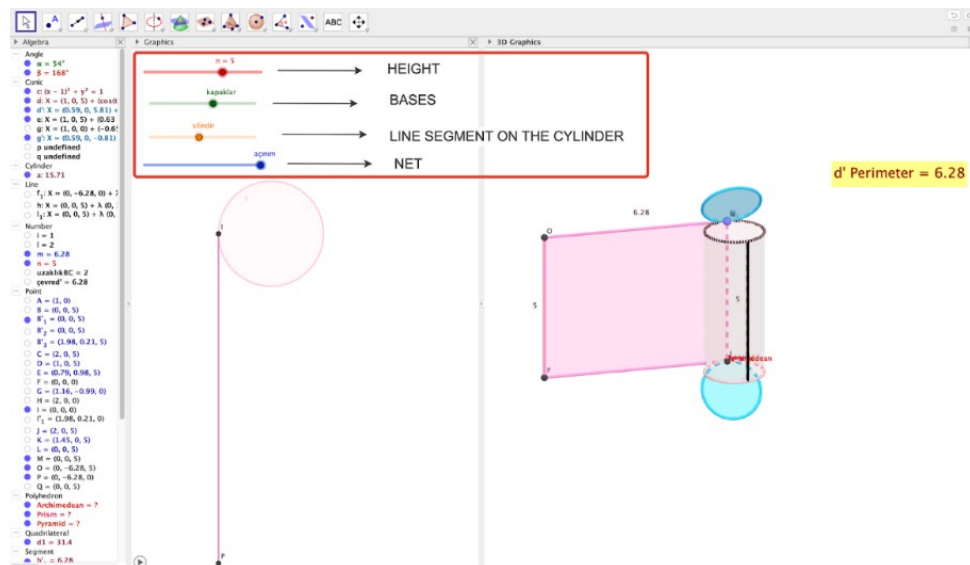


Figure 1: Mira's task on the area formula of a cylinder

For the data analysis, we conducted a qualitative data analysis considering the PMT's instrumental orchestration she planned to use, in particular exploitation mode of the technology-based task. In her plan, we examined how Mira planned to use GeoGebra, to guide her students to reach a generalisation. Specifically, we analysed what questions she planned to ask and how she planned to ask those questions sequentially. We also analysed the video record of Mira's micro-teaching to evaluate how she implemented her plan, particularly focusing on the exploitation mode of her teaching. We watched the video and identified the crucial moments characterising the orchestration types in the literature (Drijvers et al., 2010). While identifying the crucial scripts, we focused on how she orchestrated the class while teaching the parts and area of the cylinder with GeoGebra and we transcribed the selected scripts. This allowed us to reveal the outstanding points that occurred in her teaching. Considering Mira's planning and teaching, data analysis indicated that coordination in the use of 2D and 3D screens, using sliders for different purposes, zooming in and out the screens, using

dynamic texts and different colours in the task and tracing on were the outstanding points that were important in teaching the cylinder concept. In this paper, we will only present the analysis of coordinated use of 2D and 3D screens due to space issues.

Results

In this section, we particularly focused on Mira's discuss-the-screen orchestration and its exploitation mode considering her plan and micro-teaching of her technology-based cylinder task.

Mira planned to encourage students to investigate how the circle in the 2D screen was related to the long side of the rectangle, which wrapped the cylinder, in 3D screen. In her task (see Figure 1), there was a circle in 2D and attached a slider to the circumference of the circle while the cylinder in 3D unwrapped. The main question in her plan was "What do you think about the relationship between the circle in 2D and sides of the rectangle in 3D?". In her micro-teaching, in order to encourage the fellow PMTs to investigate how the circle and the long side of the rectangle related to each other, she started to discuss how the line segment in 2D screen is related to the cylinder in 3D screen, particularly to the shape (a rectangle) formed by the rotation of the height of the cylinder. In this process, she animated the sliders (one represented the line segment belonging to the circumference of the circle and the other unwrapped the cylinder) and asked the fellow PMTs to observe the screen. The following verbatim represents her teaching of this episode:

Mira (M): Now I am going to do something on the screen, and I want you to observe what is happening there. Then we will discuss your observations (she animated the sliders for unwrapping the line segment on the circle in 2D screen and the rectangle on the cylinder in 3D screen).

(The fellow PMTs observed the screen for around 15 seconds)

PMT: Where should we look?

M: Where should you look? Good question. Now this line segment (pointing the black line segment on the cylinder), what is this?

PMT: Height

M: Height of the cylinder. Now the height of the cylinder turns around the cylinder, right? (moving to the slider attached to the height of the cylinder) So you need to look at this line segment and the shape that I constructed here in 2D. I want you to observe if there is a relationship between these.

PMT: In 2D shape, there is a line segment moving to down.

M: Do you mean here (while showing the part by her finger)?

PMT: What is that line segment coming down from the point P? There is a line segment coming out of P, what is it?

M: This is that. There is a line segment OP coming out of here (she indicated the short line segment of rectangle), that line segment is the form of that in 2D plane. It's coming this way (showing the rectangle that is unwrapping the cylinder) and it is coming this way (she showed the line segment surrounding the circle by using her hand). In the 2D plane, that line segment is this.

PMT: From which side are we looking at the cylinder?

M: You are looking at the cylinder now from the front (moving the cylinder left and right).

PMT: From the left side, what side are we looking at the cylinder?

M: What do you mean with the left side?

PMT: In 2D.

M: Where are you looking from this side (showing 2D screen), from here (she rotated the cylinder in 3D screen). See, it's the same (she obtained the similar shapes in 2D and 3D screens).

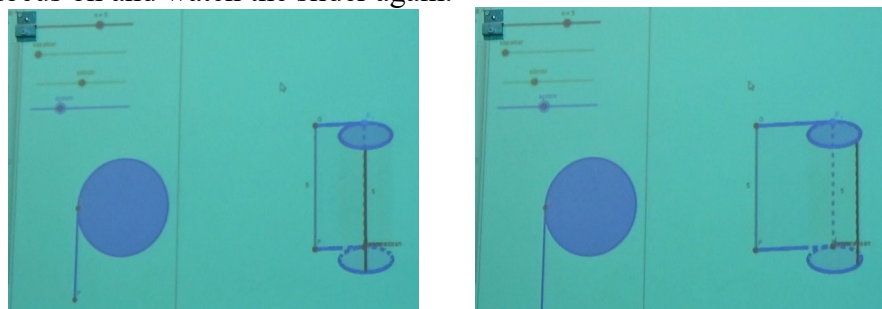
- PMT: From the top?
M: Since it is 2 dimensional, we are looking at it from the top without adding height.
PMT: OK. Isn't that the top view?
M: Yes, so let's just focus here for now. What is the relationship between these line segments? Does this line segment form a shape? If it does, why do you think it forms such a shape? Let's discuss these.

As seen in the above verbatim, because of her inexplicit and complicated questions/prompts, the fellow PMTs did not understand which parts of the screens they should observe as well as the role of the objects on the task. Additionally, she was not able to explain correctly how shapes in 2D and 3D screens were related to each other. For instance, Mira wanted the fellow PMTs to discover the relationship between the movement of the cylinder and the constructed shape in the 2D screen; however, with her questions and directions, the PMTs did not understand where to focus on which parts of both screens. The classroom discussion indicated that the fellow PMTs struggled to relate the circle on the 2D screen and cylinder on the 3D screen. Therefore, Mira directly rotated the cylinder to obtain the top view of the cylinder aiming to have the same shapes in both screens. In the individual interview, she explained the process as:

I was not expecting that they would focus and question the 2D screen so much. That's why I did not examine 2D part before the lesson. To be honest, I started considering 2D part in the micro-teaching... I answered students' questions by focusing on the 3D screen and rotated the top and the bottom of the cylinder whose appearance in 2D, which I realised during the class.

Later in her micro-teaching, to point out the fact that the bases of the cylinder were the same as the circle in the 2D screen, she used the visual features of the GeoGebra and coloured the shape using the blue ink on both screens. The verbatim transcript represented how she discussed the relationship between the long side of the rectangle and the circumference of the circle by using 2D and 3D screens coordinately relying on the features of the software:

- M: I'm going to make a change here and you'll understand where to focus on (she coloured (as blue) and thickened the long side of the rectangle and the bases of the cylinder on the 3D screen, and the IP line segment on the 2D screen). Now, only focus on these parts that are the same colour, why did I make them the same colour? Let's focus on and watch the slider again.



- PMT: There are things rotating in the cylinder (indicating the line segment turning around the cylinder), it looks like the height.
M: What do you mean rotating inside? This? (she showed the black line segment)
PMT: Yes.
M: That was already there at first, wasn't it? Now, you are going to observe its movement.

- M: Yes. Now if you look at the lengths over there, I mean between the circle at the bases of the cylinder and those lengths (she showed the bottom base of the cylinder and one of the long sides of the rectangle).
- PMT: Do we only look at the base?
- M: We're only looking at the coloured parts. Now here's the line, here (she showed one of the long sides of the rectangle). What is happening to this? (she animated the slider for netting the cylinder). It's getting shorter, but what happens, does it disappear?
- PMT: Like it's wrapping.
- M: Isn't it? Well, then what kind of relationship should it be between them to be able to wrap it?
- PMT: They have to be equal.
- M: If they are not equal, for example if this is longer (she showed the long side of the rectangle) what would happen? There will be an extra line segment in there, right? Or if it is shorter, it cannot wrap the cylinder completely.
- PMT: Won't it wrap the cylinder again? Two times, three times?
- M: Then, does its length change around the circle? For example, we say 2π , 4π , don't they mean the same thing? We are going back to the starting point again. It's the same thing as the circumference of the circle, even if it's on top of each other. So, what do you think about these two, now? [The fellow PMTs did not immediately answer the question]
- M: What does it need to be fully wrapped?
- PMT: They need to be equal.
- M: Yes, they should be equal.

The verbatim showed that she explained both screens in a better way by using the visual features of the software as well as indicating each part she discussed by using her hands and gestures. However, the content of the task, i.e., the line segment moving on the cylinder, prevented the fellow PMTs from focusing on the parts of the cylinder. In the individual interview, she stated that:

While this rectangle was wrapped around the cylinder, I was trying to point out that the long side of the rectangle was wrapped around the circle. At that time, I did not hide the height of the cylinder, so the height was rotating. It was a bit confusing for them in terms of where to look, and they wondered if there was a relationship between them.

In discussing both screens, because of her prompts and questions, she was still limited in linking both screens since the fellow PMTs did not immediately realise what she wanted them to investigate. In addition, while she was explaining or discussing the screens, the Topaze effect was occurred, in which she mostly answered the questions by herself without giving time to the fellow PMTs to think and provide an answer. In the group discussion, the fellow PMTs stated that "The task was good, but she did not provide enough examples for the different forms of the shape by using the sliders".

Conclusion

This study examined one PMT's planning and micro-teaching of a technology-based task focusing on the concept of cylinder in the context of a practicum course aiming to develop PMTs' technology integration particularly from the perspective of Instrumental Orchestration model. The results of this study particularly showed the difficulties that the PMT experienced while relating 2D and 3D screens in her micro-teaching. This indicated her lack of didactic reference points (Haspekian, 2017) to teach this topic using GeoGebra's representations of cylinder. At this point, micro-teaching provided some of these lacking reference points and particularly allowed her to adjust and find accurate questions

and prompts that were important in this session, which potentially guide her future teaching of this task. Particularly, micro-teaching experiences helped her realise the importance of proposing questions and prompts (Hollebrands & Lee, 2016) and also explaining and discussing both screens (Drijvers et al., 2010) since the fellow PMTs in micro-teaching did not understand where to focus on the screen to answer her questions for a couple of times. In this sense, this study highlights the importance of using micro-teaching set-up in PD programmes to develop PMTs' didactic reference points, which support PMTs' planning of instrumental orchestrations by reducing the instrumental distance they potentially have (Haspekian, 2017).

References

- Artigue, M. (2002). Learning mathematics in a CAS environment: The genesis of a reflection about instrumentation and the dialectics between technical and conceptual work. *International Journal of Computers for Mathematical Learning*, 7(3), 245–274. <https://doi.org/10.1023/A:1022103903080>
- Bozkurt, G., & Yigit Koyunkaya, M. (2022). Supporting prospective mathematics teachers' planning and teaching technology-based tasks in the context of a practicum course. *Teaching and Teacher Education*, 119, 103830. <https://doi.org/10.1016/j.tate.2022.103830>
- Drijvers, P., Doorman, M., Boon, P., Reed, H., & Gravemeijer, K. (2010). The teacher and the tool: Instrumental orchestrations in the technology-rich mathematics classroom. *Educational Studies in Mathematics*, 75(2), 213–234. <https://doi.org/10.1007/s10649-010-9254-5>
- Drijvers, P., Tacoma, S., Besamusca, A., van den Heuvel, C., Doorman, M., & Boon, P. (2014). Digital technology and mid-adopting teachers' professional development: A case study. In A. Clark-Wilson, O. Robutti, & N. Sinclair (Eds.), *The mathematics teacher in the digital era* (pp. 189–212). Springer. https://doi.org/10.1007/978-94-007-4638-1_9
- Haspekian, M. (2017). Computer science in mathematics new curricula at primary school: New tools, new teaching practices. In G. Aldon & J. Trgalova (Eds.), *Proceedings of the 13th International Conference on Technology in Mathematics Teaching (ICTMT 13)* (pp. 23–31).
- Hollebrands, K., & Lee, H. (2016). Characterizing questions and their focus when pre-service teachers implement dynamic geometry tasks. *The Journal of Mathematical Behavior*, 43(3), 148–164. <https://doi.org/10.1016/j.jmathb.2016.07.004>
- Trocki, A., & Hollebrands, K. (2018). The development of a framework for assessing dynamic geometry task quality. *Digital Experiences in Mathematics Education*, 4(2–3), 110–138. <https://doi.org/10.1007/s40751-018-0041-8>
- Trouche, L. (2004). Managing the complexity of human/machine interactions in computerized learning environments: Guiding students' command process through instrumental orchestrations. *International Journal of Computers for Mathematical Learning*, 9(3), 281–307. <https://doi.org/10.1007/s10758-004-3468-5>
- Yin, R. (2018). *Case study research: Design and methods* (6th ed.). Sage.