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# A novice teacher's work with resources to enhance mathematical competencies

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*For almost two decades mathematical competencies have witnessed a great interest in mathematics education research, both in learning and teaching mathematics. This study is also a product of such interest and aims to analyse a novice teacher's work with resources to enhance mathematical competencies while teaching mathematics. For this analysis, we utilised the Documentational Approach of Didactics (Gueudet & Trouche, 2009) as a theoretical framework. We designed this study as a case study to analyse the resources and their usage schemes in detail. We collected data from the recordings of interviews, lesson preparations and observations of the teacher and the schematic representation of his resource system. In conclusion, we resulted that the teacher utilised his resources from the undergraduate programme and merged them with the usage schemes he developed during his teaching experience to enhance the mathematical competencies.*

*Keywords: Mathematical competencies, novice teachers, resources.*

## Introduction

Mathematical competence can be defined as “someone’s insightful readiness to act appropriately in response to all kinds of mathematical challenges pertaining to given situations” (Niss & Højgaard, 2019, p.12). While mathematical competence includes the response to all kinds of mathematical challenges, a mathematical competency focuses on a specific sort of mathematical challenge. In this perspective, mathematical competencies are more related to school mathematics because in school mathematics students need to meet specific challenges to develop specific competencies (Koeppen et al., 2008). Because of this need, the role of mathematics teachers who would organise the related mathematical challenges is significant. Novice teachers who are in the first five years of their teaching career utilise various resources, both the resources from their undergraduate programme and the resources from the new community they try to fit in (Le Cornu, 2013). So, their resources and usage schemes to support their students to enhance mathematical competencies draw attention to research. This study is a product of such attention aiming at analysing the usage schemes of a novice teacher while he tries to develop his students’ mathematical competencies.

## Research question

The research question leading this study is:

- How does a novice teacher use his resources to support the development of his students’ mathematical competencies?

## Conceptual framework

In this study, we utilised both the Documentational Approach of Didactics (herein after DAD) as the theoretical framework (Gueudet & Trouche, 2009) and Competencies and the Learning of

Mathematics (herein after KOM) framework as a tool to understand the mathematical competencies (Niss & Højgaard, 2019). DAD is used for understanding teachers' interactions with their resources. And KOM framework is used for defining the relationship between the resource and the competencies.

DAD utilizes some special processes, such as instrumentation, instrumentalization and documentation. Through all of these processes defined in DAD, with their usage schemes (Vergnaud, 1998), teachers create documents which is also a special concept in DAD. A scheme is a teacher's stable organization of their activities, for specific situations. A scheme has four components: (i) the aim of the activity, (ii) operational invariants (theorem-in-action and concept-in-action), (iii) rules of action, (iv) possibilities of inferences. While all the resources of a teacher create their resource system, usage schemes and the resources create a document system together (Ruthven, 2007). In this study, we define the schemes with their operational invariants and rules of action. Because, when we identify the stability in teachers' actions we could define the aim of that activity and realise the possibilities of inferences. Then, we matched them with the competencies the teacher tried to enhance.

The KOM framework explains mathematical competencies and their definitions to allow researchers realise them in practice. KOM framework takes the mathematical competencies into two categories: (i) Posing and answering questions in and by means of mathematics and (ii) Handling the language, constructs and tools of mathematics. There are four sub-categories in the first category: (i) mathematical thinking, (ii) mathematical handling, (iii) mathematical modelling, and (iv) mathematical reasoning. The second category also has four sub-categories: (i) mathematical representation, (ii) mathematical symbols and formalism, (iii) mathematical communication, and (iv) mathematical aids and tools.

## **Research design**

We designed this research as a qualitative case study to analyse a novice teacher's usage schemes of his resources for a specific aim. Case study is a research method that allows deepening analysis of a chosen concept or process (Yin, 2009). In this study, we accepted the novice teacher as a case and tried to understand his usage schemes when he aimed to support the development of mathematical competencies of his students.

### **The participant of this study: Anil**

Anil was a novice teacher of two years in eastern Turkey when the data of this study were collected. Before that, he was an undergraduate student at the institution where the authors worked at the time. He was known as a student who drew attention to mathematical competencies. After being a teacher, he was defining himself as a teacher who gives importance to development of mathematical competencies of his students. In addition, he was willing to share his resources and experience with his students.

### **Data collection method and tools**

We utilised the reflective investigation method which is suggested by Gueudet and Trouche (2009) for those to use DAD. There are some principles of the method (Trouche et al., 2020, p. 6):

- Broad collection of the resources,

- Long-term follow-up,
- In- and- out-of-class follow up,
- Reflective follow up,
- Confronting teachers' views on the documentation work and the materiality of it.

According to these principles we utilized semi-structured interviews, schematic representation of resource system (SRRS), interviews on the lesson preparation, reflective notes of Anil, lesson observations, and recall sessions. We tried to match the requirements of the reflective investigation and our data collection methods in Table 1. For example, we utilized SRRS diagram of Anil to be able to reach broad collection of his resources.

**Table 1: Data collection methods according to the reflective investigation method**

Reflective Investigation Steps	Data Collection Methods
Broad collection of the resources	SRRS diagram
Long-term follow-up	25 lesson (25x40 min.) observations
In- and- out-of-class follow up	Reflective notes of Anil
Reflective follow-up	Interviews & Observations of the lessons and lesson preparations
Confronting the views on the documentation work and the materiality of it	Recall Session

### **Data collection procedure**

One of the researchers (the first author) conducted the data collection. She (the first author) first requested Anil to draw a SRRS diagram and interviewed with him during his drawing about the meaning of his arrows, the places of the resources and the relationship between the resources. Then she conducted semi-structured interviews about his lesson preparations and observed 25 lessons of his. The observed lessons were the 6th, 7th and 8th grade students' mathematics and mathematics applications lessons. During the lesson observation, she sat in the back rows like a student, video-recorded the lessons and took notes and did not interfere with the process. The notes taken during the lesson observation, about the critical points she considered to ask for confirmation of Anil. After all the lesson preparations, observations, and interviews, she organized a recall session in order to realize reflective remarks of Anil.

### **Data analysis**

One of the researchers (the first author) transcribed the audio-recordings and video-recordings of the interviews and lessons. Initial coding was based on the resources and their usage schemes related to the mathematical competencies. We (two of the authors) then, focused on DAD's key processes- instrumentation and instrumentalization.

First, in the interviews, lesson preparations and observations, we traced his resources that he mentioned in his SRRS diagram. We tried to see how he utilises his resources in his lessons and whether it is a usage scheme or just a usage for that one lesson. When we realize that there could be such a scheme, we coded it accordingly and checked it whether it could be related to a mathematical competency or not and discussed it deeply in the recall session (cf. Table 2).

**Table 2: An example of data analysis**

Mathematical Competency	Concept-in-action	Theorem-in-action	Rules of Action	Anil's Resource
Thinking and Reasoning	Critical thinking	Students should be free to answer the questions and to discuss on their ideas	<p>“Lead the students’ class discussion as the professor led before”</p> <p>“Ask the students to defend their ideas, when they are not sure about their answers”</p> <p>“Let students to explain their answers and justify it”</p>	University Professors

Then, we followed the instrumentation and instrumentalization processes during the coding. We generated an analysis frame to do systematic coding (Yin, 2009). We used various data collection tools in order to enhance the trustworthiness of the findings. We employed triangulation between the interviews, lesson observations, and recall sessions to confirm the conclusions.

## Findings

We summarized the findings in Table 3, according to the data gathered from SRRS diagram, lesson preparations, observations and interviews. We analysed the resources and their usage schemes with all three components (theorem-in-action, concept-in-action and rule of action), however we did not mention all of them in Table 3. We mentioned only the concept-in-action to indicate the usage scheme.

As seen in Table 3, Anil has more than one usage scheme for some of his resources, for example, the one that he mentioned as *university professors* in the SRRS diagram (cf. Figure 2). He prepared his students a class environment which they could express their thoughts freely. He explained this situation in the recall session as an effect of one of his university professor’s approach. Since that professor cared about the group work in which students could discuss their ideas with both their peers and the teacher, he used the exact same approach to develop the students’ reasoning. Another usage scheme of the resource *professor* is about the technique of developing critical thinking of his students. He encouraged the students who are shier than others to defend their thesis against other students with different ideas. He also wanted his students to think of other probabilities according to different assumptions. He allowed classroom discussions as well as group discussions.

Anil: (to one of his students) Why don't you defend your idea? Maybe your solution is correct and easier than your friend's! Please explain what you think. And let's see how you two conclude that.

Student 1: OK. I think we can solve this problem by multiplying the steps he took for each floor and the number of the floors.

Student 2: I solve it not by multiplying but addition of each floor. But then we had to find the same result, because the multiplying is just a faster version of addition. But we didn't find the same number.

(Both are checking their calculations.)

Student 2: I did an arithmetical mistake while adding the last floor. So, our results should be the same, now. Your way is also correct I think.

Anil: See, you should defend your solution! And you should explain your way, even if it is wrong, it is your solution.

**Table 3: Mathematical competencies and related usage schemes of Anil**

Mathematical Competencies	Anil's Usage Schemes (concepts-in-action)	Anil's Resources
Thinking and Reasoning	Reviewing students' mistakes Free class environment Finding a common feature Discussing by assumption Establishing a cause-effect relationship Critical thinking Using correct expressions Student discovery	University Professors My Mathematics Atelier (the additional common textbook) Peers from university
Problem solving	Using different types of exercises Teaching extra-curriculum concepts Creating new questions Using the questions similar to national exams	National Exams Curriculum University Professors
Representation	Using different representation methods	Internet Pages and Social Media
Others	Recalling the previous lessons Using reinforcement Accessibility of homework Active participation	Common Classroom Resources (e.g. Worksheets)

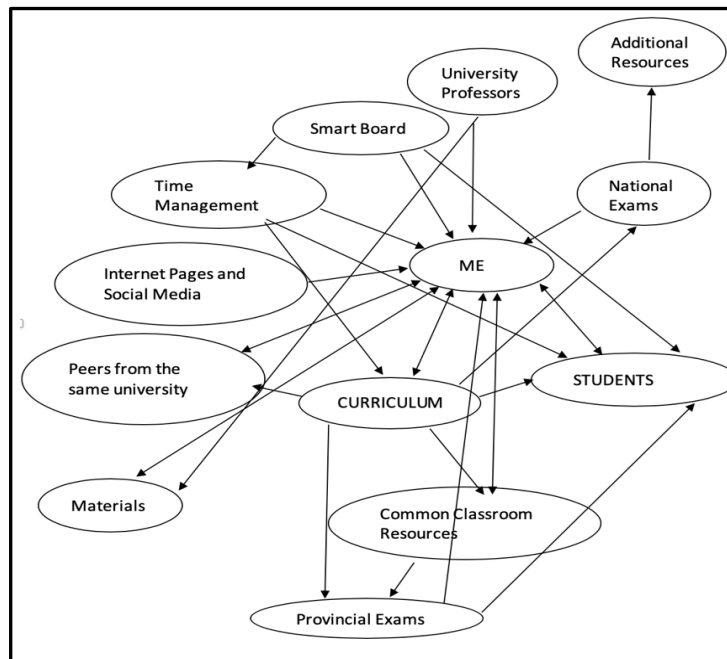
In this study, the SRRS diagram of Anil and the interview on it gave us a holistic understand of his resources. As mentioned above, he mentioned some people in his life as resources of his teaching, such as university professors, peers from university, his students and himself. We also discussed on his consideration during the recall session and he explained the thought behind it:

Researcher: You drew the university professors and your peers from university as your resources in SRRS diagram. Can you explain how you utilise these resources?

Anil: When I prepare a lesson I always think about our professor and ask myself: How would he teach this lesson? And my answer is: He would not teach, he would create a task and ask us to study on that and then he would take our opinions about the work we did. At the end, we would understand what was the subject to be taught. His techniques are one of my initial resources to shape my lesson.

Researcher: What about your peers?

Anil: When I couldn't find an appropriate task to start my lesson, I ask my former classmates how they planned their lessons. We didn't cut off the communication and we try to help each other to plan our lessons. Also, I always follow my students' lead in my lessons, I observe them and I care their learning velocity. That's why I want them to talk to me and their friends more in the class, so I could understand where they are in their learning path.



**Figure 2: Anil's SRRS diagram**

According to Figure 2, it is possible to infer from the expression in capital letters that Anil consider himself, students and curriculum as main resources of his practice. When we discuss this in recall session he mentioned that the students and curriculum “lead him how to teach”:

Researcher: How do you think curriculum and students lead you to teach? Can you explain this a little more?

Anil: I already explained the students, I always check them in the class if they understand or not. If they did not understand properly, I explain it more and I am using then additional textbooks for different easier exercises.

Researcher: And the curriculum?

Anil: As you know we need to follow the curriculum and curriculum has its own recommendations about how to teach that issue. What the limitations are and which parts we should focus on more are explained in the curriculum. So, I take care of those recommendations in my lessons.

As in the explanations of Anil, he gave a task to his students and let them study on this task in groups freely. After they completed the task, he continued with another related task. After implementing two tasks, he managed to make students define the prime numbers themselves. In this lesson, he showed the concept-in-action: student discovery and related theorem-in-action: a teacher should guide

students to discover the concepts themselves. His intention to make students study on the tasks freely and define the mathematical concept themselves were about a mathematical competency; reasoning. We could infer that he used the following resources together to provide thinking and reasoning: university professors, peers from university, and My mathematics atelier textbook.

During his lesson preparations, we observed that he used different types of exercises from different textbooks. He started with easier exercises and then continued with the problems similar to the national exams. When he needed he created some questions that are similar to national exams. He used national exams, curriculum and university professors as resources to develop problem solving competencies of the students.

As seen in Table 3, he used internet pages and social media to find different representations that other teachers used in their lessons for several mathematical issues. For example, he used multiplier flower technique to show the multipliers of a number. He also used branch-leaf method that he required from the internet pages for the same reason. He explained this situation as follows:

Anıl: I used different representations to show the multipliers of a number. Since we don't know how it would be represented in national exams, I wanted my students learn every way to recognise and able to explain those ways properly.

According to Table 3, he used common classroom resources, such as worksheets, to provide recalling the previous lessons, reinforcement, accessibility of homework for every student and active participation to the lessons. We did not match these usages of him with specific competencies, however we believe these usage schemes support the mathematical competence of the students.

## **Discussion and Conclusion**

The aim of this study was to analyse how a novice teacher use his resources when he aimed to enhance his students' mathematical competencies. For this reason, we studied with a novice teacher Anıl who cares about developing mathematical competencies of his students. According to our analysis, we resulted that Anıl attempted to develop the following mathematical competencies: (i) Thinking and reasoning, (ii) problem solving, (iii) representation. In this study, we also met some of his schemes that does not serve a specific mathematical competency, but helping the development of mathematical competencies. Those schemes were about recalling the previous lessons, using reinforcement, providing equal access to homework and increasing active participation to the classes.

Another result of this study is that Anıl having some *human* resources. He considers his classmates from university, university professors and students as his resources that shape his lessons. The consideration of Anıl has some similarities with the teacher Pierre in Gueudet et al.'s study (2012). He utilised these resources with different usage schemes for each resource. For example, he used his professor's approach in creating a free class environment and he also used it for providing critical thinking and he used it while creating new questions for his students. In this way, he used the same resource for enhancing different mathematical competencies with different usage schemes. When we analyse these usage schemes we could see the similarities between them and the indicators that are reported by OECD (2003). The indicators are more definite signs to detect if the student has that mathematical competency, while Anıl's usage schemes of his resources were more likely the attempts of his to trigger students' abilities.



In this study, we analysed only one novice teachers' work with resources on a special aim. One teacher might not be a representative of all novice teachers but we believe that this analysis of his resources and usage schemes would give an idea of novice teachers' early career experiences.

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## References

- Gueudet, G., Pepin, B., & Trouche, L. (Eds.) (2012). *From text to 'lived' resources: Mathematics curriculum materials and teacher development*. <https://doi.org/10.1007/978-94-007-1966-8>
- Gueudet, G., & Trouche, L. (2009). Towards new documentation systems for mathematics teachers? *Educational Studies in Mathematics*, 71(3), 199–218. <https://doi.org/10.1007/s10649-008-9159-8>
- Koeppen, K., Hartig, J., Klieme, E., & Leutner, D. (2008). Current issues in competence modeling and assessment. *Zeitschrift für Psychologie/Journal of Psychology*, 216(2), 61–73. <https://doi.org/10.1027/0044-3409.216.2.61>
- Le Cornu, R. (2013). Building early career teacher resilience: The role of relationships. *Australian Journal of Teacher Education*, 38, 1–16. <https://doi.org/10.14221/ajte.2013v38n4.4>
- Niss, M., & Højgaard, T. (2019). Mathematical competencies revisited. *Educational Studies in Mathematics*, 102, 9–28. <https://doi.org/10.1007/s10649-019-09903-9>
- OECD (2003). The PISA 2003 assessment framework: mathematics, reading, science and problem solving, knowledge and skills. <https://www.oecd.org/>
- Ruthven, K. (2007). Teachers, technologies and the structures of schooling. In D. Pitta- Pantazi & G. Philipou (Eds.), *Proceedings of the Fifth Congress of the European Society for Research in Mathematics Education CERME5* (pp. 52–67). Larnaca. <https://doi.org/10.1007/978-3-319-73253-4>
- Trouche, L., Gueudet, G., & Pepin, B. (2020). The documentational approach to didactics. arXiv preprint arXiv:2003.01392. <https://doi.org/10.48550/arXiv.2003.01392>
- Vergnaud, G. (1998). Toward a cognitive theory of practice. In A. Sierpiska & J. Kilpatrick (Eds.), *Mathematics education as a research domain: A search for identity* (pp. 227–241). Kluwer. [https://doi.org/10.1007/978-94-011-5194-8\\_15](https://doi.org/10.1007/978-94-011-5194-8_15)
- Yin, R. K. (2009). *Case study research: Design and methods* (4<sup>th</sup> ed.). Sage. <https://doi.org/10.33524/cjar.v14i1.73>