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An introduction to TWG14: University mathematics education

Irene Biza¹, Olov Viirman², Matija Bašić³, Ignasi Florensa⁴, Ghislaine Gueudet⁵, Mathilde Hitier⁶, Igor' Kontorovich⁷, Athina Thoma⁸ and Megan Wawro⁹

¹School of Education and Lifelong learning, University of East Anglia, UK; i.biza@uea.ac.uk

²Department of Education, Uppsala University, Sweden

³Department of Mathematics, Faculty of Science, University of Zagreb, Croatia

⁴Escola Universitària Salesiana de Sarrià, Univ. Autònoma de Barcelona, Spain

⁵Université Paris-Saclay, UR EST, France

⁶Université de Montréal, Canada

⁷Department of Mathematics, the University of Auckland, New Zealand

⁸Southampton Education School, University of Southampton, UK

⁹Department of Mathematics, Virginia Tech, USA

This paper summarizes in six themes the papers and the posters contributed to the TWG14: University Mathematics Education (UME) at CERME13. Also, we provide an overview of the three thematic discussions and the final concluding session of the group with indications for future research.

Keywords: University mathematics learning and teaching, resources, teaching interventions, mathematics for non-mathematicians, mathematics students' identity and experiences.

Introduction

TWG14 was launched at CERME7 in 2011 and, since then, a number of UME-related activities have taken place, some as a direct consequence of the work in TWG14. Only between CERME12 and CERME13, we have seen four special issues of research journals – Teaching of Mathematics and its Applications (TEAMAT) on [Calculus and pre-Calculus](#); International Journal of Research in Undergraduate mathematics Education (IJRUME) on [Calculus at the Intersection of Institutions, Disciplines and Communities](#) and [Definite Integrals](#); Educational Studies in mathematics (ESM) on [transition to UME](#) –, two specialized conferences – fourth ERME topic conference of the International Network for Didactic Research in University Mathematics ([INDRUM2022](#)) and the Learning and Teaching of Calculus across Disciplines conference ([CalcConf2](#)) – and a book on the [Teaching and Learning of Calculus](#) (IMPACT series). In the CERME13 TWG14 call for papers, we highlighted this development, and invited research proposals on a wide variety of topics, including transition issues, the teaching and learning of specific mathematical topics, the role of resources, inclusion and equity, and student experiences. As a result of this call, TWG14 received 55 paper and 12 poster submissions. The review process resulted in 37 papers and 17 posters being presented at the conference, of which 36 papers and 15 posters are published in CERME13 proceedings. After a slight decrease from 35 presented in CERME11 to 25 papers presented in CERME12 (online), the number increased again to the previous level. The large number of contributions led to the decision to have parallel sessions in two groups (TWG14A and TWG14B) addressing six themes: *students'*

learning of mathematical topics and practices; students' identity and experience; teachers, teaching and interventions; the use of mathematics by non-mathematics specialists; resources, curriculum, tasks and technology and theoretical advances and theorizing. While many papers and posters could fit in more than one theme, their classification into the six themes helped their presentation in this paper. In addition to the parallel sessions, we also held three plenary thematic sessions on: *UME for non-mathematics specialists; digital resources in UME; and the role of theory in research and practice in UME.* We offer a summary of these thematic sessions before concluding with reflections and potential ways forward.

Themes and paper and poster contributions

Students' learning of mathematical topics and practices

Fourteen papers and four posters were classified under this theme. Three studies look at the learning of functions. Palamioti et al. find that the different narratives and routines concerning sequences and functions in school and university discourses cause commognitive conflicts¹ for first-year students. Bergwall investigates students' understanding of growth of two-variable functions, comparing students who calculate directional derivatives with those who rely on properties of the gradient vector. Ergene and Çaylan Ergene, on the other hand, explore students' problem-posing concerning functions of two variables, finding that students asked to solve their posed problems made fewer errors and generated more solvable problems than the students who only posed problems. There are also several studies on students' learning of other Calculus and Analysis concepts. Borji et al. investigate how activities involving story problems can support university students in developing their understanding of exponential and logarithmic statements in real-world situations. Berggren et al. use ATD to study students' solving approaches in optimisation tasks in Calculus. Dreyfus et al. explore high school students' meaning for integral in relation to the calculation of area and suggest a disconnection between students' views on the area and their proficiency with integral computations. Bašić and Milin Šipuš examine the transposition from taught to learned knowledge to better understand students' perspective in autonomous learning of the Inverse Function Theorem. Rogovchenko and Rogovchenko provide a praxeological analysis of two approaches to exact differential equations, suggesting potential conflict factors for students as they encounter the content in different settings. Lepellere investigates the evolution of spatial ability of engineering students during a Mathematical Analysis II course that widely used [GeoGebra](#). Dibbs and Celik study how students in a flipped complex analysis course think about the relationship between complex multiplication and derivative. They found that students struggled with the geometric interpretation of the complex derivative.

Concerning Linear Algebra, Wallach and Kontorovich utilize the construct of learning and teaching agreement to investigate instructional interactions and the impact on the roles of students and instructor, within the context of an online interactive first-year tutorial. Piroi presents the results of a pilot study on the teaching and learning of eigentheory in a first-year mathematics course for engineers, characterizing the ecological conditions of the new setting and its viability. Wawro et al.

¹ Due to space limitations, we do not provide explanation and references to theoretical terms (e.g. commognitive conflict, ATD, Activity Theory, etc.). We would invite readers to find more in the corresponding contributions in the proceedings.

explore Linear Algebra students' reasoning about determinants using GeoGebra. Pleština and Milin Šipuš investigate students' mathematical praxeologies concerning polynomials in secondary school and at university. Radmehr and Taghizadeh Bilondi study students' understanding of the concept of tree in graph theory, showing how they move between sketching, manipulating formulas, and formal proving when solving problems. In the context of topology, Miranda reports on an activity where students were asked to generate examples of specific concepts, with the aim of assisting students to make conjectures about concept properties. Finally, two studies focus on students' learning of mathematical practices. Seibold et al. compare how collaboration and individual work impact prospective teachers' ability to solve familiar and novel problems, finding that individual study time improves performance on familiar problems while collaboration improves performance on novel problems. Nedaei investigates how the goal set for learning influences an engineering student's learning approach: being asked to teach a topic led the student to a more focused and deeper learning than when simply learning to be tested.

Students' identity and experience (including equality/equity)

Four papers and one poster were classified under this theme. Picariello et al. investigate the evolution of students' identity in the context of a problem-solving based course, showing how it led students to see themselves as mathematicians. With a focus on inclusion, Locke et al. study the case of a first-year international student in New Zealand, highlighting how linguistic and cultural differences challenged her participation in collaborative tutorials. Combining ATD and Joint Action Theory allows Texier-Picard and Gueudet to observe different kinds of epistemic asymmetries between one male and two female students. Gender is also a topic for Mayerhofer et al., who explore trends in expectancy for success and value beliefs at the secondary-tertiary transition in STEM fields, finding significant gender differences in favor of male students. Also concerning transition, Jiménez and González propose design principles for Calculus tasks that address prospective teachers' experience of Klein's double discontinuity in moving from school to tertiary mathematics and back.

Teachers, teaching and interventions

Three papers and one poster were classified under this theme. Focusing on university teachers' perspectives on student identity, Karavi and Mali identify explicit and implicit metarules in teacher discourse concerning students' expected participation in university mathematics. Kontorovich et al. introduce a model for knowledge mobilization between university mathematics teachers and mathematics education researchers revolving around teacher-led instructional innovations that researchers assist in studying in a disciplined manner. Concerning interventions, Mullen and Cronin present an action research project involving MathsFit, a set of hybrid resources aimed at students facing their first non-specialist mathematics course in University College Dublin. Sagredo-Sanchez presents preliminary findings from an action research project focusing on the teaching of a large-scale flipped course in mathematical modelling for engineering students.

The use of mathematics by non-mathematics specialists (including pre-service teachers)

Two papers and three posters were classified under this theme. Two of the studies concern mathematics in STEM. Hitier and González-Martín undertake a praxeological analysis of a pair of analogous tasks on derivatives, one in mathematics and one in physics, illuminating the challenges

students face in discerning the similarity between the tasks. Meanwhile, Peters and Hochmuth discuss and propose further investigation of a dialectic view between mathematics for and in engineering courses. The remaining three studies focus on secondary mathematics teacher education. González-Martín et al. compare Tanzanian secondary mathematics content with content in university mathematics courses for future teachers, finding little explicit overlap while secondary mathematics teachers see their mathematical training as unrelated to their teaching practice. The studies of Huo and Nižňanská both investigate how specific ways of addressing mathematical content, real numbers and limits respectively, can support prospective mathematics teachers in their practices as teachers.

Resources, curriculum, tasks and technology

Nine papers and four posters were classified under this theme, the majority of which focus on the use of (digital) resources. Two concern the use of the interactive theorem prover [Lean](#): Thoma and Iannone analyze schemes developed by first-year students writing a mathematical proof using Lean, while Garnelo and Liebendörfer propose the design of a learning environment that employs Lean to improve undergraduates' proof-generating skills. Meanwhile, Donatiello et al. find that using [MATLAB](#) Live Script as a “black box” in a Linear Algebra course tends to induce students to shift from a purely instrumental to a more relational approach. Fleischmann investigates how *Inverted Classroom Teaching*, and in particular video lectures, can be combined with more traditional teaching approaches in a university course on axiomatic geometry. Also focusing on video lectures, Feudel et al. highlight differences between the teacher's intentions and expectations regarding students' use of video-recorded lectures in a Linear Algebra course and students' reported use of them. Biza and Nardi explore students' use of online resources in mathematics education courses, showing students' agentivity and the need for pedagogical practices supporting it. Kock et al. explore what students perceive to learn in *Challenge-Based Education* (CBE) courses involving mathematical modelling, in the light of resources used. Flores and Vandebrouck present a study on limit calculations in the secondary-tertiary transition, analyzing limit calculations when the graph of the function is available. A few studies focus more on curriculum or design. Brandl et al. present results from the design of a digital environment (Digital Interactive Mathematical Maps) connecting secondary and tertiary mathematical content, while Kula and Akkaya present an online bridging course designed to help first-year students move from pre-university to university-level mathematics. Kinnear et al. analyze students' reasoning about functions from example-generation tasks, offering insights into students' (mis)understanding of function concepts (e.g., surjectivity and injectivity). Viirman and Jacobsson present strategies for task design aimed at promoting student shift from ritual to exploratory participation in mathematical discourse. Finally, an ATD analysis of mathematics exams in engineering programs from 1994 to 2020 lead Florensa et al. to conclude that the mathematical activity remains stable even though the description of the syllabi has evolved significantly.

Theoretical advances and theorizing

Four papers and two posters were classified under this theme. The studies by Hausberger and Hochmuth and Laukert et al. belong to the same project, and build on ATD to develop the notion of pre-structuralist and structuralist praxeologies in the context of Real Analysis. Focusing on the secondary-tertiary transition, Günther and Hochmuth propose a new theoretical framework for the

study of this transition by considering the enculturation into university mathematics as a process of transformative education, while Doukhan uses the case of courses in probability for biology students in secondary school and at university to illustrate how the networking of Activity Theory and ATD can support the study of transition. Körtling and Eichler draw on interviews with first-year university students to identify the utilization of language in the context of theorem formulation. Mavrommatis proposes using Deleuze and Guattari's notion of 'assemblage' to move beyond the micro-macro binary when studying the collaboration between mathematicians and mathematics educators.

Thematic Discussions

UME for non-mathematics specialists

Mathematics for non-mathematics specialists (including prospective teachers) is a highly vibrant research area addressed recently in special issues (e.g., Biza et al., 2022; di Martino et al., 2023; Pepin et al., 2023), as well as in conferences (e.g., CalConf2, INDRUM2022). Also, at least eight TWG14 contributions this year address aspects related to UME for non-mathematics specialists and triggered this thematic discussion that was organized around three interrelated sets of questions: (1) *Epistemological issues*: What mathematics is important in other disciplines, and what role does it play? When is mathematics considered a tool to be applied, and when is it more central in the discipline content development itself? (2) *Collaboration with other disciplines*: How can we collaborate with educational researchers in other disciplines? What conference venues and journal outlets exist for this collaborative work? and (3) *Research and design tasks*: Do we lack a shared vision of problems to investigate, and how does this impact research? What is the extent of curricular alignment across departments? What theoretical differences exist in conducting research in the various disciplines?

Beyond what mathematical topics are important in other disciplines, the discussion concerned what type of knowledge (e.g., procedural or conceptual) might be needed in those disciplines, and for what purpose (e.g., to solve problems or to develop models). Collaboration between UME researchers and educational researchers in other disciplines could help us understand how to strengthen students' transdisciplinary skills (e.g., quantitative reasoning), to create authentic applications, and to detach from a purely mathematical point of view. Furthermore, aspects such as beliefs and confidence in mathematics are also important for researchers to grasp, and these can also be explored through collaborative research. Thus, learning from educational research in other disciplines, for instance by reading relevant journals, is necessary. Interdisciplinary collaborations could be facilitated by inviting speakers from other disciplines to UME research conferences, as was done in the recent CalConf2 conference. In CalConf2, the metaphor of "different disciplines speak different dialects", emerged, which indicates the need for an open communication between disciplines. For such communication, new approaches in research might be beneficial. For instance, new approaches on peer collaboration between mathematicians and mathematics educators (Hernandez-Martinez et al., 2023) can also suggest common grounds in collaborations across disciplines that benefit from each other's expertise.

Digital resources in UME: Potential impact on (task) design and epistemology

Digital resources were a central theme in contributions submitted to TWG14, with around 16 contributions having digital resources as the main focus or the context of study. In these contributions,

a variety of digital resources are considered: theorem provers; educational resources like GeoGebra; digital curriculum resources for students and for teachers; online applications (e.g., Zoom platform); but, also, resources found online and not designed for a teaching purpose. The contributions explore, amongst others, students' mathematical practices when using these digital resources; modifications to students' mathematical practices due to the use of digital resources; and the design of teaching using digital resources. Such observations triggered a thematic discussion that was organized around two questions: (1) *Epistemological aspects*: How do digital resources modify mathematics and what could be considered as a legitimate mathematical activity? (2) *Teachers' professional activities*: How should/could teachers modify the design of their teaching by taking into account possibilities (for teachers or for students) opened by digital resources? Nevertheless, these two questions are closely connected, since instructional design is highly dependent on the mathematical activity the teacher intends students to engage in. As seen below, this connection was reflected in the discussions.

Digital resources open up new possibilities for mathematical activity (if their sometimes complex functionalities are sufficiently mastered) and at the same time shape this activity. These observations are not new (e.g., Hoyles & Lagrange, 2010). However, they seem to be taking a particular form in UME (e.g., Gueudet & Pepin, 2018) with the recent growing presence in mathematics teaching of programming, theorem provers and Artificial Intelligence (AI). Is it legitimate for university students to use a theorem prover for writing a proof? The answer to this question depends very much on who decides what is legitimate and what is not. In the case of university mathematics, the expert practices of mathematicians are an important reference point. Do university courses reflect recent changes in the activity of mathematicians, and propose to students practices similar to those of experts? This question may also arise for other professions, for example in courses for future engineers.

In the current state of technological development, it seems that a large part of mathematical activity remains mostly 'human'. Also, so far, the use of potentialities offered by digital resources, especially AI, is limited. However, students use more and more online resources, including AI tools such as ChatGPT; thus, teaching design must consider and shape such use. Students need to develop digital literacy skills (including providing relevant prompts to AI tools) and critical thinking skills in order to make the most of the resources found and the answers obtained. This means that university teachers have to design new tasks that support mathematical learning and increase students' agential use.

The role of theory in research and practice in UME

Theory usage in UME research has shifted over time. In the pre-CERME7 days of the Advanced Mathematical Thinking group, theoretical perspectives were mainly cognitive with related influential theories (e.g., concept definition/concept image; APOS; process-object duality). Since the launch of TWG14 in CERME7, the group has seen a broadening of theoretical perspectives and frameworks developed within the larger Mathematics Education community (e.g., ATD, commognition) have been adapted for use in UME research (see an overview in Nardi, 2017). The current state of UME research could best be described as one of theoretical diversity. The role of theory differs in each research project and for each individual researcher. Theory can form the backbone of the whole project or grow out of the analysis of empirical data. As researchers we can be strongly devoted to a particular theoretical perspective or take a more pragmatic stance by choosing the framework best

suitable for investigating a research topic. Accordingly, the thematic discussion was organised around two related sets of questions: (1) *Choice of theory*: What have been the gains and losses from the shifts over time in the theoretical orientations of UME research? What impact have these shifts had on UME practice and research, both in UME and Mathematics Education more generally? (2) *Use of theory*: Assuming a dialectical interplay between research questions/objects of study/findings and theoretical perspectives taken, to what extent is there a similar interplay between UME practice and UME research? Can we have practice without theory?

In the discussion, the idea of a “continuous shift of theories” was challenged, as theories do not disappear or emerge swiftly, but rather develop and gradually become more widespread. For UME researchers there is an apparent dialectic interaction between theory and questions asked. We might say that choosing a theory is an individual/idiosyncratic process for each researcher, driven by their wider interests, the type of considerations they wish to pursue and their participation in particular research communities, but the operationalization of research through research questions is formulated within a chosen theoretical framework. It was also claimed that the dialectic interaction between theory and practical outcomes is quite prominent also for UME practitioners. For instance, there can be a tension between research (academic exercise) and implications for practice, originating in the expectation of universities to have research- (and thus theory)-based tools that are easily applied. Examples of steps in mitigating these tensions are projects that bring together UME researchers and UM teachers towards the promotion of inquiry-based learning (e.g., [PLATINUM project](#)) or collaborations with researchers in physics and science education, who use different theories that (at least from the outside) might seem more accessible. Judging from this discussion, and the work of TWG14, there is a need to strengthen research papers by making the rationale behind the choice of theory more transparent, but also to make basic tenets of theories more accessible to newcomers. This may be pursued by organizing joint inter-institutional activities for learning and sharing, such as workshops and graduate-level courses.

Reflections and ways forward

In CERME13, TWG14 contributions addressed themes similar to those identified in previous CERMEs with *students’ learning of mathematical topics and practices* following by the *resources, curriculum, tasks and technology* themes being most popular. Many papers addressed more than one theme, though. We discussed, for example, how digital resources support (or even alter) students’ learning of specific topics and how they are used in teaching experiments. Transition, although not a theme on its own, was the educational problem underpinning several studies. Furthermore, a small number of studies addressed the role of cultural and social issues (e.g., students’ background) in mathematical activities. Occasionally, the discussion around the situational character of the mathematical activity under consideration – e.g., activities for mathematics or non-mathematics specialists – challenged a universal view of mathematics across educational contexts.

TWG14 discussions suggested further investigation in areas such as: (1) the role of socio-political and gender aspects in university mathematics access, participation and retention (e.g., diversity, equity, class, gender, de-colonization) with mathematics practices maintaining a central position in those investigations; (2) the learning and teaching of advanced mathematical topics that go beyond

typical first-year courses; (3) the role of digital resources in mathematical practices; and (4) the interaction between research and practice (e.g., interventions). The group seeks more collaborations between UME researchers and mathematicians or/and teachers of mathematics as well as researchers of STEM education. Also, in the future, we envisage more exchanges with other TWGs. We believe that research in University Mathematics Education has been solidified as a distinctive area in mathematics education research with significant theoretical, methodological and developmental contributions. Finally, we expect that next CERMEs will bring more insight into learning and teaching at university, what makes university studies distinctively different from other educational levels, how, and to what extent, university studies are accessed and how transition is experienced not only to and from university studies but also across disciplines, communities, and institutions.

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