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Zooming into instructional interactions online: A case of an interactive tutorial in the pandemic

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University mathematics education has made considerable efforts to embrace active learning and student-centered pedagogies. There is a need for empirical studies into what these processes look like online. During the pandemic, a group of six students in a first-year linear algebra course chose to partake in interactive online tutorials, instead of the traditional lecture-based format. Using the commognitive framework, we characterize student-student and student-instructor interactions in these tutorials. The analysis revealed three types of interactional patterns: lecture-ish, instructor-oriented and cross-student. The types differ in their learning-teaching agreements, which determined the instructor's and students' roles in the interaction. We discuss how the instructor's technological affordances to write for the whole group to see and the group expectation for formal mathematics predisposed lecture-ish interactions and confined students' learning opportunities.

Keywords: Commognitive framework, instructional interactions, learning-teaching agreement, linear algebra, online learning and teaching.

Introduction

In the last two decades, considerable efforts have been made to challenge lecturing as a predominant mode of instruction in university mathematics education. Much of these efforts unfolded under the call for active learning and student-centered pedagogies (CBMS, 2016). These umbrella terms embrace a broad range of instructional practices, many of which foreground students' mathematical communication, group problem solving, and receiving and providing feedback from both instructors and peers (e.g., Ng et al., 2020). Research shows that institutionalization of these practices is not easy, partially because instructors struggle to provide substantial opportunities for students to mathematize autonomously and agentively (e.g., Nardi & Barton, 2015; Paoletti et al., 2018). The struggle to provide such opportunities is not restricted to lectures but is also in settings where students apply their mathematical knowledge from lectures to solve problems (e.g., Kontorovich & Ovadiya, 2023).

The COVID-19 pandemic shifted higher education online, drawing attention to learning and instruction on digital communication platforms. Spontaneous interactions, instructors' "reading the classroom", students' sharing their work, mathematical writing – these are just a few examples of synchronous practices whose online versions are radically different from in-person ones (e.g., McKenna & Green, 2002). Although learning and teaching worldwide mostly returned to physical classrooms, the online modality is unlikely to disappear soon. It was used in distance education before the pandemic, and many universities capitalize on online affordances to pursue their internationalization agendas (Møgelvang et al., 2023). In the beginning of 2023, the University of Auckland oscillated between online and on-campus studies due to severe weather. This might be an illustration of a broader reframing of online instruction as an educational modality that can replace

in-person teaching and learning, when needed. Accordingly, understanding how active learning and student-centered pedagogies operate online remains relevant to contemporary educational realities.

Research has acknowledged the affordances of online education (e.g., Møgelvang et al., 2023). Yet, most research taps into these aspects drawing on students' academic outcomes and reflective interviews (e.g., Møgelvang et al., 2023). This work aims to explore active learning and instructional processes as they unfold online, with a focus on student-student and student-instructor interactions.

Theoretical Framework

We study human interaction as it occurs in communication spaces with the commognitive framework (Sfard, 2008). Commognition is a socio-cultural framework that has been acknowledged for its potential to explore communication and learning processes through the mathematical talk and actions of students and instructors, as well as their cooperation with each other (e.g., Nardi & Barton, 2015).

From the commognitive standpoint, learning is associated with one's participation in a discourse that is renowned as mathematics. Visible markers of one's participation involve using specific *keywords* (e.g., "equation") and *visual mediators* (e.g., " $|■|$ "), producing *narratives* generally endorsed as true (e.g., "a positive number less than 1 to a natural power is less than 1") and performing conventional *routines* (e.g., proving). In practice, students' learning is mostly occasioned by instructors who set *task situations* – circumstances that call for a certain kind of action (Lavie et al., 2019). Each action is viewed as a performance of a routine with two elements: a *task* that one pursues, based on the performer's interpretation of the task situation, and a *procedure* as a sequence of steps (Lavie et al., 2019). We add that a task situation, and thus the task, changes with each action that a student or an instructor takes, opening the space for a performance of another routine.

Sfard (2008) maintains that students' learning requires proactive scaffolding of instructors as the "oldtimers" of the target discourse. This special student-instructor interaction is governed by a *learning-teaching agreement* (LTA). Sfard (2008) contends that "to attain their respective goals, [students and instructors] need to be unanimous, if only tacitly, about [...] the *leading discourse*, their own respective *roles*, and *the nature of the expected change*" (p. 283, italics in the original). The first LTA component focuses on whose discourse is regarded as setting the standards: students' or instructors'. For the second component, Sfard (2008) distinguishes between instructors' role as discourse leaders and learners who "show confidence in the leader's guidance and are genuinely willing to follow in the oldtimers' discursive footsteps" (p. 284). Research shows a rich palette of finer-grain roles that instructional interactions entail (e.g., Strijbos & De Laat, 2010). Each role constitutes a duty or a specific sort of action filled by a group member. The final component concerns agreement on "the final goals of the process of learning and as to the manner in which the learning is likely to occur" (Sfard, 2008, p. 285). This is another umbrella construct that summons context-sensitive operationalizations. In the university setting, undergraduates are expected to become capable of generating written problem solutions that abide by discourse-specific rules sanctioned by the course instructors as experts of the target discourse.

Methods

Our data comes from a linear algebra course for mathematics and computer-science majors in a large university in Israel. The course is traditionally taught through lecturing (e.g., Paoletti et al., 2018). The tutorials, where the instructor solves problems on the whiteboard, are also taught in this manner. In the second semester of 2020, when the course was given online due to the pandemic, six students volunteered to participate in an alternative tutorial format advertised as promoting mathematical discussions and student interaction (see Nardi & Barton, 2015 for a similar design of “low lectures”). Before joining the tutorial sessions, students were expected to engage with the course lectures (“live online” or recorded), watch recorded tutorials from the previous semester, and work on homework problems. Their tutorials were led by Mia (pseudonym) – an experienced university instructor working towards a PhD about encouraging student participation in mathematics courses.

The sessions were held on the Zoom platform twice a week over a 13-week semester, 39 academic hours in total. Each session started with questions that students had about the course material for that week and about homework problems. This was followed by mathematical problems that Mia prepared. All students had their video cameras and microphones open, which allowed them to contribute to the discussion at any moment, not very different from in-person tutorials. Mia used an external camera to project the problems and her writing for the whole group to see. The students could have written or annotated anonymously on the shared screen, yet they did not.

Our data consisted of the screen-recordings of the tutorials. A discussion of a problem constituted the unit of our analysis. We transcribed the group interactions to identify the LTA in place. Its first component was rather evident since it was clear who leads the discourse: Mia or the students. This allowed us to divide each problem discussion into episodes. In each of them, we scrutinized the interactions to delineate the roles in-play. We first asked, “what are the roles that this setting offers students and the instructor?”. To identify the expected changes, we attended to markers that the work on the problem has concluded. We then asked, “what changes do students and the instructor pursue as part of their interactions in this setting?”. Finally, we systematically compared the episodes until patterns of interactions emerged from the data.

Findings

We first present the roles offered by the setting, the intermediate changes pursued in the discussions and the three idealized types of instructional patterns that emerged from the analysis. Then, we share an analysis of one discussion to showcase how the group transitioned between the three types of interactions. This case illustrates the dominance of the lecture-ish interactional pattern.

The roles offered by the setting

Each discussion opened with *setting the group problem* that is, the main task situation the group would work on. Some of the problems were proposed by the students and some problems came from Mia. To solve the problem, the protagonists *proposed and performed* mathematical routines. A single routine was rarely sufficient to reach a solution, and thus a sequence of routines (or sub-routines) was needed. Not every proposed routine (and accordingly, a subtask and a subprocedure) was taken by

the group. Thus, another set of roles pertains to *setting* the course of mathematical work, i.e. determining which proposal will be followed and which one will be abandoned.

For example, in one of the sessions, a student set up the group problem, asking to determine which of the four given sets are subspaces. Mia responded with “what do we need to prove it?”. With this, Mia put forward the need for a proof (a subtask) and alluded that it can be constructed with some previously covered material (procedure). Three students took the role of routine proposer by suggesting proving that “there exists a zero”, “zero and linearity”, and “linear combinations”. Mia asserted that the first routine is the simplest one, which set the course of the group action. Then, another student performed the routine, reporting that “It looks like only one set doesn’t have [a zero]”.

Three additional roles are noteworthy. One is of *mathematical endorser* who sanctions the validity of the mathematical narratives. Another is a *scribe* who records the group work in writing. Mia was the scribe, even though the students could annotate on the shared screen. The last role is *calling for a discursive change*. The discussions tended to start with students sharing rather diverse and informal mathematical narratives before a call was made to change the nature of the discussion. Usually, Mia was the one to call to “write things down”. Occasionally, students asked “what would be considered an acceptable answer on an exam?” as if confirming whether the generated mathematical narratives would be endorsed in assessments and implicitly calling to switch to mathematics of this sort.

The expected changes pursued

The students initially presented their ideas orally, through the use of microphones, and the final narratives were written by Mia on the screen. The analysis of the changes that the group pursued revealed that their interactions were targeted at a generation of a written solution-narrative that is cohesive (i.e. contains no internal contradictions), well-substantiated (i.e. each sub-narrative is justified), formal (i.e. utilizes conventional terminology and notation), and comprehensive (i.e. considers all cases). As a scribe and an endorser of students’ narratives, Mia never allowed her writing to deviate from the first two. Regarding the third, Mia often reformulated students’ narratives replacing deictic talk and colloquialisms with conventional keywords (see an example below). Deviations from the fourth one were rare and seemed unintentional. For example, in one discussion, the group problem concerned a claim about all natural numbers, but the written solution proved it for integers larger than 2. A student asked about $n = 1$, challenging the comprehensiveness of the written narrative. This reopened the discussion and led to the amendment of the written solution.

The group rarely endorsed the written narrative as a solution explicitly. Typically, Mia completed the writing with “Questions?” as if inviting students to seek further explanations. This move can be seen as Mia endorsing the solution and signaling students that the goal of their discussion was achieved, at least in her view. Students endorsed the written solution-narrative by not objecting to it.

Three types of instructional interactions

Contrasting episodes from different discussions resulted in three idealized types of instructional interactions. The first one emerged from long sequences of Mia’s utterances, where she led the work on the problem. This interaction unfolded alongside Mia writing a solution-narrative, occasionally interrupting herself with questions. Each question was tightly linked to the previous solution step,

such as “why is that true?” and “what do we get?”, and summoned a certain short answer. We refer to this instructional pattern as *lecture-ish* due to its core similarity to a traditional lecture format. We refer to the second type of instructional interaction as *instructor-oriented*. There, students generated mathematical narratives and questions directing them to Mia as a mathematical authority. For example, one student suggested, “can we prove it by summing vectors?” asking Mia to assess its appropriateness. With such questions as “what do you mean?” and “why would that work?”, Mia turned the narrative into a short exchange with the student. The students’ open cameras and microphones facilitated this dyadic interaction, as it was clear with whom the instructor was interacting. The *cross-student* interaction took place when the instructor kept silent and two or three students conversed with each other, leading the discussion on a problem. The exchanges consisted of several turns, where the students acknowledged and built on each other’s narratives.

All group discussions lead to the lecture-ish interactional pattern

The excerpt that follows illustrates that the groupwork on each problem constituted a mixture of the idealized instructional patterns presented earlier. We also use the excerpt to showcase that for all problems, at some point, the groupwork transformed to a lecture-ish interactional pattern.

In Session 6, about complex numbers, a student (Sarah) set the problem for the group: “ $z^{200} = 1 + i$. True or false: The equation has a root whose modulus is greater than 1”. Roi opened the discussion.

- 628 Roi: I am, I am simply saying, if this to the power of 200... its modulus is the square root of 2.
- 629 Mia: Let’s find the modulus of the root. What is the modulus of the root?
[Writes $|z_k| =]$
- 630 Roi: Then... its 2 to the power of 1 divided by 400 [Mia continues writing $2^{1/400}$]
- 631 Mia: And now is it a number that’s larger than 1 or smaller than 1?
- 632 Leo: Larger than 1.
- 633 Roi: It must be larger than 1 because if I put an exponent on it...
- 634 Zohar: Larger than 1. Since the minimum at infinity is 1 but its 1, its larger than 1.
- 635 Roi: Yes. Larger than 1.
- 636 Zohar: If its smaller than 1... If its smaller than 1 to the power of 200 ... to the power 200 then... then it will come out a tiny number.
- 637 Roi: That’s it! I am saying I am saying let’s say the modulus of the root was half [...] and I put it to the power of 200 it will never get to the square root of 2.
- 638 Mia: (a) Let’s write it. I am writing. (b) Let’s give it a name like R [completes $|z_k| = 2^{1/400} = R$]. (c) And we know that R to the power of 400 is equal to [Writes $R^{400} =]$ <pause> Two [Writes 2]. (d) And then if something to the power 400 is larger than 1, what do we know about the exponent base?
- 639 Zohar: Larger than 1.
- 640 Mia: Larger than 1. Yes. Because a number smaller than 1 to any power will always be smaller than 1. Thus, we know that R is larger than 1. Ok?

This excerpt can be divided into three episodes, each corresponding to the presented instructional patterns. The exchange in [628-630] is the closest to an instructor-oriented pattern. In [628], we see Roi making a step toward finding the modulus of the equation’s roots. Mia’s utterance in [629] follows on Roi’s. It involves the keywords of “root” and “modulus” that he mentioned and appears as if spelling out the task that Roi pursued for the rest of the group. Yet, Mia does not exactly

“revoice” what Roi said. Her narrative replaces Roi’s deictic “this” and “it” with the term “root” and uses notation that signals that there is more than one root to the equation. Even more importantly, Roi’s narrative was concerned with the modulus to the power of 200, while Mia sets the subtask of determining the modulus of the root. Roi performs the new subtask in [630] to obtain $2^{1/400}$. Then, Mia responds with the next subtask – to estimate whether this number is larger or smaller than 1. We see the instructor breaking down the main task by setting up a chain of three smaller ones.

The last subtask sparked a cross-student interaction in [632-637]. Three students maintained that the number is larger than 1 and substantiated it using different procedures (routine setting and performing). Students not only shared their narratives, but they also built on them. After Zohar notes that a number that is smaller than 1 to the power of 200 “will come out a tiny number” ([636]), Roi chooses half as an example to argue that “it will never get to the square root of 2” ([637]). This is a development compared to his substantiation in [633], where he wanted to “put an exponent on it”.

The lecture-ish part of the interaction [638-641] opens with Mia calling to “write it” and claiming the role of scribe. As in [628-630], she appears to operate with what was suggested by the students. But there is a more sophisticated discourse leadership in-play. In [638b], she denotes the root modulus, making the manipulation easier. Then, instead of estimating the modulus, as Zohar and Roi did, Mia raises R to a power. Accordingly, Mia proposes a new procedure and selects it by executing it on the answer sheet. The students are given an opportunity to calculate R^{400} , but once an answer does not emerge instantly, Mia performs the calculation herself (see [638c]). In [638d-640], she returns to the task of estimating R , while drawing attention to the base of the exponent. Zohar estimates correctly in [639] but provides no substantiation. Mia endorses Zohar’s response, but after not receiving the intended substantiation, she generates it herself. Notably, Mia a priori positioned the substantiation as something “we know”, and it did not find its way into a written solution-narrative.

Summary and Discussion

Given the TWG14’s interest in active learning and student-centered pedagogies, it appears imperative to understand what can facilitate and obstruct these practices online. This study makes a modest step in this direction by offering a fine-grain analysis of interactions that unfolded in an online tutorial in the first pandemic year. We acknowledge that the presented findings are inseparable from the specificity of the explored setting. For instance, the students self-selected for the study, they kept their cameras and microphones open, and interactive mathematics instruction was foreign to Mia and to the students. Yet, we believe that the findings contribute to the existing research, which mostly draws on learning outcomes and reflective interviews (e.g., Møgelvang et al., 2023).

The tutorial brought together an experienced and well-intended lecturer and a small group of students who volunteered to partake in a format that was non-traditional to this university. The format was advertised as promoting mathematical discussions and student interactions around problems. Throughout the semester, students engaged with the course content and worked on the assigned problems before partaking in the sessions. Hence, it seems the ingredients for the tutorial to deliver on its promises were in place. While our analysis is still ongoing, it is already clear that what we dubbed as lecture-ish instruction dominated the sessions, when cross-student interactions were rare, short, and did not go as far as one would hope. Indeed, the instruction reverted to the lecture-ish type

of interaction, which was more familiar to the instructor, at some point in each problem discussion. There, the instructor was the one to generate a formal solution, drawing on students' contributions.

We discuss these findings with two components of the LTA that was established in the tutorial: the protagonists' roles and the nature of pursued changes. Regarding the former, the analysis resulted in a set of roles pertaining to proposing, setting, and performing mathematical routines. The problem solutions emanated from the execution of these roles. As a discourse oldtimer, the instructor could fulfil each of these roles more successfully, efficiently, and quickly than the students. We believe that all our protagonists were aware of this. Compared to performing, proposing and setting routines involves a higher degree of mathematical agency since they summon autonomous decision-making, substantiation, and consideration of what move to take next. In this sense, the roles afforded substantially different opportunities for the students' mathematical agency, making students' learning inseparable from roles that the protagonists relinquished, took, and offered to each other.

The analysis showed that some roles were enacted by the instructor and the students on different occasions. For instance, in each session, the instructor voluntarily renounced the role of the group problem setter and proactively invited students to fill it. In turn, in lecture-ish and the instructor-oriented interaction, the role of a routine setter was mostly preserved for the instructor, leaving the role of routine performer to the students. This is consistent with previous findings from in-person tutorial classrooms (e.g., Kontorovich & Ovadiya, 2023; Ng et al., 2020).

Regarding the second LTA component, the group expected their discussion to result in a comprehensive, cohesive, well-justified, and formal written narrative that solved the focal problem. The aspect of formality taps into the strengths of the instructor as an oldtimer to the discourse of university mathematics. She was also the group member with technological tools to write conveniently and visibly. This explains why the students unanimously endorsed her as a group scribe. However, in the switch from an oral to a written modality, the instructor formalized previously shared narratives, broke down tasks into smaller ones, selected routines to pursue, and performed many of them herself. This is how she grew solution-narratives with the expected qualities through writing.

We acknowledge that in the sessions, the students were engaged, generated narratives that were key to problem solutions, interacted with the instructor, and occasionally with each other. These are major shifts toward active learning and student-centered teaching, compared to the lecture format that dominated the particular institutional setting. These shifts would not occur without the instructor opening the communication space for them. Nevertheless, we maintain that a tacit LTA between the instructor and the students regarding how they partake in a mathematical discourse, when, and for what purpose, predisposed their interactions to a significant extent.

This study's contribution to the existing literature is three-fold. Theoretically, we adapt Sfard's (2008) construct of LTA to the context of university-level mathematics education. To our knowledge, this is the first study to utilize the construct for data analysis. Empirically, we identified three types of instructional patterns in online tutorials. Research on tutorial teaching and learning is on the rise (e.g., Kontorovich & Ovadiya, 2023), and the identified patterns may be extendable beyond this setting. The lecture-ish patterns aligns with the classrooms described by Paoletti and colleagues (2018), where lecturers frequently asked questions but did not allow students opportunities for contributing content.

Practically speaking, during the pandemic, the focal university equipped instructors with digital tools that were especially useful to write mathematics for all students to see. By that, the university predisposed the instructors to the role of an online scribe. Our findings illustrate that when instructors, who are expected and used to leading a mathematical discourse, are provided with advanced writing tools, it may be challenging for them to scribe only. This makes us wonder whether allocating the same tools to students might have impacted the roles that they are allocated and take.

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