

# Ostensives and the development of mathematical praxeologies – the case of polynomials

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## Introduction, research question, theoretical framework, and methods

The anthropological theory of the didactic introduces the concept of *mathematical praxeology*, as the basic unit for analysing mathematical activity. The *praxis block of mathematical praxeology* answers the question of *knowing how*, while the *logos block* answers the question of *knowing why*. Every mathematical praxeology starts with the manipulation of perceptual objects, *ostensives*, whereby this manipulation is guided by concepts to which ostensives refer, *non-ostensives* (Arzarello et al., 2008). Ostensives have a dual role: semiotic through the connection with non-ostensives, and instrumental as a tool for mathematical practice. Apart from this ostensive/non-ostensive dialectic, the question is whether there is a connection between the development of logos blocks of mathematical praxeologies that students are equipped with and the role of ostensives (symbols) in students' mathematical activities. The posed question is investigated in the case of polynomials as an object of knowledge. A questionnaire<sup>1</sup> was created based on the analysis of *knowledge to be taught* and *taught knowledge* about polynomials in Croatian gymnasium education and undergraduate programme in mathematics and was conducted among students of all six semesters of the said undergraduate programme. Development of students' mathematical praxeologies is observed concerning *knowledge to be taught* and *taught knowledge* in the analysed institutions. In gymnasium education, a polynomial is defined as an algebraic expression of the form

$$p(t) = a_n t^n + a_{n-1} t^{n-1} + \dots + a_1 t + a_0 \quad (1)$$

where  $a_0, a_1, \dots, a_n \in \mathbb{R}$ , and as a real (complex) function. During undergraduate studies, students are equipped with praxeologies about formal polynomials (finite formal sum of powers (1) where  $a_0, a_1, \dots, a_n$  are from the arbitrary field – first semester; a sequence that only on finitely many places has elements from a ring that are different from zero – fourth semester) and praxeologies about polynomials defined as real (or complex) functions. The theory that unites the algebraic approach and functional approach to the notion of a polynomial is developed in the fourth semester through the course about algebraic structures. Due to the dual approach to the notion of a polynomial, the first

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<sup>1</sup> 1. Define the notion of a polynomial. 2. Circle the letter in front of the expression that defines (some) polynomial. Explain each of your choices. (a)  $t^2 - t + 1$ , (b)  $p(t) = \frac{t^2-9}{t+3}$ , (c)  $t^2 - 3t + 2 = 0$ , (d)  $t \rightarrow t^n + 1$ , (e)  $\cos(\arccos x)$ , (f)  $p(x) = (x - 1)^2$ , (g)  $p(x) = \sin(\arcsin x)$ , (h)  $p(t) = 4t^6 - t^3 + \sqrt{3}t - 2$ , (i)  $p(t) = t^2 - 2it + 1$ , (j)  $p(z) = z^2 - 2iz + 1$ , (k)  $p(x) = \begin{cases} x^2, & x \in \langle -\infty, 0 \rangle \\ x^2 + 1 & x \in \langle 0, +\infty \rangle \end{cases}$ .

task in the questionnaire examines the definitions adopted by the students, which should be the basis of the logos blocks of students' praxeologies on the notion of a polynomial. The expressions, i.e., ostensives in the second task are taken from the practical blocks of praxeologies, which students have already been equipped with through their gymnasium education. The second task requires the students to question the definition of a polynomial that they have stated and provides a deeper insight into the logos blocks of their praxeologies about the notion of a polynomial.

## Results and conclusion

Five categories are observed in analysing students' answers to the first question from the questionnaire: C1 – a polynomial is not defined, C2 – a polynomial is an algebraic equation, C3 – a polynomial is an algebraic expression, C4 – a polynomial is a function, C5 – a formal polynomial. The percentage of students' responses at the beginning of the first semester in categories C1 (44%) and C2 (29%) can be explained by the dominance of practical blocks over logos blocks of praxeologies in secondary education. Students in C2 interpreted expressions containing the ostensive = as equations, and the ostensives  $x$  and  $t$  as unknowns to be found. Solving algebraic equations is the most prominent type of tasks about polynomials in secondary education; therefore, it is not surprising that 90% of first-semester students whose answer is classified as C3 (23%) also identified polynomial and algebraic equation (c). At the beginning of the first semester, only 5% of students defined a polynomial as a real function (C4). Still, in the second task, their arguments did not refer to the function as a non-ostensive, but to the form of the expression. The argument that the expression under (a) does not define any polynomial because  $p(t)$  is missing is not only a characteristic of first-semester students in C4 but is the very present argument up to the fourth semester (second semester 89%, third semester 69%, fourth semester 68%). From the fourth semester, students' answers are distributed only in two categories, C4 and C5. Their arguments became more focused on non-ostensives (domain of the function, (dis)continuity of the function, definitions of the field and ring, and others), and they showed better-developed logos blocks (definitions closer to the discipline of mathematics, more properties of polynomials). Although students are equipped with praxeologies about formal polynomials already in the first semester, their answers do not show the development of these praxeologies until the fourth semester. Moreover, even after the fourth semester, most students (94% in the fourth, 79% in the fifth and 90% in the sixth semester) primarily view the polynomial as a function whose domain and codomain is a ring (most often fields  $\mathbb{R}$  and  $\mathbb{C}$ ). Definition of a polynomial as a finite formal sum of powers (1), where  $a_0, a_1, \dots, a_n$  are elements of a ring, is *syntactic*, focused on the form of the expression, and does not provide students with a straightforward answer to the question of what  $t$  is (as opposed to the definition of a polynomial as a sequence). Thus, many students do not interpret the ostensive  $p(t)$  as an abbreviation for the formal sum  $a_n t^n + a_{n-1} t^{n-1} + \dots + a_1 t + a_0$ . Furthermore, the ostensive  $p(t)$  in that definition evokes function as non-ostensive, which is the dominant notion in modern mathematics and mathematics education.

## References

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