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# Towards a designated undergraduate students' mathematics identity through a thinking group approach

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*We present the first findings of a study on the construction of mathematics learner identity during a university-level problem-solving experience. To shed light on the impact of the activities on the student's self-perception as a mathematician and on how this influences the attitude towards mathematics, we analyse some students' protocols from which some changes in the awareness to perceive themselves as mathematicians emerge.*

*Keywords: Mathematics identity, problem solving, creativity, undergraduate students.*

## Introduction

The identity of the mathematician is widely studied (Radovic et al., 2018) and developed starting from the definition of identity constructed by Sfard and Prusak (2005) which makes use of the concept of story as an element that gives shape to the identity of the individual thus preserving its fluid and dynamic nature. The study that we present here is only the first step of a much wider ongoing research and it is based on an experience conducted at the University level, with the aim of improving the problem-solving skills of university students. From an initial analysis of the students' answers, the words of some students that had explained in a quite clear way a change in perceiving themselves "as mathematicians" together with a change in the view of mathematics and of doing mathematics, had struck our attention. This led us to investigate the nuances with which students perceived a change in the self-perception of their mathematical identity.

## Background Literature

In Sfard and Prusak (2005, p. 16) authors suggest that "Identities may be defined as collections of stories about persons or, more specifically, as those narratives about individuals that are reifying, endorsable, and significant". From the perspective of Sfard and Prusak, identity cannot be separated from the social context: the stories that constitute an individual's identity can be stories that the individual tells about themselves, stories that others tell about the individual, stories that the individual tells *themselves*. These narratives can be divided into two categories: *actual identities* and *designated identities*. Actual identities are those stories that refer to a current situation and are almost always told in the present tense (i.e. "I am a good driver"); the designated ones are those stories that the individual believes will become actual one day and are often told in the future tense (i.e. "I have to be a better person"). The designated narratives are therefore mainly linked to the expectations that that person has (or makes) of herself and have a great influence on the individual, guiding her actions in such a way that, in most cases, the actuals move towards the designated ones. The wider and more lasting the gap between the actuals and the designated, the more likely that person is dissatisfied. The great influence these stories have on the individual, often acting as self-fulfilling prophecies, is reflected on learning and on how this process and its result are perceived by the individual themselves (Sfard and Prusak, 2005). The study of mathematics learners' identity at university level fits well in the

illustrated scenario. In this context situations are recurring in which the student “as a mathematician” is expected to appreciate certain mathematical entities, as definitions or theorems, and the related concepts or proofs, or think in a certain way fostering the generation of mathematical knowledge by themselves. A gap is thus created between the actual identity— “I am a mathematics student” —and the designated identity— “what kind of mathematician I am expected to be” —which could generate dissatisfaction and hinder the construction of one’s own mathematics identity. Our research focuses on the problem of the gap between the student's current mathematical identity and the student's expected mathematical identity and how it is possible to reduce it. The study is part of a wider research project that was born to develop university-level skills and creativity through the design and experimentation of suitable learning activities.

### **Theoretical framework and research goal**

The theoretical lens of our study is inspired by the one introduced by Sfard and Prusak (2005) in a general context, as regards the reading of the stories students tell about their self-view as mathematicians and by considering the definition of mathematical identity introduced by McGee & Martin (2011). We refer to mathematics identity as follows:

Mathematics identity encompasses the dispositions and deeply held beliefs that individuals develop about their ability to participate and perform effectively in mathematical contexts and to use mathematics to change the conditions of their lives. A mathematics identity encompasses a person’s self understanding as well as how they are constructed by others in the context of doing mathematics. Therefore, a mathematics identity is expressed in narrative form as a negotiated self, a negotiation between our own assertions and the external ascriptions of others. Math identities are always under construction (McGee & Martin, 2011, p. 1350).

Our research goal is to highlight how much the implemented problem-solving activity has been functional in allowing students to get to know and recognize themselves as mathematicians, to change their vision of themselves and of mathematics, thanks to a greater ability to produce an original mathematical meaning.

### **Methods**

To address our research goal, we analysed the answers provided and the essays produced by second year students of the degree course in mathematics who participated in planned activities designed in such a way to develop mathematical creativity to produce original mathematical meanings. Authors faced the issue of promoting the development of problem-solving competencies through structured opportunities to practice and become aware of one’s cognitive and affective processes.

### **The experiment**

The methodology applied in the activities, focusing on problem solving and creativity, arises from the intertwinement between the constructs of Digital Interactive Storytelling in Mathematics (DIST-M) (Albano et al., 2021) and Thinking Classroom (Liljedahl, 2016). DIST-M originates from theories on storytelling and digital storytelling, and is developed by organizing the cognitive functions involved during the resolution of a problem into cognitive roles (Boss, Promoter, Peste and Blogger), each of which is represented by a character that students play within story-problems that they are

asked to solve in a digital environment (Albano et al., 2021). On the other hand, Thinking Classroom (TC) is conceived and developed by Liljedahl (2016, p. 364) and consists of

a classroom that is not only conducive to thinking but also occasions thinking, a space that is inhabited by thinking individuals as well as individuals thinking collectively, learning together and constructing knowledge and understanding through activity and discussion.

In a TC, students solve appropriately chosen problems on vertical blackboards arranged along the walls of the classroom, working in randomly organised groups. The Thinking Group model (TG) inspiring the design originated from the intertwinement of the TC model (Liljedahl, 2016) with the DIST-M one (Albano et al., 2021) through the personification in a group of the cognitive functions being activated when a mathematician faces a problem. “A Thinking Group is a TC in which each student performs an assigned cognitive role, as a Solver or as an Onlooker” (Miranda, 2022). Metacognitive, cognitive, and affective impact of the activities on students’ learning were investigated and gave some satisfying outcomes (Albano et al., 2022; Miranda, 2022).

Albano et al. (2021) outlined some mental processes that mathematicians should activate when solving a problem (e.g., looking for paths, questioning themselves, organising themselves, systematising the findings...), and identified each cognitive function with a cognitive role to personify. In the model, each student had a specific role in relation to which specific actions/processes had to be performed according to the cognitive function they personified: *Boss*, *Promoter*, *Critical mind*, *Blogger*. Our design foresaw any cognitive role, corresponding to a specific cognitive function, played on two levels of engagement in the problem-solving process: the solving level and reflecting level. At the solving level, the Solver group was devoted to collectively solve the problem by acting according to the cognitive function to perform. At the reflecting level, an Onlooker group observed how the Solver group was working by reflecting on how a specific student acted with respect to both the mathematical problem and their individual role. According to the TC model (Liljedahl, 2016), the activities took place in a university classroom, in an environment in which the student can move freely, and in which physical, paper whiteboards were mainly used, on which every sign remained and every gesture took on a non-negligible meaning. Each group was thus engaged in a TG, “a space inhabited by thinking individuals” performing a cognitive role as well as individuals thinking collectively and performing a group role, to discuss and construct the solution in a problem-solving activity. This allowed the groups to learn together and construct knowledge and understanding through activities and discussions.

The experience involved thirty-six students attending the second year of a Bachelor degree in mathematics, within the course Geometry III, held in University of Salerno during the period from March to June of the academic year 2021-2022. Besides acquiring content knowledge, dealing with the fundamental concepts of general topology, the main educational goal was to construct students’ mathematical reasoning skills, through analysing and exploring problems, with an efficient use of topology concepts and results. The experimental design provides that the participants have been split into four groups, named WG1, WG2, WG3, WG4, each consisting of two subgroups, a Solver and an Onlooker. Each student has been associated with a role-pair (subgroup role, individual role). The values assumed by ‘individual role’ correspond to the cognitive functions performed through a

cognitive role while those assumed by collective role acted as Solver or as Onlooker. Along the course, students have been involved in four activities CWi ( $i=1,2,3,4$ ). To foster the awareness of all the cognitive functions, the students changed roles with each new activity, also changing between Solver and Onlooker groups and permuting the Onlooker groups, and simultaneously, the problem to be focused on, according to a planned calendar.

### A sample of task

The tasks are designed to promote students' construction of knowledge by themselves, going from the example generation, to acquire concepts, to the free production of conjectures and proofs. The tasks may deal with the notions of boundary, continuity, compactness, or connection. In every CWk activity, the structure of the problem-solving task consisted of three problems (see Figure 1): the first two concerned a given definition and required the construction of examples satisfying that definition under given constraints; the third one asked to provide some characterisation related to the property that was being investigated. Figure 1, dealing with the notion of topological continuity, gives a flavour of the kind of problems the students were asked to face.

#### PROBLEM n.1

- 1.a Construct two topological spaces  $(S, \tau)$ ,  $(S', \tau')$  where  $S$  and  $S'$  are non-empty sets,  $\tau$  and  $\tau'$  are topologies on  $S$  and  $S'$ , respectively, different from those studied during lessons (invent them !!), such that there exist a function  $f_1 : (S, \tau) \rightarrow (S', \tau')$  which is continuous and a function  $f_2 : (S, \tau) \rightarrow (S', \tau')$  which is not continuous.
- 1.b Consider the topological spaces you have defined in the previous problem (at point 1.a). Denote with  $\tau_1$  the topology  $\tau$ , with  $\tau_2$  the topology  $\tau'$ , with  $\tau_3, \tau_4$  the *trivial topology* on  $S$  and the *trivial topology* on  $S'$ , respectively, and with  $\tau_5, \tau_6$  the *discrete topology* on  $S$  and the *discrete topology* on  $S'$ , respectively. Determine if  $f : (S, \tau_i) \rightarrow (S', \tau_j)$  is continuous varying  $i \in \{1, 3, 5\}$  and  $j \in \{2, 4, 6\}$ .

**Figure 1: Problem n.1, Task CW2 - Topological continuity**

### Data collection

All the data concerning the learning activity have been digitally stored via a Moodle platform set up for the course: 1) two Google Docs at the end of each activity, one personal and the other one shared with the subgroup, to report the reflections concerning the individual roles played and the collective mathematical process applied by the subgroup, respectively; 2) a questionnaire related to the experience students lived that they were required to answer at the end of all the activities.

### Data analysis

Starting from the collected data, we focus our qualitative analysis on the nuances to which students perceived their identity as mathematicians (students' self-perception of mathematical identity) looking at students' answers to the following questions, selected from the questionnaire: *What sensations did you feel during the activities? Have they changed over time? In which way? Beyond the exam, what did you have this experience leave you? Would you recommend a friend of yours to have a similar experience? Why? Tell us.*

We considered the students' answers to all the above questions as stories about the student that the students tell themselves and analysed them according to the definition of mathematical identity considered in our study. More in detail, we looked for whether changes emerged in the aspects

characterising the mathematical identity. To do this, we identified four themes, characteristic of the definition of identity considered, and we went to observe the occurrence of these components in the answers provided by the students. We called these themes: *cooperative working*; *comparison with others*; *problem-solving skills*; *creativity*.

*Cooperative working* refers back to the aspect of participation and the abilities which come from it; *comparison with others* is related to the negotiation as the process of identity construction between an individual's self-understandings and others "ascriptions"; *problem-solving skills* are linked with using mathematics as a tool to enhance a person's conditions of life. Recent developments on the study of the creative process in mathematics have highlighted the link between creativity and problem solving, in particular Levenson defines mathematical creativity as: "a disposition to promote the ability to generate several solutions and solution paths, to change directions of focus, and to produce novel and original solutions" (Levenson, 2022, p. 470). In this way, problem solving and creativity have become, over time, part of the aspects characterizing the identity of the mathematician. This led us to also consider *creativity* as a theme in our analysis. Obviously, these themes are not clearly separated from each other, in the sense that each captures nuances on the impact of the activities experienced on the perception of student's identity *in fieri*.

## Findings

The first responses that caught our attention were always accompanied by accounts of improvement in one's 'perceived mastery' and characterised by positive emotions:

- S35: I felt like a budding little mathematician, it was fun.
- S15: These activities promote learning in an active way and make you understand how a mathematician works.
- S2: It helped me to have a new view of things, a vision that I perceived more mature in mathematics in general.

In many of the students' answers, the theme of *comparison with others* emerges strongly: the construction of one's own identity as a "future mathematician" takes shape in a continuous exchange with others, therefore also through a "negotiation" with others.

- S19: The best part of this kind of activity was that I was able to engage with colleagues with whom I am not particularly familiar, confronting opinions outside my "comfort zone" was a very valuable experience for me in terms of growing as a future mathematician.

For S19 it seems that this continuous confrontation, from an emotional point of view, has led to the leaving of their "comfort zone", therefore a sort of temporary emotional imbalance, but recognised by themselves as a growth moment. It seems that signs of a transition towards their mathematics identity are beginning to appear. From S6's words the intensity of the discussions and comparison emerges, which in the perception of the student has contributed to the deeper understanding of the mathematical contents covered in the course:

- S6: Thanks to the CW activities, I was able to compare myself with my different colleagues and this allowed me to better understand the concepts covered in class. Very often during the activities, real debates arose as each of us strongly believed in our own ideas. Thanks to these debates, many observations arose that turned out to be very interesting.

In S13 and S21's words it seems to emerge how the comparison with others, in the process of building one's own mathematical identity, has favoured a change in the perceived competence. In the case of S13 there is the overcoming of "shyness" and "fear of making mistakes". For S21 it seems as if their perceived competence grew thanks to the perceived competence of the other groupmates, more confident than they were initially.

S13: It allowed me to get involved and bring out my skills even in moments or situations where I would have avoided interventions due to shyness or fear of making mistakes. This experience has helped me a lot in this respect; in fact, I feel I have grown a lot in this respect.

S21: [...] There were moments when the paths we chose didn't get us anywhere, we were stuck on the ideas proposed and although my idea was to change strategy, they [the classmates] were persevering and confident in their abilities, so slowly we were able to find and try the various solutions.

In S14 and S8 protocols positive emotions emerge, closely linked to the content of the mathematical activities. Both speak not only of content knowledge but S14 of formalizing one's ideas and S8 of construction of reasoning and communication competencies, aspects recalling the fundamental competencies for the construction of mathematical competence (Niss, 2003).

S14: I previously mentioned my initial scepticism due to my introverted nature. Contrary to what I expected, however, I really enjoyed comparing myself with my teammates, listening to the ideas of others and looking for a clear formalisation of my own ideas so that the other team members could understand them.

S8: [...] it is not only about learning the subject and thus preparing for the exam, but it was an opportunity to test one's reasoning skills, and communication skills with other group members, as well as an opportunity for cultural 'exchange'. It reminded me how fun and at the same time productive it is to work with others.

In many of the students' answers, the theme of *cooperative working*, that is how group work has influenced self-perception emerges. In S22 working with others seems to help overcome lack of self-confidence and, therefore, in a certain sense to improve the sense of self-efficacy. This seems to lead to a perception of improvement in one's "mathematical creativity" (the "intuitions", the "conjectures" seem to come more spontaneously) (Liljedahl, 2013), accompanied by positive emotions.

S22: Working in a group was enjoyable from the beginning: in spite of the initial insecurity, it was easy to feel that if one did not arrive, the other was ready to step up. Insights, conjectures then almost always came spontaneously, and this created an astonishment that gave the CWs their fascination.

Looking closely at an excerpt of the collective document describing the solving process of WG4 Solver Working Group to task CW2 (Figure 1) as elaborated by the Blogger, we can observe that, to answer the first problem, they considered  $\mathbb{C}$  as a support for their topology, thus placing themselves in a new and unexplored environment showing an improvement of the attitude towards the complex case, together with greater confidence in the creation of mathematical objects.

[...]Our support is therefore the set  $S=\mathbb{C}$ . We have also defined an ordering, since the set  $\mathbb{C}$  is not ordered. Let  $x$  and  $z$  be two complex numbers  $x = a + ib$  and  $z = c + id$ , and we have that  $x < z \leftrightarrow \sqrt{a^2 + b^2} < \sqrt{c^2 + d^2}$  and we have hence defined the distance that associates the real positive number  $\sqrt{(a - c)^2 + (b - d)^2}$  and we considered the topology induced by the metric described above. Let  $T1$  be the topology. A set  $A$  belongs to  $T1$  iff  $\forall x \in A, \exists r > 0$  s. t.  $S(x, r) \subseteq A$ . [...]The

promoter therefore suggested another topology T2 defined as follows: a set  $A$  belongs to T2 iff  $x = a + ib \in A \Rightarrow -a - ib \in A$ , and urged by the critical mind he went to verify that this was actually a topology.

The *creative* act implemented by WG4 continue to manifest in the process of building the functions required by the task.

[...]We must therefore find two functions between the two spaces, one continuous and one non-continuous. The blogger therefore deemed it appropriate to recall the definition of continuous function, i.e. a function is said to be continuous when the counterimage of each open set of the codomain is an open set of the domain. We note that the identity map with domain in  $(\mathbb{C}, T2)$  and codomain in  $(\mathbb{C}, T1)$  is continuous. In fact we note that  $x$  and  $-x$  have the same norm and consequently if  $x$  belongs to a spherical neighborhood of radius  $r$ , also  $-x$  belongs to that neighborhood, so the counterimage of an open in T1 is a set  $A$  s. t.  $\forall x \in A \Rightarrow -x \in A$  and therefore it is open in T1. The promoter then followed the advice of one of the gurus who suggested us to exploit a piecewise function, and proposed a function with domain in  $(\mathbb{C}, T1)$  and codomain in  $(\mathbb{C}, T2)$  which associates 0 to  $x$  if  $a + b \in Q$  and 1 if  $a + b \in R \setminus Q$ . This function is not continuous, in fact the counterimage of  $\{-1,1\}$  is  $\{x = a + ib \in \mathbb{C} \text{ s. t. } a + b \in R \setminus Q\}$  which is not open in T1.

The theme of *problem-solving skills* emerges also in the protocols of S5 and S30. Students recognise that to face a problem in a Thinking Group setting is an opportunity for personal growth in both their self-regulated learning and emotional engagement:

- S5: Many things remain from this experience: one among all, a great opportunity for growth (both from a personal point of view and in relation to collective interaction and from the point of view of learning the subject and the most disparate forms of problem solving). And then a lot of fun, a lot of emotional involvement.
- S30: [this experience left] Having internalised the importance of comparing and working in group, as well as the development of the ability to solve a problem, through elaborating a strategy based on the tools at our disposal.

In some students' protocols it seems to emerge very strongly a rethinking of themselves as mathematicians, as problem solvers, as if activities of this kind gave free expression to *creativity* in "doing mathematics", with positive emotions accompanying a surprising self-discovery:

- S14: At the end of the activities I realize the effectiveness of this experience as it immerses you completely in the solution process, bringing you [...] to a creativity that you would not have touched otherwise. A similar experience leads you to spontaneously question the validity or otherwise of what you study, also looking for real evidence.
- S27: I would recommend doing such an activity because, if done in the right way, it is able to bring you into a new light and make you discover that you are more intuitive than you thought.

## Conclusions

The study we conducted is part of a wider research project concerning the development of cognitive, metacognitive and affective skills in problem solving at university level. For this purpose, suitable teaching activities focused on the work of Thinking Groups were designed and tested (Miranda, 2022). From the students' answers it seems that these activities have changed their awareness of perceiving themselves as mathematicians: immersing themselves in these activities has allowed them



to become aware of the meaning both of “doing mathematics” and “being a mathematician”. This is often accompanied by a change in the attitude towards mathematics and in the perception of their abilities in the creation of mathematical meanings. This is only the first step of an ongoing study on mathematics identity that could go in several directions, as for example, deepening how creativity affects learners’ and teachers’ identity and further analyses on mathematical identity with respect to other definitions in the literature.

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