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First-year students' commognitive conflicts concerning sequences and functions

Nikoletta Palamioti¹, Theodossios Zachariades¹, Irene Biza² and Despina Potari¹

¹National and Kapodistrian University of Athens, Greece; nikipalam@math.uoa.gr

²University of East Anglia, Norwich, UK

We present part of an ongoing study aiming to explore the discursive activity of first-year students in Calculus. We focus on evidence of students' commognitive conflicts regarding the realisation of sequences. These commognitive conflicts emerged from contradictory narratives regarding the transition from school to university mathematical discourse. From the data of our study, contradictory narratives emerged about the graph of a sequence and the relationship between the functions and sequences. The different narratives and routines concerning sequences and functions between school and university discourses lead to the emergence of commognitive conflicts.

Keywords: Commognitive conflict, narratives, sequences, functions.

Introduction

Tertiary mathematics education research has attracted increasing attention in recent years, focusing on students' transition from school to university environments, as well as the learning and teaching in university. According to Sfard, university context may present challenges for first-year students as the "words fail to make much sense" (2014, p.199) in terms of the context of lectures, the interactions between learners and teachers as well as the mathematics itself. In particular, the mathematical discourses at the university differ from those at the school level where differences depend on the educational contexts (Thoma & Nardi, 2018); as Sfard (2014) mentions, students need to shift to the new discourse. In addition, as Biza (2021) argues, first-year students' previous experiences have an effect on their subsequent work with mathematics. Thoma and Nardi (2018), studying the transition from school to university, highlighted a conflict between students' algebra and university discourse about Set theory since students recalled the school algebra in a Set Theory task in university. Concerning calculus, Güçler (2013) explored the first-year students' discursive shifts about the limit, and Güçler (2016) studied the learners' realisation of the function as learners conceived it either as a process or as an object. In Greece, where this study is conducted, function is an essential mathematical object in the school mathematical discourse, in contrast to sequences. However, in university discourse there is an implicit shift and the object of sequence becomes fundamental. This shift might influence students' mathematical discourse (Güçler, 2013; 2016) creating a need for further research on students' discursive activity about functions and sequences as they shift to university from school discourses. In our study, we focus on first-year university students' discursive activity concerning functions and sequences during the transition from school to university mathematics.

Conceptual Framework

In this paper, learning and thinking are approached from a participationist and discursive perspective drawing on the Commognitive Framework (Sfard, 2008). According to this framework, *thinking* is seen as the "individualized version of interpersonal communication" (ibid, p.81), *mathematics* as a

discourse, *mathematical learning* as the initiation into the discourse of mathematics and learners' substantial discursive shifts, and the *teaching* of mathematics as a facilitator of these shifts. In this way, Sfard considers mathematical communication to be based on the realisation of, and incessant transition between, signifiers, such realisations are "perceptually accessible entities" (2008, p. 155). Signifiers' realisation can be of a different type, such as verbal (spoken words) and visual (e.g. written words, symbols, images, or gestural).

Under this framework, mathematics, as a discourse, can be described through four specific characteristics: *word use* as the way that a word is used in the discourse (e.g. how the word *function* is used); *visual mediators* as the non-verbal means of communication (e.g. graphical representation of a function, symbols); *endorsed narratives* as the set of utterances describing mathematical objects and their relationships (e.g. definitions, mathematical terminology); and, *routines* as the repetitive activities-practices that someone does (e.g. proving, defining, graphing) (Sfard, 2008). The mathematical discourse is established in a community and reflects the practices of this community. In this sense, the school mathematical discourse differs from the university mathematical discourse even in the same mathematical area. Three characteristics distinguish university mathematical discourse from that of school:

first, this [university mathematical] discourse's extreme objectification; secondly, its reliance on rules of endorsement that privilege analytic thinking and leave little space for empirical evidence; and thirdly, the unprecedented level of rigor that is to be attained by following a set of well-defined formal rules. (Sfard, 2014, p. 200)

These characteristics are essential regarding students' initiation into the university mathematical discourse, especially in their transition from school.

Of special interest in the transition between discourses is the different meaning of the same signifier within those discourses (e.g. has the word *function* the same meaning?). Sfard mentions that different discussants might "use the same mathematical signifiers (words or written symbols) in different ways or perform the same mathematical tasks according to different rules" (2008, p.161). Such mismatch may create discursive conflicts, *commognitive conflict*. In other words, a commognitive conflict occurs when conflicting narratives originate from incommensurable discourses (Sfard, 2008). For example, seeing a sequence as a function might be in conflict with narratives of functions that are always continuous and defined in an interval of real numbers. Tabach and Nachlieli (2016) point out that meta-level discussions – namely a discussion about the rules that underpin the discourse – about the use of words reinforce the process of resolving commognitive conflicts. In this process, discussants need to (a) recognize a disagreement that originates from using words differently, (b) explicitly express their ways of using the word, (c) listen to other uses and identify the differences in expressed uses, and finally (d) agree upon one acceptable use, which should be that of an expert or an insider to the discourse (Sfard, 2008).

In acknowledgment of learners' previous discursive experiences, Lavie et al. (2019) redefine routines as task-procedure pairs and emphasize the role of previous experience in a task situation. When a learner faces a new task situation, she returns to precedent events, namely, past situations that she interprets as similar to the new one and repeats what was done then (Lavie et al., 2019). Precedents

arise from the bulk of learner's previous discursive experiences that shape the *precedent-search-space*. In the transition from school to university such space is shaped not only by learners' experiences at lectures but also by their school curricular experiences – what Biza (2021) calls *discursive footprint*. A mismatch between school and university discourses can generate conflicts which in this paper we study in the case of functions and sequences through the research question: *What commognitive conflicts emerge when first-year university students deal with sequences?*

Methodology

Context of the study

In the Greek educational system, students who are planning to continue with tertiary studies on mathematics participate to a national examination at the end of Grade 12. The mathematical component of this examination includes elements of Calculus that cover limits and continuity of functions, derivatives and integrals with an emphasis on computations and methods. According to the school curriculum, the school narrative promotes that function is a correspondence between two sets of real numbers with an interval or a union of intervals as a domain. However, in most of function cases students engage in, the domain is an interval of real numbers and the function graph is an uninterrupted continuous curve. Overall, the emphasis is on computations (e.g., calculations of limits and derivatives) rather than on the production and interpretation of function graphs. In the Grade 10 algebra syllabus, students have the opportunity to be introduced to sequences as a correspondence from natural to real numbers in the context of arithmetic and geometrical progressions. Again, not so much emphasis is given either to sequence graphs or to connections between functions and sequences.

In contrast, in mathematics undergraduate studies at the institution in which this research has been conducted, first-year Calculus is a proof-based compulsory course with elements of Real Analysis. During the first semester, students are introduced to the foundations of real numbers and define functions, sequences, limits and continuity on the grounds of these foundations. In contrast to school, in this course, a sequence of real numbers becomes an essential object for the first time. In addition, the domain of a function can be any subset of real numbers and not necessarily an interval or a union of intervals of real numbers. Moreover, the limit and continuity of function are based on the ε - δ definition of limit and ε - δ definition of continuity, respectively.

Methods

In this paper, we present preliminary findings of the first author's ongoing doctoral study that focuses on the mathematical discourse that first-year undergraduate mathematics students engage in while participating in lectures, seminars and using course materials. The study is a multiple case study of 14 first-year mathematics students (8 female - 6 male, average age 18) who attend a first-year Calculus course at a university in Greece. At the beginning of the academic year, all first-year students were invited to complete a questionnaire about their school and mathematical background. The questionnaire consisted of fifteen questions about students' school experience (e.g. why they decided to study mathematics, what their school mathematical study practices were) and three mathematical questions (e.g. Let $A \subseteq \mathbb{R}$. What is a real function with domain A ?). All the students who met the following two criteria were invited to participate in the study: 1) be first-year students enrolled in the course during the autumn semester (i.e. students who retook the course were excluded) and 2) express

in the questionnaire their willingness and availability to participate in the study. The researcher (first author) divided the 14 students who accepted the invitation into three groups to ensure that students in each group had diverse mathematical and school backgrounds based on the information gathered by the questionnaire. Each group had five research sessions in which students discussed mathematical topics related to the course that were initiated by the researcher. The sessions were based on pre-prepared questions and tasks inspired by incidents from the lectures and research in university mathematics education focused on Calculus. In each session, the discussion was initiated by the researcher with a question inviting students' narratives around a mathematical object, such as "What are your thoughts about sequences?". Tasks required proving a proposition, commenting on a proof, or solving exercises from the course textbook. The role of the researcher during the sessions was to pose questions, to encourage discussion without confirming students' mathematical assertions, and to prompt further explanations. Students were invited to work in their groups without researcher intervention. During the group discussion, students were reassured that their work would not be assessed and were able to express themselves freely either verbally or by writing. The meetings were scheduled two to three weeks after students had been taught the relevant mathematical objects in lectures to ensure connections between the issues discussed in the sessions and the lectures.

The data for this paper are taken from one episode that took place at the second meeting of a group of four students: Elena, Myles, Mat, and Ivy. The session was about the mathematical object of sequence and its convergence. Students' activity during the group discussion was audio recorded and transcribed, and their written answers were collected. This episode on sequences has been chosen because of the mismatch between what students had experienced about sequences and functions in school and what they were experiencing in the university Calculus course. We note that students had already been taught in university lectures about sequences and continuous functions at the time of this session. The analysis of the episode first considered the objects of function and sequence in the school and university discourses, respectively. Then, students' verbal utterances and written communications were characterized in the light of school and university discursive practices. Word use (e.g., continuous, sequences, functions) was seen in relation to visual mediators (e.g., function graphs) in order to identify students' narratives and routines. This led to the identification of contradictions that may evidence commognitive conflicts within individualized discourses or within the group (e.g., "continuous" might mean either continuous function or uninterrupted graph).

Results

The episode, which has been translated from Greek into English, started when the researcher asked students "What are your thoughts about sequences?". Myles responded that "... a sequence is a function from natural numbers to real ones" and Mat and Ivy agreed with him. It seems that most of the students have endorsed the definition of sequences as it was introduced in the lectures. On the contrary, Elena expressed her confusion stating "Probably I do not understand the sequences [like others] as I was thinking something entirely different" and she claimed that sequence is "like a curve... which goes to the infinite and has neither end nor beginning" and she drew an uninterrupted curve (Figure 1).

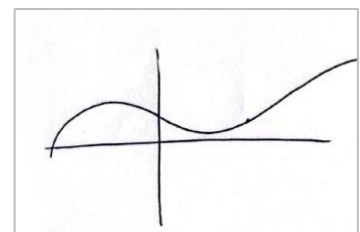


Figure 1: Elena's graph (1)

Afterward, the researcher encouraged all students to provide the graph of a known sequence and Elena proposed drawing the graph of $a_n = \frac{1}{n}$, which all students did (Figures 2-5). We see those drawings as students' visual realisation of sequences against the verbal one provided by Myles earlier.

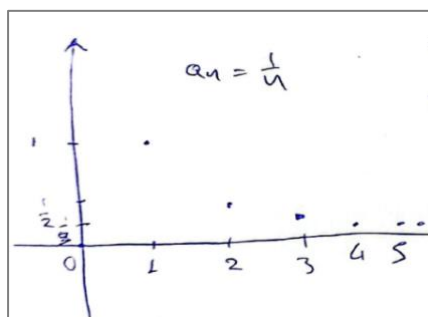


Figure 2: Mat's graph

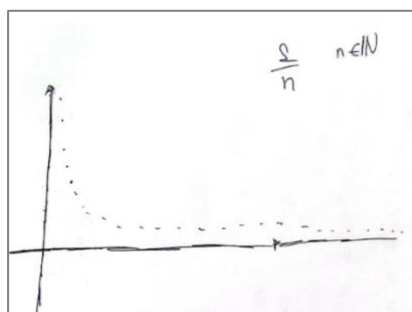


Figure 3: Ivy's graph

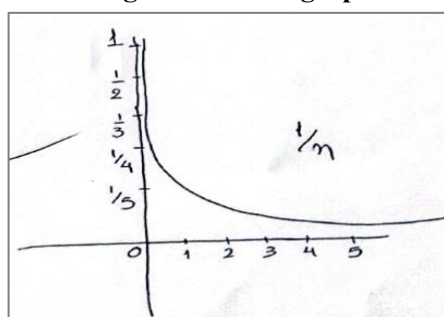


Figure 4: Elena's graph (2)

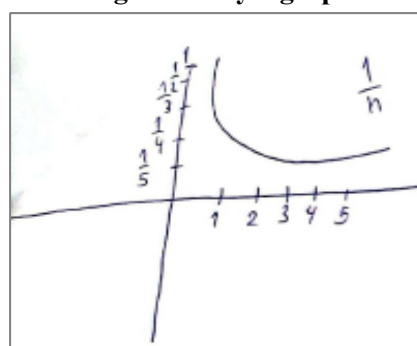


Figure 5: Myles' graph (1)

On the one hand, Mat and Ivy drew the sequence as a discrete point curve. In particular, Mat's graph (Figure 2) provides a point-wise view of the sequence since he represents the correspondence of the domain to the co-domain of the sequence. In contrast, Ivy drew (Figure 3) a dotted curve without individual coordinates. We interpreted Ivy's graph as a holistic view of $a_n = \frac{1}{n}$ with strong resemblance to the graph of the function $f(x) = \frac{1}{x}$, which is known from school. In addition, we consider Mat's and Ivy's graphs as evidence of different graphing routines probably originated both from their school and university experiences: point correspondence for Mat and holistic realisation of the curve for Ivy.

On the other hand, Elena (Figure 4) and Myles (Figure 5) drew an uninterrupted curve in the first quadrant of the axes. The graphs of the sequence $a_n = \frac{1}{n}$ are very similar to the graph of the function $f(x) = \frac{1}{x}$ when $x \in (0, +\infty)$ suggesting that their graphical realisations of sequences and functions are very similar. In addition, Elena and Myles represent the domain and the co-domain of the sequence in the xx' and yy' axis respectively. However, they did not take into account in their graphs that the domain of the sequence is natural numbers, indicating that their realisation of the sequences' domain and the domain of functions that they experienced in school are similar.

Later, when Myles saw Mat's and Ivy's graphs, he recalled the $(-1)^n$ sequence that they had seen in lectures: "I think I got it. When we discussed convergence, the $(-1)^n$ does not converge. It is 1 or -1. Therefore, it is with dots. One dot to 1, one dot to -1". Afterward, Myles drew the sequence $a_n = \frac{1}{n}$ as a graph of distinct points (Figure 6) stating that

Therefore in the graph, the dots should be only in natural numbers... not everywhere, it should be simply dotted. Right? That is what I understand. Be only one to one, two to one half, three to one-third... so the dots must be far from each other.

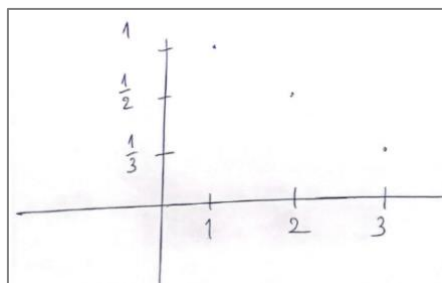


Figure 6: Myles' graph (2)

We interpret that there is evidence of commognitive conflict between Myles' endorsed narrative about the definition of sequences and the visual mediator (Figure 5) that he proposed as a realisation of sequences. However, Myles realised the conflict when he came across Mat's and Ivy's graph leading him to recall a case of sequence that was familiar from lectures so his revised visual mediator (Figure 6) took into account the correspondence of natural numbers to real ones. We conceive that the example $(-1)^n$ is the mediator for the realisation of sequences as a distinct point curve since its co-domain is a set of two elements, a countable subset of real numbers. Therefore, his narrative concerning the definition of sequences and his visual mediator seemed to be aligned with the university discourse about sequences, indicating evidence of shift to the university discourse. The fact that the presented example comes from students' experiences from university lectures reinforces such shift.

Afterward, the dialogue below followed.

171. Elena: I just want to say... I do not know if it is relevant but the $(1/n)$ is considered as a continuous function. Isn't it? Therefore, why do we distinguish it when is a sequence and when is a function since it is always continuous?
172. Myles: I would like to say ... Now there are three [he referred to the three points in Figure 6] ...so it should be point by point?
173. Mat: Yes.
174. Ivy: Otherwise, if it was a continuous line, why are we talking about a sequence and not a function?
175. Myles: But the sequence is a function. Isn't it?
176. Ivy: Yes, but... I am not sure... In a function, we are talking about an interval, as I understand, of numbers ...
177. Elena: Yes...
178. Ivy: Like this [Figure 6], from one to three ... we say now for an interval, so it takes all the intermediate.
179. Interviewer: Do you refer to the domain of the function?
180. Ivy: Yes, the domain of the function. So, the x or n , any variable that we choose... A sequence gets a set of numbers that differs... Here [points to the graph in Figure 6] it takes the one, two, three, and four... It is more specific, it ends up in a specific [element of the set].

In the extract above, the use of the words "sequence" and "function" are used with different meanings in Elena's and Ivy's responses indicating different narratives. In [171] Elena uses the words "sequence" and "function" as realisations of the same signifier since she does not recognize any

difference between the two mathematical objects. On the other hand, in [174] Ivy seemed to use the words “sequence” and “function” as realisations of different signifiers. Specifically, Ivy uses the word “function” firstly to describe that function as an uninterrupted graph [174] since she relates it to the “continuous line” and then in [176 & 178] to relate the graph with the domain which is an interval, as she was familiar with from school. In [180] the word “sequence” signifies that the domain is the natural numbers. As a result, the word used indicates evidence of a commognitive conflict between Elena’s and Ivy’s narratives about sequences and functions.

Moreover, students use the word “continuous” differently, creating contradictory narratives. In [171] Elena refers to the sequence as a continuous function, meaning the continuity of the sequence. In contrast, in [174] Ivy claims that the function is a continuous line, denoting that the graph is uninterrupted.

Afterward, Elena observed Myles’ new representation (Figure 6) stating that

“Yes...If what Myles said is ... If it is true from natural numbers to real ones, then we have done it wrong and it is not a line. ... Right? Why do I strongly believe that the sequence has to do with the continuity?”

In this extract, Elena seems to realise the contradiction in her narrative and her visual mediator (Figure 4), influenced by Myles’ graph (Figure 6). However, Elena uses the word “continuous” relating the sequences with the continuity of the functions, without being explicit about what this word signifies so her conflict remains unresolved.

Discussion

From the above analysis, evidence of commognitive conflicts emerged regarding sequences and functions generated by drawing a graphical representation of a sequence. As the literature mentioned, first-year students need to shift from school to university discourse (Sfard, 2014; Thoma & Nardi, 2018). Our analysis points out that the translation of the endorsed narrative of the sequences into a visual realisation indicates evidence of commognitive conflict and that the discursive footprint was manifested in students’ narratives, visual mediators, and routines (Biza, 2021). In addition, the different use of the words “function”, “sequence” and “continuous” in school and university discourses indicates evidence of commognitive conflict. Particularly, the use of the word “function” in the definition of sequences and students’ school experience about functions might lead to a commognitive conflict between visual mediators related to sequences and functions. The use of the word “continuous/continuity” signifies a conflict between the graph being uninterrupted and the continuity of the function. Moreover, students’ routines that concern drawing graphs of sequences are influenced by the routines of drawing graphs of functions as students engage in school discourse. As a result, students’ precedent events have an effect on the shift to the university discourse, as mentioned by Lavie et al. (2019).

Furthermore, we observed that students who initially drew the sequence as an uninterrupted curve produced different narratives as the discussion evolved. Myles seemed to shift into the university’s discourse, influenced by the fact he came across different visual mediators, such as Mat’s and Ivy’s. In addition, Myles’ shift was encouraged by recalling the example of the sequence $(-1)^n$ from the

lectures, whose formula cannot be misinterpreted as a function formula known from their school experience, so the precedent events from the university reinforce his shift.

In contrast, Elena did not shift to the university discourse. Elena's narrative seems to be influenced by the school discourse since she does not consider the difference in the domain of the sequence and function. Moreover, the contradictory use of the word "continuous" in her narrative evidences her conflict, which can be based on both the fact that she had been taught the continuity of functions in lectures and the sequences are continuous functions based on the ε - δ definition of continuity, according to the university curriculum. It seems that Elena's commognitive conflict stays unresolved.

In conclusion, the above findings illustrate that the role of the precedent events was either to reinforce the resolution of a commognitive conflict or to reinforce the creation of one. We propose further research that investigates the role of the discursive footprint in the creation of commognitive conflicts and the resolution of those in Calculus in the first year of tertiary studies.

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