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# Novel ultrahard carbon allotropes from crystal chemistry and first principles: rhombohedral ene- $\mathrm{C}_{21}$ and yne- $\mathrm{C}_{24}$ 

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#### Abstract

Rhombohedral ene $-\mathrm{C}_{21}$ and yne- $\mathrm{C}_{24}$, characterized by the presence of $\mathrm{C}=\mathrm{C}$ and $\mathrm{C} \equiv \mathrm{C}$ bonds, respectively, inserted into the 9 R diamond polytype $\mathrm{C}_{18}$, are proposed as novel ultrahard carbon allotropes from crystal chemistry and first principles. Like $9 \mathrm{R} \mathrm{C}_{18}$, they belong to the cfe topology characteristic of layered SiC polytypes. With ultrahard properties approaching those of diamond and lonsdaleite, ene $-\mathrm{C}_{21}$ and yne $-\mathrm{C}_{24}$ are dynamically stable with phonon signatures identifying $\mathrm{C}=\mathrm{C}$ and $\mathrm{C} \equiv \mathrm{C}$ high frequency vibrations similar to molecular allene and acetylene. The electronic band structures correspond to insulating $\mathrm{C}_{18}$, metallic ene- $\mathrm{C}_{21}$ and semiconducting yne- $\mathrm{C}_{24}$.


Keywords: diamond polytypes; hybridization; DFT; hardness; phonons; electronic structure

[^0]
## Introduction

Diamond, known mainly in the cubic form, has a less common hexagonal form called lonsdaleite. Such structures are also observed in binary SiC , and the different phases are called polytypes, which are distinguished by the stacking of layers, usually labeled $\mathrm{A} / \mathrm{B} / \mathrm{C}$ : AB in 2 H (2 layers; hexagonal system), ABC in 3C (3 layers; cubic system), ABCB in 4H (4 layers; hexagonal system), etc. [1]. Silicon and carbon are isoelectronic with respect to the valence shell, and diamond has analogous polytypes, with the best known being 3 C , which corresponds to cubic diamond, and 2 H , which corresponds to lonsdaleite. Other diamond polytypes: 4H, 8H, 9R to 21R ( R for rhombohedral) have been identified with calculated X-ray diffraction data published by Ownby et al. in 1992 [2]. Recently, it has been shown that a novel class of machine-learning-based interatomic potentials can be used for random structure search and readily predicts several hitherto unknown carbon allotropes [3], allowing them to be identified with topology labels such as dia (diamond), lon (lonsdaleite), and for polytypes: cfc for 4H diamond, cfe for rhombohedral 9R. In view of the enormous number of carbon allotropes, experimentally found and theoretically predicted, a database "SACADA" has been created to store them with topologies and identifiers; 9R allotrope ( $\mathrm{hRC}_{18}$ in Pearson notation) studied here and classified with cfe topology and database code: 447 [4].

In 3 C and 2 H diamond, as well as in the extended lattice polytypes, the valence 4 electrons $\left(2 \mathrm{~s}^{2}, 2 \mathrm{p}^{2}\right)$ arrange in tetrahedra with $\mathrm{sp}^{3}$-like hybridization; note that this term, as well as other carbon hybridizations ( $\mathrm{sp}^{2}$ and $\mathrm{sp}^{1}$ ), are common in organic chemistry, and less so in the solid state chemistry; nevertheless, they are useful for establishing the chemical description of the bonding and the resulting physical and chemical properties.

For applications such as electronics, the physical properties of diamond can be modified by introducing trigonal $\mathrm{C}\left(\mathrm{sp}^{2}\right)$ - double $\mathrm{C}=\mathrm{C}$ bonds as in ethene - and linear $\mathrm{C}\left(\mathrm{sp}^{1}\right)-$ triple $\mathrm{C} \equiv \mathrm{C}$ bonds as in acetylene - on the one hand, and by working on the particle size as in nanodiamonds, on the other hand. Indeed, recent works have shown that nanodiamonds with mixed $\mathrm{C}\left(\mathrm{sp}^{2}\right)-\mathrm{C}\left(\mathrm{sp}^{3}\right)$ hybridization play an important role in designing advanced electronic materials [5]. Zhai et al. highlighted electrochemical applications of hybrid diamond/ $\mathrm{sp}^{2}-\mathrm{C}$ nanostructures [6]. Furthermore, regarding other mixed carbon hybridizations, $\mathrm{sp}^{3}$-sp ${ }^{1}$ were recently identified in superhard so-called "ynediamond", which is classified as semi-metallic [7].

In this context, we considered the reduction of the $\mathrm{sp}^{2} / \mathrm{sp}^{3}$ extended stoichiometries allotropes, and $\mathrm{C}_{10}, \mathrm{C}_{14}$ and $\mathrm{C}_{18}$ were proposed as derived respectively from $4 \mathrm{H}, 6 \mathrm{H}$ and 8 H diamond polytypes, respectively, and also characterized by ultrahardness close to diamond, and semi-metallic electronic structure due to the presence of $\mathrm{C}\left(\mathrm{sp}^{2}\right)$ [8]. The purpose of the present work was to study the changes in the mechanical and electronic properties of cfe $9 \mathrm{R}\left(\mathrm{hRC}_{18}\right)$ allotrope, referred to as $\mathrm{C}_{18}$ in the present work, by the insertion of $\mathrm{sp}^{2}$ and $\mathrm{sp}^{1}$ carbon atoms in two stoichiometries, based on crystal chemistry rationale and subsequent first-principles ground state energy calculations of stable structures and derived quantities within the well-established quantum mechanics framework of the density functional theory DFT $[9,10]$. The two novel allotropes: ene $-\mathrm{C}_{21}$ with $\mathrm{sp}^{3}-\mathrm{sp}^{2}$ mixed
hybridizations and yne- $\mathrm{C}_{24}$ with $\mathrm{sp}^{3}$ - $\mathrm{sp}^{1}$ mixed hybridizations retain the same cfe topology of the original $\mathrm{C}_{18}$. ene- $\mathrm{C}_{21}$ and yne- $\mathrm{C}_{24}$ were found unique from the CCDC curation and provided with corresponding DOI references [11,12].

## Brief presentation of the computational framework

The methodology was developed in earlier work (cf. [8] and the papers cited therein). Essentially, the search for the ground state structures with minimal energies was performed with unconstrained geometry optimizations within the DFT-based plane-wave Vienna Ab initio Simulation Package (VASP) [13] using the projector augmented wave (PAW) method for potentials [14]. Within the DFT, the effects of exchange and correlation (XC) were treated using Generalized Gradient Approximation (GGA) scheme [15]. Investigation of the mechanical properties was based on the calculations of the elastic properties by performing finite distortions of the lattice and deriving the elastic constants from the strain-stress relationship. The calculated elastic constants $\mathrm{C}_{\mathrm{ij}}$ were then used to obtain the bulk $(B)$ and the shear $(G)$ moduli using the Voigt averaging method [16] based on uniform strain. Besides the mechanical properties, the dynamic stability was determined from the calculations of the phonons and illustrated with the corresponding band structures in the reciprocal space (Brillouin zone) using the "Phonopy" interface code based on the Python language [17]. The structures were plotted using the VESTA graphical program [18]. For the evaluation of the electronic properties, the band structures were obtained with the all-electrons augmented spherical wave method ASW [19].

## Crystal chemistry results

The 9R diamond polytype ( $\mathrm{hRC}_{18}$; space group $R-3 m$ No. 166) is shown in Fig. 1a, in two representations: ball and stick (left) and tetrahedral stacking (right). C4 tetrahedra are corner sharing within 9 layers like 9 -layered SiC polytype. Table 1 listes the structural parameters showing agreement between literature and presently calculated values of the lattice constants and the z values of three ( 6 c ) $0,0, \mathrm{z}$ sites. The shortest C-C separations have two $\mathrm{d}(\mathrm{C}-\mathrm{C})$ values: $1.54 \AA$ and $1.56 \AA$, in contrast to diamond with a unique $\mathrm{d}(\mathrm{C}-\mathrm{C})=1.54 \AA$. Nevertheless, the close relationship to diamond is established by the identical cohesive energy of $\mathrm{E}_{\text {coh }}=2.479 \mathrm{eV} /$ atom.

The introduction of a trigonal $\mathrm{sp}^{2}$ carbon was achieved by inserting of a $4^{\text {th }}$ carbon at the (3a) $0,0,0$ site resulting in a $\mathrm{C}_{21}$ stoichiometry. The resulting constrained structure with short C -C distances was relaxed along successive cycles of geometry optimization until reaching the ground state configuration whose parameters are shown in Table $1,2^{\text {nd }}$ column, characterized by the shortest distance: $1.46 \AA$ assigned to the $\mathrm{C}=\mathrm{C}$ bond, larger by $0.11 \AA$ than that in the ethylene molecule $(\sim 1.35 \AA)$. The perturbation of the 9R diamond polytype ( $\mathrm{hRC}_{18}$ ) leads to a decrease of the cohesive energy $\mathrm{E}_{\text {coh }}=2.061 \mathrm{eV} /$ atom. The structure shown in Fig. 1b exhibits a different stacking of tetrahedra and a less compact structure quantified by the decrease in density from $\rho=3.52 \mathrm{~g} / \mathrm{cm}^{3}$ in
$\mathrm{C}_{18}$ to $3.39 \mathrm{~g} / \mathrm{cm}^{3}$ in $\mathrm{C}_{21}$. Note that the configuration of $\mathrm{sp}^{2}$ carbon in the present ene- $\mathrm{C}_{21}$ with $\mathrm{C}-\mathrm{C}-\mathrm{C}$ motifs is closer to the propadiene $\mathrm{C}_{3} \mathrm{H}_{4}$ molecule, $\mathrm{H}_{2} \mathrm{C}=\mathrm{C}=\mathrm{CH}_{2}$.

To create the carbon allotrope with $\mathrm{C} \equiv \mathrm{C}$ bonds, the (3a) $0,0,0$ position of ene- $\mathrm{C}_{21}$ was changed to (6c) $0,0, \mathrm{z}$ resulting in a $\mathrm{C}_{24}$ stoichiometry. Following the same geometry relaxation protocol as above, the final structure with zero strains is shown in Fig. 1c and the lattice parameters are given in Table $1,3^{\text {rd }}$ column. As expected, the structure is now characterized by a very short C-C distance of $1.21 \AA$, close to that observed in the acetylene molecule. The more open structure in Fig. 1c, with a different arrangement of the $C 4$ tetrahedra, has a low density $\rho=3.13 \mathrm{~g} / \mathrm{cm}^{3}$. The cohesive energy is reduced by an order of magnitude $\mathrm{E}_{\text {coh }} /$ atom $=-1.923 \mathrm{eV}$. The two novel ene- $\mathrm{C}_{21}$ and yne- $\mathrm{C}_{24}$ allotropes have been identified with cfe topology like original $\mathrm{C}_{18}$.

## Charge density projections

A further qualitative illustration of the different types of hybridization is obtained from the projections of the charge densities. Figs $2 \mathrm{a}, 2 \mathrm{~b}$ and 2 c , which reproduce the crystal structure sketches of Fig. 1, highlight the volumes shown in yellow. In diamond-like $\mathrm{C}_{18}$ (Fig. 2a), the sp ${ }^{3}$-type hybridization is clearly observed from the tetrahedral volumes around the C atoms. Like diamond, the rhombohedral 9 R polytype is a perfectly covalent system with similar physical properties (mechanical, dynamic, etc.) and electronic structure as shown below. Changes are observed in the hybrid ene- $\mathrm{C}_{21}$ (Fig. 2b), where the $\mathrm{sp}^{3}$-type hybridization is now accompanied by the $\mathrm{sp}^{2}$-type one observed with the yellow volumes along the C-C-C unit aligned along the vertical hexagonal $c$ direction. In Fig. 2c, representing the projections in yne- $\mathrm{C}_{24}$, besides the $\mathrm{sp}^{3}$ tetrahedral shapes, a remarkable charge density is observed along the aligned C-C-C-C unit with strong localization for the middle C - C clearly featuring the $\mathrm{C} \equiv \mathrm{C}$ triple bond. Therefore, the mixed hybridized novel allotropes are further illustrated by the charge density projections.

## Mechanical properties

The evaluation of the mechanical properties was carried out by calculating the elastic properties by performing finite distortions of the lattice. The elastic constants $\mathrm{C}_{\mathrm{ij}}$ were then derived from the strainstress relationship, assuming statistically isotropic solid on a large scale. The $\mathrm{C}_{\mathrm{ij}}$ were then used to obtain the bulk $(B)$ and shear $(G)$ moduli using the Voigt averaging method [16].

The calculated sets of elastic constants are given in Table 2. All $\mathrm{C}_{\mathrm{ij}}$ values are positive, and their combinations obey the rules of mechanical stability of the system:

$$
\mathrm{C}_{11}>\left|\mathrm{C}_{12}\right| ;\left(\mathrm{C}_{11}+\mathrm{C}_{12}\right) \cdot \mathrm{C}_{33}>2 \mathrm{C}_{13}^{2} ; \mathrm{C}_{44}>0 ; \mathrm{C}_{66}>0
$$

The bulk $(B)$ and shear $(G)$ moduli were calculated from elastic tensors within the Voigt averaging scheme using ELATE software [20]. Diamond-like $C_{18}$ has the largest $B_{V}$ and $G_{V}$ close to the
accepted values for diamond $\left(B_{V}=445 \mathrm{GPa}\right.$ and $G_{V}=550 \mathrm{GPa}$ [21]). For hybrid $\mathrm{C}_{21}$, large $B_{\mathrm{V}}$ and $G_{V}$ are obtained but they are of smaller magnitudes compared to the original polytype. The trend of smaller $B_{\mathrm{V}}$ and $G_{\mathrm{V}}$ is confirmed for $\mathrm{C}_{24}$. The two hybrid systems with lower densities are increasingly more compressible.

Vickers hardness $\left(H_{\mathrm{V}}\right)$ has been predicted using four modern theoretical models. The thermodynamic (T) model [22], which is based on thermodynamic properties and crystal structure, shows surprising agreement with available experimental data [23] and is therefore recommended for hardness evaluation of superhard and ultrahard phases [24]. The Lyakhov-Oganov (LO) approach [25] takes into account the topology of the crystal structure, the strength of covalent bonding, the degree of ionicity and directionality; however, in the case of ultrahard phases of light elements, this model gives underestimated hardness values [23,24]. Two empirical models, Mazhnik-Oganov (MO) [26] and Chen-Niu (CN) [27], use the elastic properties. Fracture toughness ( $\mathrm{K}_{\mathrm{Ic}}$ ) has been evaluated within the Mazhnik-Oganov model [26]. The results for the currently proposed hexagonal carbon allotropes and others from the literature are summarized in Tables 3 and 4.

Table 3 shows the crystal parameters, density and Vickers hardness obtained from the thermodynamic model. For pristine $\mathrm{C}_{18}$, the values of density $(\rho)$, hardness $\left(H_{\mathrm{V}}\right)$ and bulk modulus $\left(B_{0}\right)$ have magnitudes close to those of diamond and lonsdaleite. This is quite expected since $9 R$ carbon is considered to be a diamond polytype. The two hybrid allotropes show lower values closer to $\mathrm{C}_{18}$.

Table 4 displays the hardness values along the different models in addition to the thermodynamic model presented above. Again, the trends of decreasing values along the series are observed, with the highest value obtained for $\mathrm{C}_{18}$. However, the Pugh modulus ratio $(G / B)$ for all three allotropes is almost the same ( $\sim 1.15$ ).
The fracture toughness of $\mathrm{C}_{18}\left(6.3 \mathrm{MPa} \cdot \mathrm{m}^{1 / 2}\right)$ is between the corresponding values for diamond and lonsdaleite, while two hybrid allotropes show lower values, i.e. 5.7 and $4.4 \mathrm{MPa} \cdot \mathrm{m}^{1 / 2}$ for ene $-\mathrm{C}_{21}$ and yne- $\mathrm{C}_{24}$. Similar trends are also observed for Young's modulus ( $E$ ) and Poisson's ratio (v).

These trends in mechanical properties are consistent with the decrease in density and increase in openness of the structure upon introduction of $\mathrm{C}=\mathrm{C}$ and $\mathrm{C} \equiv \mathrm{C}$ bonds in ene- $\mathrm{C}_{21}$ and yne- $\mathrm{C}_{24}$ respectively.

## Dynamic properties from the phonons

Other stability criteria, namely dynamic stability, were obtained by studying phonons, defined as quanta of vibrations. The phonon energy is quantized by the Planck constant h multiplied by the frequency $\omega$, giving the phonon energy $\mathrm{E}=(\mathrm{h} / 2 \pi) \omega$. The calculations of the phonons for $\mathrm{C}_{18}$, ene $-\mathrm{C}_{21}$ and $y n e-\mathrm{C}_{24}$ are illustrated with the corresponding band structures shown in Fig. 3. In all three panels all frequencies are positive signaling stable systems. Marked differences between the three carbon
allotropes are observed for the highest frequencies of the optical modes. In $\mathrm{C}_{18}$ they culminate at $\omega \sim$ 40 THz , a magnitude observed for diamond by Raman spectroscopy [29]. In ene- $\mathrm{C}_{21}$ they culminate at $\omega_{\max } \sim 42 \mathrm{THz}$, a magnitude also observed for ene-like $\mathrm{C}_{10}, \mathrm{C}_{14}$ [8], as well as in tetragonal $\mathrm{C}_{6}$ with 3 tetrahedral and 3 trigonal carbons, where the flat band was attributed to antisymmetric C-C-C stretching in the allene (propadiene) molecule [30]. The phonon band structure of yne-C $\mathrm{C}_{24}$, exhibits flat bands at 70 THz , which can be assigned to $\mathrm{C} \equiv \mathrm{C}$ characterized by much shorter distance than in ene- $\mathrm{C}_{21}$.

## Electronic band structures

The electronic band structures shown in Fig. 4 were obtained using the ASW method [17] with crystal inputs of the calculated crystal parameters from Table 1. The bands develop along the main directions of the primitive rhombohedral Brillouin zone. For the diamond-like 9R $\mathrm{C}_{18}$, Fig. 4a shows an insulating behavior with a large band gap $\sim 5 \mathrm{eV}$ separating the filled valence band VB from the empty conduction band CB. The energy level along the vertical line is with respect to the top of the valence band $(\mathrm{VB}), \mathrm{E}_{\mathrm{V}}$. In contrast, ene- $\mathrm{C}_{21}$ is metallic-like with a band that crosses the Fermi level $\mathrm{E}_{\mathrm{F}}$, and the energy is now with respect to $\mathrm{E}_{\mathrm{F}}$. Such behavior can be explained by the delocalized nature of the $\pi$ electrons resulting from the $\mathrm{C}=\mathrm{C}$ bonds. Finally, yne- $\mathrm{C}_{24}$, shows a semiconducting behavior and the zero energy is with respect to $\mathrm{E}_{\mathrm{V}}$. The localization of the charge at the triple $\mathrm{C} \equiv \mathrm{C}$ bond can explain the semiconducting behavior.

## Conclusions

The aim of this work was to give diamond new properties by introducing additional carbon atoms forming $\mathrm{C}=\mathrm{C}$ and $\mathrm{C} \equiv \mathrm{C}$ bonds into the rhombohedral 9 R polytype $\mathrm{C}_{18}$. The resulting new structures were geometry-optimized to the ground state and found to be cohesive, though less than pristine 9 R diamond polytype. Like $9 \mathrm{R} \mathrm{C}_{18}$, ene- $\mathrm{C}_{21}$ and yne- $\mathrm{C}_{24}$ belong to the cfe topology characterizing layered SiC polytypes. From mechanical characterization, the two hybrid allotropes have been identified as ultrahard and dynamically stable. They exhibit high-frequency vibrational phonon modes close to the molecular $\mathrm{C}=\mathrm{C}$ and $\mathrm{C} \equiv \mathrm{C}$ bonds characteristic of as allene and acetylene molecules, respectively. The electronic band structures are indicative of the insulating $\mathrm{C}_{18}$, metallic ene- $\mathrm{C}_{21}$ and semiconducting yne- $\mathrm{C}_{24}$. It is suggested that such model systems are likely to help improve experimental studies of nanodiamonds for electronic and other applications.

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Table 1 Crystal structure parameters of carbon allotropes with cfc topology. Values for cfe $\mathrm{C}_{18}$ are calculated for the $\mathrm{C}_{18}$ structure proposed by Deringer et al. [3].

| Space group <br> $R$-3m (No 166) | $\text { cfe } \mathrm{C}_{18}$ <br> SACADA Database Code: 447 | cfe ene-C ${ }_{21}$ | cfe yne- $\mathrm{C}_{24}$ |
| :---: | :---: | :---: | :---: |
| $a, \AA$ | 2.510 | 2.4922 | 2.5935 |
| $c, \AA$ | 18.672 | 22.988 | 26.290 |
| $V_{\text {cell }}, \AA^{3}$ | 101.91 | 123.65 | 153.14 |
| $<V_{\text {atom }}>\AA^{3}$ | 5.66 | 5.97 | 6.38 |
| Density, (g/cm ${ }^{3}$ ) | 3.52 | 3.39 | 3.13 |
| Shortest bonds, $\AA$ | 1.54, 1.56 | 1.46, 1.54, 1.55 | 1.21, 1.56, 1.59 |
| Atomic position |  | C1 (6c) $0,0,0.0635$ <br> C2 (6c) $0,0,0.1782$ <br> C3 (6c) $0,0,0.24579$ <br> C4 (3a) $0,0,0$ | $\mathrm{C} 1(6 \mathrm{c})$ $0,0,0.02267$ <br> $\mathrm{C} 2(6 \mathrm{c})$ $0,0,0.0774$ <br> $\mathrm{C} 3(6 \mathrm{c})$ $0,0,0.1766$ <br> $\mathrm{C} 4(6 \mathrm{c})$ $0,0,0.2358$ |
| $\begin{aligned} & \mathrm{E}_{\text {tot }}, \mathrm{eV} \\ & \mathrm{E}_{\text {tot }} / \text { atom, } \mathrm{eV} \\ & \mathrm{E}_{\text {coh }} / \text { atom, } \mathrm{eV} \end{aligned}$ | $\begin{aligned} & -163.43 \\ & -9.079 \\ & -2.479 \end{aligned}$ | $\begin{aligned} & -181.89 \\ & -8.661 \\ & -2.061 \end{aligned}$ | $\begin{aligned} & -204.54 \\ & -8.523 \\ & -1.923 \end{aligned}$ |

Table 2 Elastic constants $\mathrm{C}_{\mathrm{ij}}$ and Voigt values of bulk $\left(B_{\mathrm{V}}\right)$ and shear $\left(G_{\mathrm{V}}\right)$ moduli (all values are in GPa ) of carbon allotropes with cfc topology.

|  | $\mathrm{C}_{11}$ | $\mathrm{C}_{12}$ | $\mathrm{C}_{13}$ | $\mathrm{C}_{33}$ | $\mathrm{C}_{44}$ | $\mathrm{C}_{66}$ | $B_{\mathrm{V}}$ | $G_{\mathrm{V}}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| cfe $\mathrm{C}_{18}$ | 1168 | 131 | 30 | 1288 | 518 | 459 | 445 | 516 |
| cfe ene- $\mathrm{C}_{21}$ | 1009 | 89 | 33 | 1423 | 460 | 420 | 417 | 479 |
| cfe $y n e-\mathrm{C}_{24}$ | 794 | 76 | 21 | 1406 | 359 | 380 | 359 | 415 |

Table 3 Vickers hardness $\left(H_{V}\right)$ and bulk moduli $\left(B_{0}\right)$ of cfe carbon allotropes calculated within the framework of the thermodynamic model of hardness [22]

|  | Space group | $a=b(\AA)$ | $c(\AA)$ | $\rho\left(\mathrm{g} / \mathrm{cm}^{3}\right)$ | $H_{\mathrm{V}}(\mathrm{GPa})$ | $B_{0}(\mathrm{GPa})$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{C}_{18}{ }^{\# 166}$ | $R-3 m$ | 2.5104 | 18.672 | 3.523 | 98 | 444 |
| ${\text { ene- } \mathrm{C}_{21}}^{\# 166}$ | $R-3 m$ | 2.4922 | 22.987 | 3.387 | 94 | 427 |
| $y^{\text {ne- } \mathrm{C}_{24}{ }^{\# 166}}$ | $R-3 m$ | 2.5935 | 26.290 | 3.126 | 87 | 394 |
| Lonsdaleite | $P 6_{3} / m m c$ | $2.5221^{*}$ | $4.1186^{\ddagger}$ | 3.516 | 97 | 443 |
| Diamond | $F d-3 m$ | $3.56661^{\ddagger}$ |  | 3.517 | 98 | $445^{\dagger}$ |

[^1]Table 4 Mechanical properties of cfe carbon allotropes: Vickers hardness $\left(H_{V}\right)$, bulk modulus (B), shear modulus ( $G$ ), Young's modulus ( $E$ ), Poisson's ratio ( $v$ ) and fracture toughness ( $K_{\mathrm{Ic}}$ )

|  | $H_{V}$ |  |  |  | $B$ |  | $G_{V}$ | $E^{* *}$ | $v^{* *}$ | $K_{\text {Ic }}{ }^{\ddagger}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | T* | $\mathrm{LO}^{\dagger}$ | $\mathrm{MO}^{\ddagger}$ | $\mathrm{CN}^{\S}$ | $B_{0}{ }^{*}$ | $B_{V}$ |  |  |  |  |
|  | GPa |  |  |  |  |  |  |  |  | $\mathrm{MPa} \cdot \mathrm{m}^{1 / 2}$ |
| $\mathrm{C}_{18}{ }^{\# 166}$ | 98 | 90 | 97 | 89 | 444 | 445 | 516 | 1117 | 0.082 | 6.3 |
| ene- $\mathrm{C}_{21}{ }^{\# 166}$ | 94 | 73 | 89 | 84 | 427 | 417 | 479 | 1039 | 0.084 | 5.7 |
| yne- $\mathrm{C}_{24}{ }^{\# 166}$ | 87 | 69 | 77 | 78 | 394 | 359 | 415 | 899 | 0.082 | 4.4 |
| Lonsdaleite | 97 | 90 | 99 | 94 | 443 | 432 | 521 | 1115 | 0.070 | 6.2 |
| Diamond | 98 | 90 | 100 | 93 |  |  | $530^{\dagger \dagger}$ | 1138 | 0.074 | 6.4 |

* Thermodynamic model [22]
$\dagger$ Lyakhov-Oganov model [25]
* Mazhnik-Oganov model [26]
§ Chen-Niu model [27]
${ }^{* *} E$ and $v$ values calculated using isotropic approximation
$\dagger$ $\dagger$ Ref. 21

(c)

Fig. 1 Ball-and-stick (left) and tetrahedral (right) representations of crystal structures of $\mathrm{C}_{18}(a)$, ene- $\mathrm{C}_{21}$ (b) and yne- $\mathrm{C}_{24}$ (c).

(a)

(b)

(c)

Fig. 2. Charge density projections (yellow volumes) in $\mathrm{C}_{18}(a)$, ene- $\mathrm{C}_{21}(b)$ and yne- $\mathrm{C}_{24}(c)$.


Fig. 3. Phonons band structures of $\mathrm{C}_{18}(a)$, ene- $\mathrm{C}_{21}(b)$ and yne- $\mathrm{C}_{24}(c)$.


Fig. 4. Electronic band structures of $\mathrm{C}_{18}$ (a), ene- $\mathrm{C}_{21}$ (b) and yne- $\mathrm{C}_{24}$ (c).


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[^1]:    * Ref. 2
    * Ref. 28
    $\dagger$ Ref. 21

