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Fair Energy Allocation for Collective Self-Consumption

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Abstract. This study explores a collective self-consumption community with several houses, a shared distributed energy resource (DER), and a common energy storage system, as a battery. Each house has an energy demand over a discrete planning horizon, met by using the DER, the battery, or purchasing electricity from the main power grid. Excess energy can be stored in the battery or sold back to the main grid. The objective is to determine a supply plan ensuring a fair allocation of renewable energy while minimizing the overall microgrid cost.

We investigate and discuss the formulation of these optimization problems using mixed integer linear programming. We show some dominance properties that allow to reformulate the model into a linear program. We study some fairness metrics like the proportional allocation rule and max-min fairness. Finally, we illustrate our proposal in a real case study in France with up to seven houses and a one-day time horizon with 15-minute intervals.

Keywords: Energy Management · Fair Allocation · Production Planning

1 Introduction

In a time when renewable energy sources are gaining increasing prominence and communities seek to reduce their carbon footprint, collective self-consumption represents a fundamental transformation in the way individuals and communities access and use energy, offering a sustainable, economically viable, and environmentally friendly alternative to traditional energy distribution models [5]. Energy management under collective self-consumption empowers groups of individuals, neighborhoods, or even entire communities to unite and jointly harness, generate, and distribute locally produced renewable energy. It offers numerous advantages, including a reduced reliance on fossil fuels, lowered energy costs for participants, and decreased stress on the grid. Furthermore, this approach fosters a sense of ownership and environmental responsibility among participants, thereby strengthening the sense of community as they collaboratively embrace

the energy transition [6]. An essential aspect within self-consumption communities is the distribution of the generated energy, which becomes the responsibility of the legal entity managing the community. This entity must communicate the distribution of the electricity generated to the distribution system operator (DSO). For most of these communities, their operation can be divided into two major systems: a technical system and an economic system. The technical system must account for the actual distribution of energy, while the economic system considers the monetary distribution that occurs after the energy has been processed. Consequently, the financial balance of each participant depends on the rules selected for the distribution of the generated energy, or allocation rules [14]. This problem has been long studied in the literature, however, its characteristics and formulation depend on the technical configuration, the goals and the social arrangements of the community [6]. The literature has focused mainly on microgrids where each user has their own DER and energy storage system, and they are allowed to trade among community members [14, 10, 7]. These works focus on price selection for exchanges between users. On the other hand, microgrids where the DER is shared among all users are less studied. Ogando-Martínez et al. [11] presents an LP model to evaluate different allocation strategies for a community with a shared ownership over a photovoltaic system. They consider the allocation rule as a coefficient, of the PV production, fixed before the optimization process. In [16], fairness is measured by minimizing the discrepancy between the cost assigned to each house and the cost it would have incurred if it had been the only house in the microgrid.

In this paper, we address the problem of energy supply planning for collective self-consumption communities, ensuring a fair allocation of renewable energy. The energy supply planning problem in a collective self-consumption, ensuring fair allocation of renewable energy consists in finding an energy supply planning that not only ensures equitable distribution of renewable energy but also minimizes the overall expenses. We consider a microgrid composed of a set of J smart houses, a renewable distributed energy resource (DER) as a photovoltaic panel (PV), a shared energy storage system as a battery, and a connection to the main power grid. Each house $j \in J$ has an energy demand to satisfy over a discrete planning horizon composed of T time steps of δ hours each, typically $\delta = 0.25$ which correspond to 15 min. The demand $D_{j,t}$, for each house j at each time step t , can be fulfilled either by using the DER, the battery, or by purchasing electricity from the main power grid. Excess energy can be stored in the battery or sold back to the main grid. The battery has a charge and discharge efficiency, e_c and e_d respectively, i.e. energy is lost when charging and discharging the battery. The battery has a maximum capacity C^s and a minimum storage limit S^{min} , the charge and discharge rates have a maximum limit, \bar{F} and \underline{F} , respectively. PV panels produce at most C_t^{PV} at each time step $t \in T$. There is a manager, who may or may not be a physical person, in charge of managing the photovoltaic energy. This manager must report at each time step the distribution of PV to the DSO, who is then in charge of issuing the bills for each user. And, the following assumptions are considered, each member

owns a smart meter that reports their actual consumption at each time step. For each time step, electricity selling price β is always strictly lower than electricity buying price B_t . The manager distributes all the PV energy at each time step. A user cannot charge and discharge energy from the battery at the same time step, and a user cannot sell and buy energy from the main grid at the same time step. Finally, the charge efficiency must be larger than the discharge efficiency.

The main contributions of the paper are the following. We first formulate the problem of energy supply planning, which does not incorporate fairness, as a mixed integer linear programming (MILP) model. Then, we show some dominance properties for this problem that allow not only to relax the integrality constraints, but also to entirely remove binary variables from its formulation, leading to a linear programming (LP) model. Finally, we study the fairness aspect of this problem, by analyzing some fair allocation rules for green energy. We propose a mathematical formulation for each allocation rule studied, and show some dominance properties for these models. Some numerical experiments on a real case study in France illustrate these properties. The rest of this article is organized as follows, in Section 2 we analyze the energy supply planning problem in collective self-consumption without fairness, and in Section 3 we study some fair allocation rules and their implementation in the energy supply planning problem in collective self-consumption. In Section 4 we present some numerical experiments over a real case study in France.

2 Energy Supply Planning Problem

2.1 Mathematical Formulation

We formulate the energy supply planning problem considering shared PV and battery as an MILP model. The daily cost is minimized by selecting the electricity output from the PV panel to each house $p_{j,t}$, from the battery $y_{j,t}$, and from the main grid $i_{j,t}$. It also selects the electricity that each house stores in the battery $z_{j,t}$, and the amount of electricity that each house sells to the main grid $g_{j,t}$. Finally, variable s_t represents the state of charge of the battery at each time step, variables $v_{j,t}$ and $w_{j,t}$ are working variables that model the activation of the charging and discharging, and the activation of the buying and selling of energy, respectively.

Equation (1) represents a well accepted function to compute the total cost of the microgrid [3] where all equipment capacities are considered given, so only operating and maintenance costs are included. The cost function consists of maintenance costs associated with battery discharge μ , the cost of purchasing electricity from the main grid B_t and the revenues received from the sale of surplus electricity β . Note that no production or maintenance costs associated with the PV are considered, this is because the total production of electricity through PV is fixed, independently of its allocation, so its operating costs become constant, and therefore negligible for our problem.

Constraints (2) set the capacity limits for the state of charge of the battery. While Equations (3) represent the electricity production of the PV. Equations

(4) indicate the state of charge of the battery at each time step, which depends on the state of charge of the previous period, the amount of electricity charged and the amount discharged in the same period. Constraint (5) shows that the state of charge of the battery at the end of the time horizon must be equal to the initial state of charge. Constraints (6) and (7) represent the limits in the charge and discharge rate of the battery at each time period.

$$\begin{aligned}
 & \left\{ \begin{array}{l}
 \min \quad \delta \sum_{j \in J, t \in T} (\mu y_{j,t} + B_t i_{j,t} - \beta g_{j,t}) \quad (1) \\
 \text{s.t.} \quad S^{\min} \leq s_t \leq C^s \quad \forall t \in T \quad (2) \\
 \sum_{j \in J} p_{j,t} = C_t^{PV} \quad \forall t \in T \quad (3) \\
 s_t = s_{t-1} + e_c \sum_{j \in J} z_{j,t} - e_d \sum_{j \in J} y_{j,t} \quad \forall t \in T \quad (4) \\
 s_T = s_0 \quad (5) \\
 \sum_{j \in J} y_{j,t} \leq \underline{F} \quad \forall t \in T \quad (6) \\
 \sum_{j \in J} z_{j,t} \leq \overline{F} \quad \forall t \in T \quad (7) \\
 D_{j,t} = p_{j,t} + y_{j,t} + i_{j,t} - z_{j,t} - g_{j,t} \quad \forall j \in J, t \in T \quad (8) \\
 z_{j,t} \leq \overline{F} v_{j,t} \quad \forall j \in J, t \in T \quad (9) \\
 y_{j,t} \leq \underline{F} (1 - v_{j,t}) \quad \forall j \in J, t \in T \quad (10) \\
 i_{j,t} \leq (D_{j,t} + C^s)(1 - w_{j,t}) \quad \forall j \in J, t \in T \quad (11) \\
 g_{j,t} \leq (C_t^{PV} + C^s) w_{j,t} \quad \forall j \in J, t \in T \quad (12) \\
 s_t, i_{j,t}, g_{j,t}, z_{j,t}, y_{j,t}, p_{j,t} \geq 0 \quad \forall j \in J, t \in T \quad (13) \\
 v_{j,t}, w_{j,t} \in \{0, 1\} \quad \forall j \in J, t \in T \quad (14)
 \end{array} \right. \quad (EPP)
 \end{aligned}$$

Constraints (9) and (10) indicate that for each house in each time period it cannot charge and discharge the battery at the same time. Similarly Constraints (12) and (11) indicate that a house cannot buy and sell power to the grid in the same time period. Finally, Constraints (8) represent the electricity balance in the microgrid.

2.2 Dominance Properties

We below show that the energy supply planning problem in collective self-consumption can be reformulated as a linear programming problem. To address this, we introduce a relaxed version of the model (EPP) where we remove constraints (9)-(12) and (14), obtaining a LP model that we call (EPP-L). Lemma 1 and Lemma 2 show some dominance properties of any optimal solution of (EPP-L) problem.

Lemma 1. *Given an instance of (EPP-L) where $B_t > \beta$ for all $t \in T$, then in any optimal solution $x^* = (s^*, i^*, g^*, z^*, y^*, p^*)$ of (EPP-L), $i_{j,t}^* g_{j,t}^* = 0$ holds for all $j \in J, t \in T$.*

Proof. Let us assume, by contradiction, that there exists $x^* = (s^*, i^*, g^*, z^*, y^*, p^*)$, an optimal solution of (EPP-L) with a cost $c(x^*)$, such that $\exists t' \in T, j' \in J : i_{j',t'}^* g_{j',t'}^* > 0$. We show that we can build a feasible solution $\bar{x} = (\bar{s}, \bar{i}, \bar{g}, \bar{z}, \bar{y}, \bar{p})$ of (EPP-L) identical to x^* except for j', t' such that $\bar{i}_{j',t'} \bar{g}_{j',t'} = 0$. We will prove that \bar{x} have a cost $c(\bar{x}) < c(x^*)$.

First we set $\bar{s} = s^*$, $\bar{p}_{j',t'} = p_{j',t'}^*$, $\bar{y}_{j',t'} = y_{j',t'}^*$ and $\bar{z}_{j',t'} = z_{j',t'}^*$. In this way, Constraints (2)-(5) are satisfied.

Now, let $\epsilon = \min\{i_{j',t'}^*, g_{j',t'}^*\}$, $\bar{i}_{j',t'} = i_{j',t'}^* - \epsilon$ and, $\bar{g}_{j',t'} = g_{j',t'}^* - \epsilon$. Thus, $\epsilon > 0$ and $\bar{i}_{j',t'} \bar{g}_{j',t'} = 0$

As we are reducing the exported and imported energy in the same amount, the balance of energy given by Equation (8) remains satisfied, and therefore, \bar{x} is a feasible solution of (EPP-L).

Furthermore, we can write the cost difference between x^* and \bar{x} as:

$$c(x^*) - c(\bar{x}) = \delta\epsilon(B_{t'} - \beta)$$

Now, as $B_t > \beta \forall t \in T$, then $c(x^*) - c(\bar{x}) > 0$, and consequently, x^* is not optimal. Hence, we have reached a contradiction, and therefore, in any optimal solution of (EPP-L) $i_{j,t}^* g_{j,t}^* = 0 \forall j \in J, t \in T$. \square

Lemma 2. *Given an instance of (EPP-L) where $B_t > \beta \forall t \in T$, and $e_c \geq e_d > 0$ then, in any optimal solution $x^* = (s^*, i^*, g^*, z^*, y^*, p^*)$ of (EPP-L), $y_{j,t}^* z_{j,t}^* = 0$ holds $\forall j \in J, t \in T$*

Proof. In the same fashion as for lemma 1, let us assume, by contradiction, that $x^* = (s^*, i^*, g^*, z^*, y^*, p^*)$ is an optimal solution to (EPP-L) with a cost $c(x^*)$ such that $\exists t' \in T, j' \in J : y_{j',t'}^* z_{j',t'}^* > 0$.

We show that we can build $\bar{x} = (\bar{s}, \bar{i}, \bar{g}, \bar{z}, \bar{y}, \bar{p})$, a feasible solution to (EPP-L), with cost $c(\bar{x})$ such that, \bar{x} is identical to x^* except for variables related to house j' at time period t' , where $\bar{y}_{j',t'} \bar{z}_{j',t'} = 0$, and we will prove that $c(x^*) > c(\bar{x})$. First, we set $\bar{s}_{t'} = s_{t'}^*$ and $\bar{p}_{j',t'} = p_{j',t'}^*$, consequently, constraints (2), (3), and (5) are satisfied. Now, considering that $\bar{s} = s^*$, it holds from constraint (4) that for both solutions to be feasible, we must have:

$$e_c z_{j',t'}^* - e_d y_{j',t'}^* = e_c \bar{z}_{j',t'} - e_d \bar{y}_{j',t'} \quad (15)$$

Now, as we are aiming at having either $\bar{z}_{j',t'} = 0$ or $\bar{y}_{j',t'} = 0$, it holds from (15) that either $e_c z_{j',t'}^* - e_d y_{j',t'}^* = -e_d \bar{y}_{j',t'}$ or $e_c z_{j',t'}^* - e_d y_{j',t'}^* = e_c \bar{z}_{j',t'}$. Thus, we can deduce the following cases:

Case 1: $e_c z_{j',t'}^* - e_d y_{j',t'}^* \leq 0$.

We set $\bar{z}_{j',t'} = 0$, and $\bar{y}_{j',t'} = y_{j',t'}^* - \frac{e_c}{e_d} \epsilon$ where $\epsilon = z_{j',t'}^*$. In this way, in solution \bar{x} , we reduce the charge $\bar{z}_{j',t'}$ and discharge $\bar{y}_{j',t'}$ of energy from the battery. Therefore, Constraints (6) and (7) are satisfied. However, the difference

in battery charging is greater than the difference in discharging. Consequently, there is a surplus of energy in the balance proposed by Equations (8):

$$\begin{aligned}
D_{j',t'} - (\bar{p}_{j',t'} + \bar{y}_{j',t'} + \bar{i}_{j',t'} - \bar{z}_{j',t'} - \bar{g}_{j',t'}) &= 0 \\
D_{j',t'} - (p_{j',t'}^* + y_{j',t'}^* - \frac{e_c}{e_d}\epsilon + \bar{i}_{j',t'} - (z_{j',t'}^* - \epsilon) - \bar{g}_{j',t'}) &= 0 \\
\Leftrightarrow i_{j',t'}^* - \left(g_{j',t'}^* + \frac{e_d - e_c}{e_d}\epsilon \right) &= \bar{i}_{j',t'} - \bar{g}_{j',t'} \tag{16}
\end{aligned}$$

By Lemma 1, we know that either $i_{j',t'}^* = 0$ or $g_{j',t'}^* = 0$. Therefore, to maintain the energy balance, we can either buy less energy from the main grid, sell more energy, or stop buying and sell energy instead. These cases can be formally distinguished as follows:

- Case 1.1: $g_{j',t'}^* \geq 0$ and $i_{j',t'}^* = 0$.

We set $\bar{i}_{j',t'} = 0$ and $\bar{g}_{j',t'} = g_{j',t'}^* + \frac{e_d - e_c}{e_d}\epsilon$. Which verifies expression (16). And, we can write the cost difference between x^* and \bar{x} as:

$$c(x^*) - c(\bar{x}) = \delta \left(\mu \frac{e_c}{e_d}\epsilon + \beta \frac{e_d - e_c}{e_d}\epsilon \right)$$

Now, as $e_d \geq e_c$ then $c(x^*) - c(\bar{x}) > 0$, which is a contradiction to optimality of x^*

- Case 1.2: $g_{j',t'}^* = 0$ and $0 < i_{j',t'}^* < \frac{e_d - e_c}{e_d}\epsilon$.

We set $\bar{i}_{j',t'} = 0$ and $\bar{g}_{j',t'} = \frac{e_d - e_c}{e_d}\epsilon - i_{j',t'}^*$. Similarly as before, this verifies expression (16). And, we can write the cost difference between x^* and \bar{x} as:

$$c(x^*) - c(\bar{x}) = \delta \left(\mu \frac{e_c}{e_d}\epsilon + (B_{t'} - \beta) i_{j',t'}^* + \beta \frac{e_d - e_c}{e_d}\epsilon \right)$$

Now, as $e_d \geq e_c$, and $B_{t'} > \beta$ then $c(x^*) - c(\bar{x}) > 0$, which is a contradiction to optimality of x^* .

- Case 1.3: $g_{j',t'}^* = 0$ and $i_{j',t'}^* \geq \frac{e_d - e_c}{e_d}\epsilon$.

We set $\bar{g}_{j',t'} = 0$ and $\bar{i}_{j',t'} = i_{j',t'}^* - \frac{e_d - e_c}{e_d}\epsilon$. Which verifies expression (16). We can write the cost difference between x^* and \bar{x} as:

$$c(x^*) - c(\bar{x}) = \delta \left(\mu \frac{e_c}{e_d}\epsilon + B_{t'} \frac{e_d - e_c}{e_d}\epsilon \right)$$

Now, as $e_d \geq e_c$ then $c(x^*) - c(\bar{x}) > 0$, which is a contradiction to optimality of x^* .

Case 2: $e_c z_{j',t'}^* - e_d y_{j',t'}^* > 0$.

Symetrically, we prove that we can build a solution \bar{x} for this case. We now set $\bar{y}_{j',t'} = 0$, and $\bar{z}_{j',t'} = z_{j',t'}^* - \frac{e_d}{e_c}\epsilon$, where $\epsilon = y_{j',t'}^*$. Then, as in the previous case, we are reducing both the charge and discharge of energy from the battery, and therefore, Constraints (6) and (7) are satisfied. Now, the reduction we made

in battery discharging is greater than the reduction we made in battery charging, also generating an energy surplus in equation (8):

$$i_{j',t'}^* - \left(g_{j',t'}^* + \frac{e_d - e_c}{e_c} \epsilon \right) = \bar{i}_{j',t'} - \bar{g}_{j',t'} \quad (17)$$

And, as well as for the previous case, by Lemma 1, we know that either $i_{j',t'}^* = 0$ or $g_{j',t'}^* = 0$. Therefore, to maintain the energy balance, we distinguish the cases: (2.1) when $g_{j',t'}^* \geq 0$ and $i_{j',t'}^* = 0$, (2.2) when $g_{j',t'}^* = 0$ and $0 < i_{j',t'}^* < \frac{e_d - e_c}{e_c} \epsilon$, and (2.3) when $g_{j',t'}^* = 0$ and $i_{j',t'}^* \geq \frac{e_d - e_c}{e_c} \epsilon$. Using arguments similar to those used above, we reach a contradiction in each case.

Hence, our initial assumption that x^* is an optimal solution of (EPP-L) is false. Therefore, in any optimal solution of (EPP-L) $y_{j,t}^* z_{j,t}^* = 0 \forall j \in J, t \in T$. \square

Corollary 1. *Given an instance of (EPP) where $B_t > \beta \forall t \in T$, and $e_c \geq e_d > 0$ then (EPP) is a polynomial problem.*

3 Adding Fairness Considerations

So far, we have focused our analysis of energy management in microgrids on the overall system efficiency. However, in the context of shared resources, such as PV energy, we are confronted with a challenge of allocating scarce resources. When considering only the overall efficiency, individual agents often find themselves in a position where they must sacrifice personal benefits to enhance the common welfare. Consequently, the notions of fairness and equity have garnered significant attention as potential tools to address the disadvantages some agents may face [9, 15, 12, 1].

Thus, several definitions of fairness have emerged in the context of resource allocation [13, 4, 2]. To implement these definitions in the resource allocation problem, allocation rules are used. Such rule can be defined as the vector $P \in \mathcal{R}^n$ composed of the resource quantities allocated to each user, where n is the number of users. The vector P must meet the following characteristics: (i) non-negativity, (ii) demand limitation, and (iii) efficiency. In our problem each component of the vector P is defined as the aggregate PV electricity allocation of each house $j \in J$ over time horizon T , i.e., $P_j = \sum_{t \in T} p_{j,t}$ for each $j \in J$.

The following section aims to define some resource allocation rules for the distribution of PV energy in microgrids: proportional allocation rule, and max-min fairness.

3.1 Proportional Allocation Rule

The proportional allocation rule consists of allocating the resource in a way that if the allocation of a user is increased it exists at least another user whose allocation decreases and the loss of this user in proportion is larger than the gain of any other one [8]. This rule satisfies the following properties: (a) each user

receives the same portion of the demand, i.e., $\lambda = \frac{P_j}{D_j} \forall j \in J$; (b) users with the same demand must receive the same amount of resource; (c) scale invariance; and (d) resource monotonicity: if we increase the amount of available resource, the users should receive at least the same allocation as before.

To implement this allocation rule in the energy supply planning problem, we add equations (18) to (EPP), which state that the portion of the demand allocated to each house must be the same.

$$\frac{\sum_{t \in T} p_{1,t}}{\sum_{t \in T} D_{1,t}} = \frac{\sum_{t \in T} p_{j,t}}{\sum_{t \in T} D_{j,t}} \quad \forall j \in J \quad (18)$$

3.2 Max Min Fairness (MMF)

In this rule, the allocation is based on the egalitarian notion and it seeks to protect the weaker users [2], it can be calculated as follows, first we order the users according to their increasing demand, i.e., $D_1 \leq \dots \leq D_n$, then we allocate the resource following Equation (19).

$$P_j = \min\left\{D_j, \frac{R - \sum_{k=1}^{j-1} P_k}{n - j + 1}\right\} \quad (19)$$

And the main property of this rules is that for any other allocation \bar{P} : if $P_s < \bar{P}_s$ for some user $s \in J$ there exists at least one user l such that $P_l \leq P_s$ and $\bar{P}_l < P_l$, i.e., to increase the allocation for a user, we have to decrease the allocation for another user whose MMF allocation was smaller than the benefited user.

To apply this allocation rule to our problem (EPP), we add constraints (20) and (21).

$$\sum_{t \in T} p_{j,t} \leq \sum_{t \in T} D_{j,t} \quad \forall j \in J \quad (20)$$

$$\sum_{t \in T} p_{j,t} \leq \frac{\sum_{t \in T} C_t^{PV} - \sum_{k=1}^{j-1} \sum_{t \in T} p_{k,t}}{|J| - j + 1} \quad \forall j \in J \quad (21)$$

3.3 Dominance Properties

We below show that the fair energy supply planning problem in collective self-consumption can be reformulated as a linear programming problem as proposed in the previous section. To this, we consider (EPP-L), and we incorporate an allocation rule, obtaining an LP model. Thus, we deduce Lemma 3.

Lemma 3. *Given an instance of (EPP-L) with proportional allocation rule (or max min fairness rule), where $B_t > \beta \forall t \in T$, and $e_d \geq e_c > 0$, then in any optimal solution $x^* = (s^*, i^*, g^*, z^*, y^*, p^*)$, $i_{j,t}^* g_{j,t}^* = 0 \forall j, t$, and $y_{j,t}^* z_{j,t}^* = 0 \forall j, t$.*

Proof. By incorporating Constraint (18) (or Constraints (20)-(21)) into (EPP-L) problem, we are limiting the quantity of PV energy being supplied to each household. This allows us to apply the same proof approach as we did for Lemmas 1 and 2, where this variable does not play a significant role.

4 Numerical Experiments

These allocation rules were implemented in Julia 1.8.1, using the CPLEX 12.10 solver. Instances created from data collected by E4C’s DrahiX demonstrator were used. These instances consider 7 zones of the building, whose electricity consumption profiles will represent 7 houses in our problem, in increasing order of their demand, i.e., $\sum_{t \in T} D_{1,t} \leq \dots \leq \sum_{t \in T} D_{7,t}$. For the production of green electricity, a PV array with a capacity of 37.5 kWc and a battery with a capacity of 10.5 kWh are considered. A time horizon of 24 hours with time steps of 15 min is used. To calculate the costs we consider the fixed electricity purchase price of 0.2062 euros, according to the current price of electricity in France, as well as for the sale of electricity we use the price established for the sale of surplus electricity in French self-consumption communities, 0.1 euros. All the technical parameters of the battery were obtained from Zhang et al. [16].

4.1 Energy Supply Planning Problem

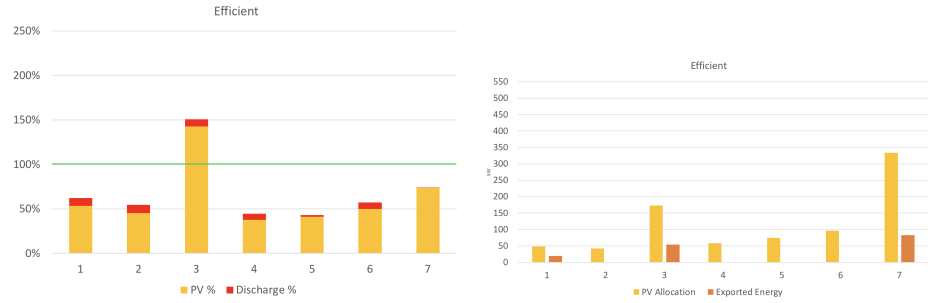
Figure 1 shows the results obtained by solving (EPP-L) without fairness. Figure 1a shows the percentage of demand that can be satisfied with the allocated PV and battery. We can see that only one house is able to satisfy the 100% of their demand, while the others are around 50%. In Figure 1b for each house, the first bar shows the amount of PV energy allocated to the house, and the other one, the amount of energy exported to the main grid. We observe that even the houses with less than 100% of autonomy are selling some energy to the main grid.

4.2 (EPP-L) With Proportional Rule

In Figure 2a, we can see that all the houses are able to satisfy the same percentage of their demand, ensuring more than a 50% of autonomy per house. In Figure 2b, we observe that all of them are selling some energy to the main grid. Furthermore, in order to ensure the same autonomy to all houses, we are forced to allocate more PV energy to those houses with a bigger demand.

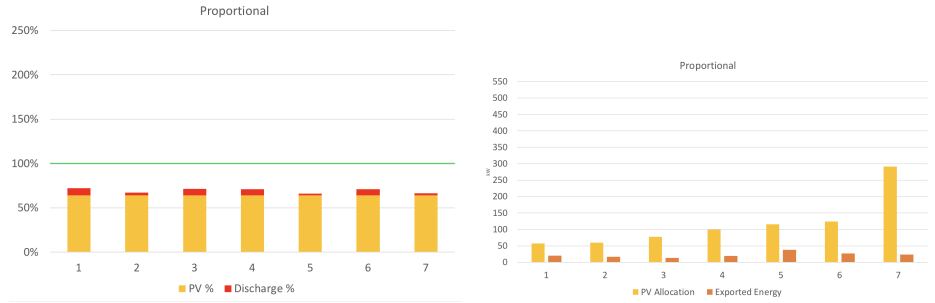
4.3 (EPP-L) With Max Min Fairness

In Figure 3 we can see that houses with a small demand are able to satisfy their full demand, and this autonomy decrease as we increase the demand. In Figure 3b, as well as we see before, some houses that are not able to satisfy their full demand, are selling energy to the main grid. Furthermore, houses with less than 100% autonomy receive the same amount of photovoltaic energy, which is higher than the amount received by those who achieve 100% autonomy.



(a) Percentage of the demand satisfied with the allocated PV energy and the battery. (b) Amount of energy exported compared to amount of allocated PV.

Fig. 1: Results obtained by solving (EPP-L)



(a) Percentage of the demand satisfied with the allocated PV energy and the battery. (b) Amount of energy exported compared to amount of allocated PV.

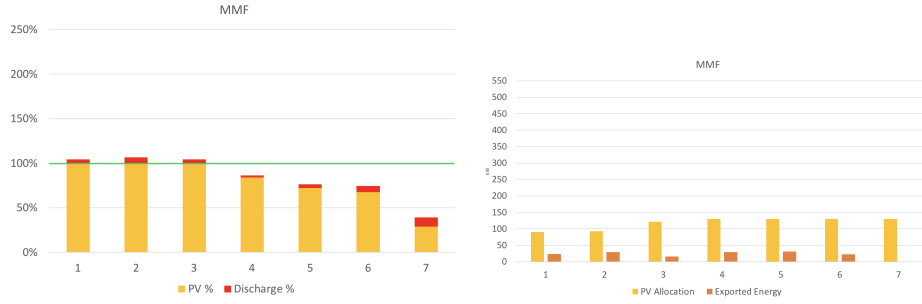
Fig. 2: Results obtained using the proportional rule

4.4 Price of Fairness

In Figure 4 we can see the percentage increase in total cost when using the fair allocation rules, compared to the cost obtained when solving (EPP-L). The proportional rule does not present an increase in costs.

5 Conclusion

This work addressed the energy supply planning problem in collective self-consumption. First we presented a mixed integer linear formulation of the problem, and we reformulated it as a linear program based on some dominance properties. Then we presented two fairness allocation rules from the literature, and we proposed their implementation into the energy supply planning problem. We



(a) Percentage of the demand satisfied with the allocated PV energy and the battery. (b) Amount of energy exported compared to amount of allocated PV.

Fig. 3: Results obtained using the max min fairness rule

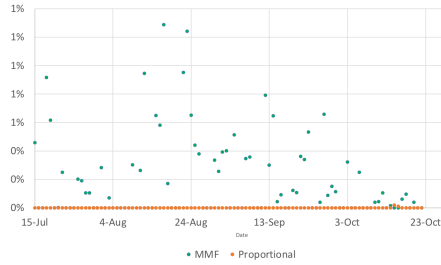


Fig. 4: Percentage increase in total cost when using the proportional and max min fairness rules.

also showed how the dominance properties from the energy planning problem holds for the problem considering fairness.

We illustrated our proposal in a real case study in France, using consumption profiles from a university. We were able to show the impact of the different allocation rules on the distribution of PV energy and on the economical efficiency of the community.

To continue this work we want to study the impact of time aggregation in allocation, and the impact of uncertainty in predictions of PV production or demand. Furthermore, we want to study the α -Fairness allocation rule, which is a collection of different allocations that captures different utility functions.

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