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Maya Ron Ezra, Esther S. Levenson. Expressing mathematical creativity: The case of mediating open-ended tasks for students with learning disorders. Thirteenth Congress of the European Society for Research in Mathematics Education (CERME13), Alfréd Rényi Institute of Mathematics; Eötvös Loránd University of Budapest, Jul 2023, Budapest, Hungary. hal-04409444

HAL Id: hal-04409444

<https://hal.science/hal-04409444>

Submitted on 22 Jan 2024

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Expressing mathematical creativity: The case of mediating open-ended tasks for students with learning disorders

Maya Ron Ezra¹ and Esther S. Levenson²

¹Tel Aviv University, Achav College, Israel; maya.ronezra@gmail.com

²Tel Aviv University, Israel

Encouraging mathematical creativity is one of the aims of mathematics education. This study examines expressions of mathematical creativity in students with learning disorders (LD). Twelve students with LD learning in special education classrooms took part in this study, working in pairs and engaged with three short tasks believed to encourage mathematical creativity. Findings indicate that completely open-ended tasks lead to more expressions of fluency, while semi-open-ended tasks lead to more expressions of flexibility. The inclusion of tasks that encourage mathematical creativity in special education classrooms requires mediation to make the tasks accessible for these students.

Keywords: Open-ended tasks, learning disorder, mediation, mathematical creativity.

Researchers and educators recognize creativity as an important competency for the twenty-first century (Partnership for 21st Century Skills, 2016). Furthermore, the 2017 OECD report emphasises that every person, regardless of their abilities, has the potential to think creatively. Different studies have shown that encouraging mathematical creativity may strengthen the connections between different mathematics topics and expand existing knowledge (Shen & Edwards, 2017). Yet, despite this recognition, very few studies have examined the encouragement of mathematical creativity in populations of students with special educational needs (SENs).

When it comes to students with SENs, there is a growing awareness of the importance of creating equal learning opportunities for each and every student (Treahey & Gurganus, 2010). Teaching methods that expose students with SENs to a range of solution strategies may encourage flexibility and creativity as well as deepen their mathematics knowledge and understanding (Peters et al., 2014). This paper presents a study of how students with learning disorders (LDs) engaged in open-ended tasks and, with the help of different types of mediation given by the researcher, were able to express mathematical creativity.

Theoretical background

A deep understanding of mathematical concepts incorporates elements of creativity and divergent thinking (Bahar & Maker, 2011). Mathematical creativity is usually assessed according to three criteria: 1. Fluency – the *number* of final solutions or solution strategies related to the given task. 2. Flexibility – the *variety* of solution strategies or approaches, divergent thinking which leads to switching between different categories. 3. Originality – the “*uniqueness*” or “*innovation*” of the solutions and solution strategies, indicating insights and associations between different topics (Levenson, 2013). Studies have shown that working in groups has a greater impact on fluency and flexibility than on originality (Molad et al., 2020). In the current study, students with LDs worked in pairs on open-ended tasks, and thus the study focuses specifically on fluency and flexibility.

An open-ended task is a task that provides a starting point with no specific instruction (Klein & Leikin, 2020). This type of task is thought to encourage mathematical creativity and will be referred

to as a “completely open-ended task”. By contrast, a “semi-open-ended task” is a task whose level of “openness” is reduced: the starting point remains the same, with no specific instruction, but the result is predetermined. Such a task stimulates divergent thinking and invites students to create a variety of paths leading to the same solution, thus encouraging creativity (Bahar & Maker, 2011). Students in the current study engage with both completely open-ended and semi-open-ended tasks.

A learning disorder (LD) is an ongoing impairment of learning functions (e.g., reading, reading comprehension, written expression, spelling, mathematical calculations, and mathematical logic), and can manifest in varying degrees of severity: severe, moderate, and mild (American Psychiatric Association, 2013). Lovin et al. (2004) mention five main areas of difficulty affecting students with LD: language, memory, motivation, and sense of competence, attention, and executive functions. In the context of mathematics, these students experience difficulties in understanding mathematical contexts, tend to use more basic, less effective strategies, and struggle to recall information from their long-term memory (Geary & Hoard, 2005). In Israel, where this study takes place, students with moderate LDs can study in a regular class, or study in a special class for students with LDs, according to the decision of their parents and the educational staff. In both instances, students with LDs learn according to educational programs that cover the same mathematical content as the general curriculum. To meet these curricular aims, teachers often present to their students a range of strategies that also encourages flexibility and creativity (Peters et al., 2014). Nevertheless, the teaching of mathematics must be made accessible and relevant for them and requires mediation tailored to their unique needs to properly establish new knowledge (Hunt et al., 2016).

Mediation is an intentional act of intervention between the student and the stimulus to which (s)he is being exposed, in order to generate the appropriate cognitive response to the situation. Exposure to a range of mediated experiences helps develop formal and informal processes and the ability to self-mediate (Tzuriel, 2013). Feuerstein and Feuerstein (1991) defined different types of mediation (see Figure 1), including: mediation of *intentionality and reciprocity* in which the mediator focuses the student’s attention on the stimulus or on a central component within it, and the student demonstrates reciprocity in response to the mediation; mediation of *meaning*, referring to creating contexts and highlighting the importance of the stimulus through verbal means (i.e., repetition, naming and explanations), and nonverbal means (i.e., gestures, facial expressions, body language); mediation of *transcendence*, which involves extending the situation beyond the “here and now” and generalising it to similar situations; mediation of *feeling of competence*, means creating situations in which students can demonstrate their proficiency and openness to new experiences, while the mediator draws attention to and clarifies the components or causes of success (Lovin et al., 2004); and mediation of *sharing behaviour*, namely the ability to share experiences with others and participate in the experiences of others, while the role of the mediator is to set an example and “echo” the situation. Students with LDs often require tailored mediation to gain and benefit the most out of their experience (Lovin et al., 2004). This study involved different forms of mediation, such as asking open questions and strategies of echoing and thinking aloud, which promote metacognitive skills such as self-regulation, monitoring, and solution analysis (Treahy & Gurganus, 2010), as well as opening new avenues of thought, essential for fluency and flexibility.

Based on the above, the theoretical framework which guided this study is presented in Figure 1.

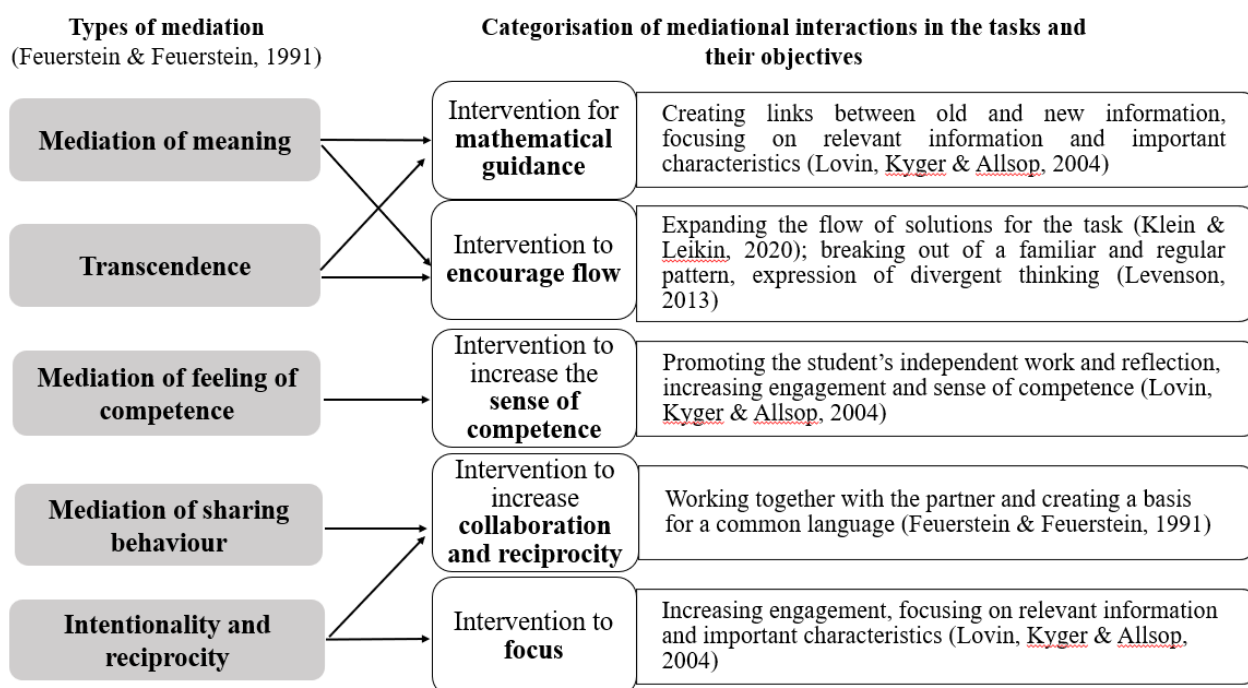


Figure 1: Theoretical framework

Considering the importance of fostering mathematical creativity for all students, and considering the benefits of mediation for students with LDs, this study examines how pairs of third-grade students with LDs, learning in special education classes, demonstrate mathematical creativity when they receive appropriate mediation. We ask: What types of mediation can help students with LDs express mathematical creativity in completely open-ended and semi-open-ended tasks? What expressions of mathematical creativity arise when students with LDs engage with completely open-ended and semi-open-ended tasks?

Methods

Participants in this study were 12 third-grade students (ages 9-10 years) with moderate LDs, learning in two special education classes located within general, mainstream schools in Israel. Notably, study participants showed no indication of dyscalculia or specific mathematical difficulties. To encourage fluency and flexibility in their interactions (Molad et al., 2020), the teacher divided the students into pairs, with careful consideration of their mathematical and social abilities, as essential factors for productive collaboration.

To examine expressions of creativity, the students were given three short tasks in fixed order with increasing levels of difficulty. The first task was a completely open-ended task: "How many mathematical expressions can you create using the numbers 0-9?" This task has an entirely open, free path with a low level of difficulty. The next two were semi-open-ended tasks: "How many mathematical sentences can you create using the numbers 0-9, whose result is 20?" This task with a medium level of difficulty, and "How many mathematical sentences can you create using the numbers 0-9, whose result is 32?" with a higher level of difficulty. Students were given an unlimited time frame for completing the tasks. Tasks were assigned to students in a quiet room at school. For solutions, the two students shared a blank sheet (unless they requested another sheet, each student

could write down answers on his or her own sheet). Four pairs performed all three tasks in a single session; one pair performed the third task in another session, and another pair did not perform the third task, due to the increase in difficulty. Students completed the first task in an average of 15 minutes, the second in 10 minutes, and the third in 7 minutes. The students' performance was documented by a video camera. At the end of each session, all worksheets were collected and coded by the researcher. Furthermore, four videos were transcribed, and the mediated interactions were extracted from them.

The researcher, who was present while the students performed the tasks, carried out mediational interactions based on the types of mediation defined by Feuerstein and Feuerstein (1991). Part of the mediation was planned according to a mediation protocol prepared in advance, and included guiding questions (e.g., is it possible to use more than one action? Is it possible to write a sentence with a different number of numbers?). Some of the mediational interactions were given spontaneously. In the process of examining the findings, the mediational interactions were divided into five different categories: *mathematical guidance*, *encouraging fluency*, *focusing*, *expanding collaboration and reciprocity*, and *promoting a sense of competence*. Each category of mediational interaction has different objectives in the context of promoting the skills required for the task (i.e., mathematical, creative, or emotional). Figure 1 depicts the connection between the theoretical framework, based on Feuerstein and Feuerstein (1991) and the analysis of mediational interactions carried out during the study.

Mathematical creativity analysis was divided to two. Element of *fluency* was measured based on the total number of correct mathematical sentences each pair wrote together. In cases where a student wrote the same mathematical sentence twice, or where each of the two students wrote the same sentence, those sentences were counted as one solution. An expression of *flexibility* was considered when the students “broke out of” a familiar pattern (e.g., an expression with a single operation, usually addition or multiplication), and produced expressions with more than one operation or sentences in which the same operation appeared more than once.

Findings

The findings section is divided into two parts. The first part discusses the types of mediation provided to students while performing the tasks, and the implications of these mediations (or the perceived resonance of the researcher’s intervention). The second part analyses the results produced by the students from the perspective of expressions of mathematical creativity.

The following episodes demonstrate the categorisation of the mediations and their effects. Episode 1 exemplifies the categories of *mathematical guidance*, *encouraging fluency*, and *expanding collaboration and reciprocity*.

Episode 1 – a conversation between the researcher and a pair of students while performing Task 1 (How many mathematical expressions can you create using the numbers 0-9?).

- Researcher: Does it have to be an expression with two numbers, or can it have more?
Student 1: (Opens his eyes with surprise and looks at his friend.)
Student 2: It can have more.
Researcher: Such as?
Student 1: Um... eight plus two plus two plus two. (This is said out loud but is not written down.)

Student 2: Whoa, wait a minute (jumps in his seat) ... eight plus two plus five plus six (writes down the expression and solves it).

Student 1: (murmuring) eight plus nine, seventeen plus two, nineteen (writes the expression).

In this situation, the researcher asks the students a question connected to prior mathematical knowledge in order to encourage fluency and flexibility. The additional question – “Such as?” – arouses in the students an additional, emotional, aspect (in the form of facial expressions, jumping up in excitement) and encourages an expansion of the discourse and reciprocity, evidenced in the subsequent conversation between the students:

Student 1: Want to know my method? Look at my method, how many digits did you do?

Student 2: Three digits.

Student 1: Do one more digit, now I can show you a way (takes the sheet from Student 2 and demonstrates what he means).

The researcher's question served to focus the students, but it unexpectedly triggered a mathematical discourse among them. This discourse involved sharing and consultation, leading the students to generate expressions with more than two elements ($8+9+2$, $8+2+5+6$) and to combine different arithmetic operations in one exercise ($5*5+5*5$).

Episode 2 provides another example of mediational interaction during a conversation between the researcher and two of the students. The initial mediation was intended to promote collaboration between the pair of students, but also led them to create connections between old and new knowledge.

Episode 2 – a conversation between the researcher and students in Task 2 (How many mathematical expressions can you create using the numbers 0-9, whose result is 20?).

Student 4: (Writes down the sentence $1+1+1+1+1+...+1+1=20$.)

Researcher: What is Student 4 doing here? (Reads the sentence aloud) One plus one plus one...

Student 3: (Addressing the other student) You know this can also be done with two, right?

Student 3: Wait, I must do twenty times like this? (Pointing to the sentence $1+1+1+1...$).

Researcher: Ah, wait, did you hear what Student 3 said? You need to do twenty times, so which sentence can we write down when we say “times”? What operation is “times”?

Student 4: Multiplication.

By “echoing” out loud what Student 4 wrote, the researcher creates a mediational interaction of the types: *focusing* and *mathematical guidance*, focusing the students on relevant information and important characteristics through which the students create connections between topics.

In the next episode, Episode 3, taken from the third task which had a higher level of difficulty, the mediational interaction promotes several objectives related to the task simultaneously. First, the researcher uses positive reinforcement, which is a mediational interaction that increases the sense of competence. Later she “echoes” the expression out loud, which is a mediational interaction of the *focusing* and *expanding collaboration and reciprocity* types. As described earlier, it is intended to generate collaboration and increase engagement, and at the same time it *encourages fluency and flexibility* in order to break out of familiar patterns and promote divergent thinking.

Episode 3 – the researcher reads the sentences aloud and provides positive reinforcement (How many mathematical expressions can you create using the numbers 0-9, whose result is 32?).

Student 5: (Writes down the expression $5+5+5+5+5+5+2$.)

Researcher: Excellent (refers to the expression Student 5 has written down and reads the exercise aloud): five plus five plus five plus five plus five plus five plus two equals thirty-two. And what did you write (addresses Student 6)?

Student 6: Five times four, twenty, plus nine, twenty-nine, plus one, thirty, thirty plus two – thirty-two.

Researcher: Listen, you both rock! I knew I could give you this challenge, I knew it!

The researcher’s statement in reference to the expressions the students produced stimulates a mediational interaction intended to increase the students’ engagement and sense of competence, as well as encourage a greater flow of solutions for the task.

We now describe the task solutions, which were assessed according to fluency and flexibility. Regarding fluency in Task 1, the six pairs produced a total of 186 correct mathematical sentences. In the other two tasks, they produced a smaller number of sentences: 56 sentences for Task 2, and 32 sentences for Task 3. Most of the students used a single arithmetic operation in their solutions (e.g., addition), which appeared once or more in the sentence, with a varying number of elements (see Figure 2, Pair A). Table 1 shows the relative frequency of sentences involving a single arithmetic operation and sentences involving multiple operations (N represents the total number of correct sentences produced by the six pairs for each task). As can be seen, different tasks elicited the use of different arithmetic operations. For example, in the first task the students used addition (40% of the sentences for this task) and multiplication (33% of the sentences for this task) more than other operations. Furthermore, the semi-open-ended tasks prompted the students to create sentences involving multiple operations (chain exercises). In the first task, 7% of the sentences produced by the students used more than one operation, compared to the third task, where the overall number of sentences they produced was lower (N=32), but the percentage of sentences with multiple operations was high (65.6%).

Table 1: The relative prevalence of exercises produced in the three tasks per operation

	Task 1 (N=186)	Task 2 (N=56)	Task 3 (N=32)
Addition	40% (75 sentences)	62.5% (35 sentences)	21.9% (7 sentences)
Subtraction	14% (26 sentences)	-	-
Multiplication	33% (62 sentences)	9% (5 sentences)	12.5% (4 sentences)
Division	6% (11 sentences)	-	-
Multiple operations	7% (12 sentences)	28.5% (16 sentences)	65.6% (21 sentences)

Due to the young age of the students with LD, an expression of flexibility was considered to have occurred when the students broke free from familiar patterns. Pair A’s solution sheet shows that the students “broke out of” the common pattern (a sentence with a single operation), and while they only used one operation (addition), the number of addends varied from one sentence to the next ($1+9+1+9$, $4+4+4+4+4$). By contrast, Pair B (Figure 3) created chain exercises from the outset, with multiple operations and a range of different operations ($1*9+9+2$). Moreover, they were the only pair that used a chain exercise involving both multiplication and subtraction operations ($4*8-8-4$). Figure 2 and Figure 3 demonstrate how flexibility was manifested in two student pairs in Task 2.

Handwritten solution sheet for Pair A. The page is filled with mathematical expressions, many of which are variations of $20 = 20$. Notable equations include: $0+20=20$, $7+1+1+1+1+1+1+1+1+1+1+1+1+1+1+1=20$, $4 \times 5 = 20$, $5+5+5+5=20$, $2+2+3+2=20$, $10+10=20$, $4 \times 9 = 36$, and $4+1+4+1+4+1+4+1+4+1+4+1=20$. A large, light-colored number '20' is written at the top center.

Figure 2: Task 2 solution sheet (Pair A)

Handwritten solution sheet for Pair B. The page contains several mathematical equations: $2 \times 8 + 4 = 20$, $9 \times 2 + 2 = 20$, $5 \times 4 = 20$, $5 \times 4 + 5 = 20$, and $1 \times 9 + 11 = 20$. On the right side, there are more equations: $7 \times 4 + 4 = 20$, $5 \times 4 + 5 = 20$, and $1 \times 8 + 12 = 20$. A handwritten '00' is visible at the bottom right.

Figure 3: Task 2 solution sheet (Pair B)

Discussion and conclusions

The inclusion of tasks that encourage mathematical creativity in special education classrooms is not to be taken for granted and requires various forms of mediation to make the tasks accessible for these students. This study has shown that students with LDs can express fluency and flexibility when they engage in completely open-ended tasks, as do their peers in mainstream classrooms (Klein & Leikin, 2020). In the first task, there were more expressions of fluency than of flexibility, manifested mainly in the use of addition combined with multiplication. In contrast, in the second and third tasks, which were semi-open-ended, there were more expressions of flexibility while the element of fluency declined. Coping with these tasks resulted in expressions of divergent thinking and in creating connections between different topics (Bahar & Maker, 2011).

A practical contribution of this study was the understanding that providing mediational interactions can play a part in making open-ended tasks accessible and suitable for students with LDs. The different episodes described in this paper demonstrate how mediational interactions can have an impact on how students engage with mathematically challenging tasks, while promoting different objectives at the same time. For example, in Episode 1, a mediation that was intended to encourage the students to break out of a familiar and regular pattern (Levenson, 2013) triggered a reflective reaction, including an expression of admiration and astonishment in one student (Lovin et al., 2004). It may be surmised that these interventions helped students recall information and enabled its processing in a manner compatible with the task (Treahy & Gurganus, 2010). A theoretical contribution of this study was adapting Feuerstein and Feuerstein's (1991) theory of mediational interactions to mathematical learning. Furthermore, acknowledging variations in cooperation among pairs of students during these tasks is crucial. Some pairs demonstrated substantial collaboration, while others had minimal or no interaction. Understanding these differences can shed light on how mediation influences students' participation and creative problem-solving processes, providing valuable insights for future interventions. We recognize that this is a relatively small study examining expressions of creativity among 12 students with LDs in three short tasks with varying levels of difficulty. Nevertheless, it supports the notion that the potential for creative thinking exists among different learners and reinforces the view that equal opportunities for learning and higher-order thinking can be created for students with LDs, including tasks that are thought to encourage mathematical creativity. The insights gleaned from this analysis can serve as an example of the support required to help make accessible tasks that encourage mathematical creativity.

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