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Mathematics teachers' professional noticing of gifted students' mathematical thinking within the context of pattern generalization

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The present research aimed to explore how middle school mathematics teachers who worked at the Science and Art Center (SAC) in Türkiye noticed gifted students' mathematical thinking. A qualitative case study approach was adopted to gain an in-depth understanding of the teachers' skills of attending, interpreting, and deciding how to respond to students' thinking. Data collection was conducted in two phases; comprising a questionnaire to assess the solutions of gifted students and semi-structured interviews with mathematics teachers. The data were analyzed using the "Professional Noticing of Children's Mathematical Thinking" framework. The findings revealed that teachers could analyse the mathematical details of gifted students' solutions and interpret their mathematical thinking. However, while one teacher could offer efficient instructional acts, another needs support in developing skills in responding to gifted students.

Keywords: Gifted students, teachers' professional noticing, pattern generalization.

Introduction

The National Council of Teachers of Mathematics has reported that gifted students are often overlooked, and their potential is not fully utilized (NCTM, 1980). This is a significant issue as these students have the potential to play a critical role in the scientific and economic development of countries (Fıçıçı & Siegle, 2008; Hannah et al., 2011). Despite the emphasis placed on gifted students and their education in the past, there remains ongoing debate and disagreement about the definition and identification of giftedness (Mihaela-Singer et al., 2016). Various terms are used to describe gifted students, such as "mathematically gifted and talented," "highly able," "intellectually precocious," "bright," and "mathematically advanced" (Mihaela-Singer et al., 2016, p. 1). Despite different labelling, researchers have found that gifted students tend to possess specific common characteristics, such as acquiring and retaining knowledge quickly, engaging in complex thinking, and exhibiting curiosity and a desire for growth and development (Johnson, 2000).

Regarding gifted students who excel in mathematics, they tend to be particularly interested in understanding the underlying concepts and principles behind mathematical content rather than simply focusing on computation. They tend to possess a high level of thinking and advanced problem-solving strategies (Krutetskii, 1976; Aydemir-Özdemir & Işıksal-Bostan, 2019). Considering that gifted students have different ways of thinking and solution strategies than other students, their teachers must understand their thinking and solutions and interpret their understanding. A thorough understanding of gifted students' thinking allows teachers to provide a challenging and inclusive education that meets their needs and helps them reach their full potential. To understand students' thinking, one of the teacher competencies teachers need to have is the ability to notice (Kaiser et al.,

2017). Thus, in this research, we investigate the extent to which teachers notice gifted students' mathematical thinking in the context of pattern generalization.

Teacher noticing as a professional vision is defined as the ability to identify and assess significant events occurring simultaneously within the classroom, manage these events effectively, and respond appropriately to the students (van Es & Sherin, 2002; Jacobs et al., 2010). Thus, teacher noticing is essential expertise to analyse the classroom environment and facilitate mathematics discourse in this environment (van Es & Sherin, 2002; Jacobs et al., 2010). Although researchers agreed on the noteworthy moments to notice, they defined teacher noticing from a different point of view (van Es & Sherin, 2002; Jacobs et al., 2010). For instance, van Es and Sherin (2002) specifically focused on teachers' three competencies: (1) the ability to identify significant occurrences in the classroom setting, (2) the ability to use professional knowledge to analyse those events, (3) the ability to link such events to teaching and education principles. Therefore, van Es and Sherin (2002) point out paying attention to classroom events and interpreting these events through teachers' existing knowledge. Moreover, van Es and Sherin (2021) added a new dimension, shaping, which refers to creating interactions that increase teachers' opportunities to attend and interpret noteworthy mathematical events. In this view, teachers manipulate interactions to gain a deeper understanding of student thinking (van Es & Sherin, 2021). Except for van Es and Sherin (2002), Jacobs and his colleagues (2010) focused on particular students' mathematical thinking rather than noticing the events in the classroom environment. For this reason, Jacobs et al. (2010) specialized the teacher noticing as "Professional Noticing of Children's Mathematical Thinking." Through this framework, Jacobs and her colleagues investigated teacher noticing under three dimensions: "(1) attending to children's strategies, (2) interpreting children's understanding, and (3) deciding how to respond on the basis of children's understandings" (Jacobs et al., 2010, p. 169).

Fıçıçı and Siegle (2008) emphasized the crucial role that teachers play in not only identifying gifted students but also in providing guidance and support for their development. To effectively support these students, teachers must be able to analyse their thinking and provide appropriate instruction through their noticing skills. However, there is a lack of research on how teachers analyse gifted students' solutions and respond to them based on their mathematical thinking. This study aims to fill this gap by exploring how teachers attend to gifted students' solutions, interpret their thinking, and respond to gifted students' basis on their thinking. Thus, the research questions for this study are as follows:

1. To what extent do teachers attend to gifted students' solutions and interpret their mathematical thinking within the context of pattern generalization?
2. How do teachers respond to gifted students based on their mathematical thinking within the context of pattern generalization?

Method

Research design

In order to obtain an in-depth exploration of teachers' skills of attending, interpreting, and deciding how to respond, the qualitative case study was used (Yin, 2009). Since in this study, there is a single

case, middle school mathematics teachers worked at the Science and Art Center (SAC) in Türkiye, and three units of analysis, which are teachers' attending to students' solutions, interpreting their mathematical thinking, and deciding how to respond on the basis of their mathematical thinking, the single-case embedded design was the chosen (Yin, 2009). SACs are institutions under the control of the Ministry of National Education that provide education to gifted students after regular school hours. These institutions aim to provide activities to students according to their interests and abilities (MEB, 2012). Teachers worked at SACs are selected among qualified teachers with expertise in a particular field and can guide gifted students according to their interests and abilities.

Participants

The participants of this study are two middle school mathematics teachers working at the SAC in Türkiye. The participants were selected through purposive sampling among voluntary middle school mathematics teachers working at the SAC. Furthermore, to make a detailed exploration, the volunteer teachers who like to speak and explain their opinion are preferred as participants.

Data collection

Based on the aim of the study, data were collected in two phases: a questionnaire to reveal 5th-grade gifted students' solution strategies and semi-structured interviews with mathematics teachers working at SAC on those particular strategies. Firstly, the questionnaire, including two problems, was given to eight 5th-grade gifted students. To align with the research aim, the questionnaire was administered to students who had been identified as gifted and attended SAC regularly. The fact that problems are parallel with the 5th-grade curriculum is another reason why 5th-grade students were selected to ask the problems. However, based on the page limitation, only the first problem was used in this study. In the selected problem (see Figure 1), gifted students were asked to realize the pattern and relationship given in the figure.

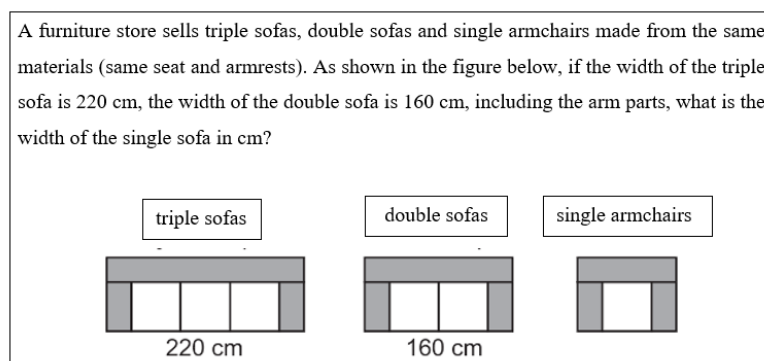


Figure 1: Problem (Kanguru Exam Committee, 2017, p. 6)

Two student solutions were selected, including mathematical details worthy of notice (Student A's and Student B's solutions). In Student A's solution (see Figure 2), the student recognized the pattern and solved the problem based on the relationship she realized. In Student B's solution (Figure 3), a student could not realize the relationship in the pattern and solve the problem by trial and error.

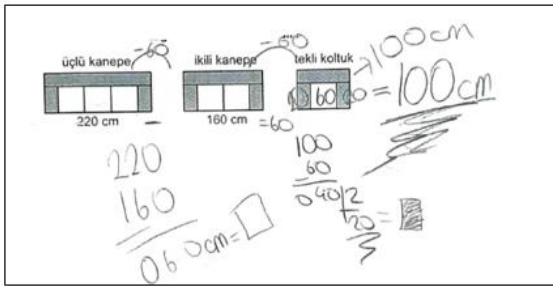


Figure 2: Student A's solution

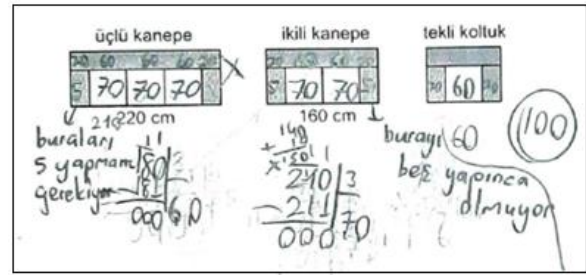


Figure 3: Student B's solution

In the semi-structured interview, student A's solution and student B's solution were given to teachers, respectively. After teachers explored the students' solutions, teachers were asked to respond to the following prompt based on student A's and student B's solutions: "(1) Please explain in detail what you think the student did in response to this problem, (2) Please explain what you learned about this student's understanding, (3) Pretend that you are the teacher of these children. What problem or problems might you pose next?" (Jacobs et al., 2010, p. 178-179). These questions were asked to identify teachers' skills of attending, interpreting, and deciding how to respond to students within the context of pattern generalization. During the semi-structured interview, based on the teachers' responses, further in-depth questions were asked about the students' solutions to make teachers' explanations more explicit. Especially regarding the third question, both teachers were asked to give detailed answers and to put forward the problems they planned (e.g. Can you give an example of the problems you plan to pose? What is your rationale for asking this problem to the student?). Although the same follow-up questions were asked, one of the teachers explained in detail what kind of problem she planned to ask, while another preferred to pose the problem she would ask the students only.

Regarding ethical concerns, before conducting the study, the aim of the research and the researchers' expectations from the participants were explained. Based on this explanation, consent forms were signed by the students and teachers. Furthermore, it was stated that participants' personal information, responses, or video recordings would not be shared. Thus, pseudonyms were assigned to ensure the anonymity of the participants. Finally, an appointment was established based on their availability to conduct the interview effectively.

Data analysis

The data of this study was analysed based on the framework of "Professional Noticing of Children's Mathematical Thinking" developed by Jacobs et al. (2010). Three categories emerged under the skill of teachers' attending to students' solutions. Based on the analysis, a new category, *limited evidence of attending* to students' solutions, was added to the attending dimension. Teachers who offered *robust evidence of attending* demonstrated the ability to accurately identify all mathematical concepts and attend to all mathematical details in the students' solutions. However, teachers who presented *limited evidence of attending* were able to determine the accuracy of the students' solutions but failed to encompass all mathematical details fully. On the other hand, teachers' explanations that demonstrated incorrect identification of the students' solutions were classified as a *lack of evidence of attending*.

The teachers' interpretation of students' thinking was categorized into three classifications: *robust*, *limited*, and *lack of evidence*. Teachers who displayed *robust evidence of interpretation* were able to comprehend students' thought processes. For example, they can interpret whether the student recognizes the pattern or realizes the relationship in the pattern. In contrast, teachers' interpretations that displayed *limited evidence* included less detail and a less distinct understanding of the student's thinking. Finally, teachers' interpretations that offered incorrect or irrelevant remarks on the students' thought processes were classified as a *lack of evidence of interpretation*.

Similar to the categorizations of attending and interpreting, the teachers' responding skills were classified as *robust*, *limited*, and *lack of evidence*. Teachers' responses that aimed to extend or reinforce the students' existing thought processes were considered to demonstrate *robust evidence*. Conversely, responses in which the teachers requested students to perform routine exercises without considering their thought processes were coded as *limited evidence*. Finally, when teachers failed to consider the students' thinking and instead proposed direct instruction, such responses were coded as a *lack of evidence of responding*.

Findings

In the present study, the findings are presented under two headings: attending to gifted students' solutions and interpreting their mathematical thinking, and deciding how to respond to gifted students.

Table 1: Teachers' skills of attending, interpreting, and responding

Teacher	Attending to		Interpreting		Deciding how to respond to	
	student A's solution	student B's solution	student A's thinking	student B's thinking	student A	student B
Emily	robust	robust	robust	robust	robust	robust
Lily	robust	robust	robust	robust	limited	limited

Attending to gifted students' solutions and interpreting their mathematical thinking

The results of the data analysis indicate that both teachers exhibited *robust evidence of attending* to gifted students' solutions, as they were able to provide detailed explanations of the students' solution processes. The following excerpt from Emily's interview serves as an illustration of this finding:

Student A is aware that one seat is different between the sofas. Based on this, she realizes that the difference between double and triple sofas will give him the width of one seat. The child finds that the width of one seat is 60 cm. Subtracting 60, that is the width of one seat, from 160, she finds 100. Then she realizes that when she subtracts the width of the seat from 100 again, the width of the armrests will remain. She subtracts 60 from 100 again and finds 40; if we divide it by two, he finds that the width of each armrest will be 20 cm.

In this instance, Emily demonstrated a thorough understanding of student A's problem-solving process, as she could identify and explain the step-by-step reasoning behind the student's actions. For example, she explained how student A arrived at the width of a single sofa, why she subtracted 60 from 160, and how she found the width of one armrest. As a result of this level of attention and

understanding, Emily's attention to student A's solution was classified as *robust evidence of attending* student's solution.

In a similar manner to teachers' attending to the students' solutions, both teachers' interpreting of students' thinking was also classified as *robust evidence*. The following excerpt from Lily's interview transcript serves as an example of this *robust evidence of interpretation*:

When the student is doing trial and error in the question, she understands what is asked in the question, but she cannot recognize the pattern. In other words, she cannot see that one seat, whose width is 60 cm, is decreased from each sofa. Moreover, she cannot think practically enough and cannot form a strategy. She makes trial and error, and I do not think she has a problem-solving strategy. She may have solved it, but not enough.

In this example, Lily accurately understood student B's thought process, noting that the student could not identify the underlying pattern in the problem and analyse the relationship between the steps. She also highlighted that the student had solved the problem through trial and error. As a result of her clear and accurate understanding of the student's mathematical thinking process, her interpretation of the student's thinking was classified as *robust evidence*.

Deciding how to respond to gifted students

Even though both teachers' attending and interpreting skills, the categorization of their responding skills varied. Specifically, Emily's responses to both students' solutions were classified as *robust evidence*, while Lily's responses were classified as *limited evidence of responding*. The following examples from Emily's and Lily's interview transcripts serve to illustrate this difference:

First, I would like to ask student B if there is a pattern between these shapes. I would ask questions such as what kind of relationship there is between the shapes, how many seats there are, and how many arms there are to help the student find that logic. I would expect a solution similar to the previous solution (student A's solution). I would guide student B to discover the relationship and solve the question based on the given information rather than by experimenting. Afterward, assuming that the student does not have difficulty and continues with understanding, I would ask more complex, more real-life, interdisciplinary questions that involve science. In other words, I would ask interdisciplinary questions enabling student B to discover the relationship between the disciplines and make inferences.

In this particular example, Emily effectively facilitated the development of student B's mathematical thinking by encouraging the student to recognize the pattern and relationship among steps through follow-up questions. She also emphasized the importance of solving the problem based on the relationship in the pattern instead of using trial and error. Additionally, she aimed to expand the student's thinking by providing interdisciplinary and real-life context problems. Due to these efforts, Emily's response was classified as *robust evidence of responding*. Another example excerpted from Lily's interview is as follows:

First of all, I would ask student A to explain this solution. She shows the pattern in the question in her solution, but I would like to listen to the student's solution again to see if she will say something extra. Afterward, I continue the flow from easy to difficult. By making the current problem more

difficult, I could ask about a similar problem. I would like to see if student A can find or recognize the pattern again in the problem I will ask about. The problem that I planned to pose to student A is as follows: “A furniture store sells triple sofas, double sofas, and single armchairs made from the same materials (same seat and armrests). If the width of the triple sofa is 180 cm, and the width of the single sofa is 70 cm, including the arm parts, what is the width of the double sofa in cm?”

In this example, while student A had correctly solved the problem by considering the relation in the pattern, Lily could not effectively build on this thinking by posing follow-up questions to extend the student’s thinking. Instead, her approach focused on posing the problem with different numbers. As a result, her response was classified as *limited evidence of responding*.

Therefore, it can be concluded that both teachers could analyse the important mathematical details of gifted students’ solutions and interpret their mathematical thought processes. However, this conclusion does not hold for their responding skills. Specifically, Emily demonstrated the ability to extend the mathematical thinking of gifted students, while Lily merely asked for drill without considering the students’ thinking.

Discussion

The data analysis demonstrated that middle school mathematics teachers who worked at the SAC displayed a strong ability to attend to gifted students’ solutions and interpret their thinking. These teachers could discern the crucial mathematical aspects of gifted students’ solutions, even if gifted students used different strategies. However, they provided different levels of responses based on students’ thinking. Stated specifically, while the teacher, Emily, provided appropriate instructional support for these students, another teacher Lily, tended to ask to drill without considering the students’ mathematical understanding. This finding suggests that teachers’ ability to respond effectively to students is more than just determined by their attending and interpreting skills. This conclusion aligns with the findings of Özel (2019), who reported no direct correlation between teachers’ skills in attending and interpreting and their ability to respond to students.

The disparity in the responding abilities of the two teachers may be attributed to their experiences at the SAC. Both teachers possess approximately ten years of teaching experience. However, Emily has taught mathematics to gifted students for four years at the SAC, whereas Lily has only taught gifted students for two years at the SAC. This difference in experience may have led to Emily’s more effective instructional support, as she has had more opportunities for interaction and lecturing with students. As noted in the study by Jacobs et al. (2010), a teacher’s experience significantly impacts their ability to attend to students’ solutions and interpret their thinking effectively. This investigation is delimited by two restrictions: firstly, it solely relied on the solutions provided by two gifted students for the teachers to notice; secondly, the data were collected through only interviews.

As a future study, it would be beneficial to expand the current research on teachers’ noticing skills by incorporating observations of teachers’ in-the-moment teaching practices in the classroom. Such observations would provide a more comprehensive understanding of teachers’ skills in attending to gifted students’ solutions, interpreting their thinking, and responding to them instead of relying solely on interviews. The suggested study could provide valuable insights into the relationship between a teacher’s experiences, instructional support, and student learning outcomes.

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