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► To cite this version:

Liina Malva, Triin Kivirähk. Developing differentiated algebra worksheets for inclusive classroom. Thirteenth Congress of the European Society for Research in Mathematics Education (CERME13), Alfréd Rényi Institute of Mathematics; Eötvös Loránd University of Budapest, Jul 2023, Budapest, Hungary. <hal-04409419>

HAL Id: hal-04409419

<https://hal.science/hal-04409419v1>

Submitted on 22 Jan 2024

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Developing differentiated algebra worksheets for inclusive classroom

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Inclusive education demands teachers to use differentiated materials in order to support all students in the classroom. Difficulties in mathematics can escalate on ISCED 2 level due to the abstract nature of algebra. This study focuses on the development process of differentiated worksheets on the topic of polynomials and was aimed to find out which design factors support or hinder students' learning. A design-based research method was followed. Six special education teachers tested the worksheets with 19 students. The results show that in the teachers' opinion students' learning of polynomials is best supported with step-by-step difficulty increase, and with visual cues. The teachers also pointed out that there is a risk of oversimplifying the tasks and overusing visual cues, which can hinder students' learning. The results and examples from developed worksheets can be a guide for teachers or researchers for future development and research of learning materials in inclusive classroom.

Keywords: Instructional materials, learning disabilities, special education, mathematics education, polynomials.

Introduction

Numerous research papers have proposed possibilities to apply inclusive education successfully. Different adaptations are necessary for an inclusive classroom, where all students are integrated in general education classes as much as possible (Burke & Sutherland, 2004). However, inclusive classroom also brings challenges on all ages and forms of education. In practice more emphasis is put on kindergarten and primary school level, leaving the young adults at risk of being “missed out” (Westwood, 2004). Difficulties in mathematics, however, could appear at different ages due to a variety of reasons – for example, when students are not given enough time to practice, the instructions are too complicated, there are not enough examples, and inadequate learning materials (Westwood, 2004). Therefore, it is important to support teachers with resources that could better help students with difficulties attending their classrooms. In the context of the current study, the term “difficulties in mathematics” is used in case a student needs any kind of extra support (i.e., adapted materials, learning support classes, assistant teacher, etc.).

Algebra is a topic that is usually introduced in primary school through one variable in simple math problems (e.g., $x + 4 = 10$). In the case of Estonia, where this current study was carried out, the proportion of algebra increases significantly in ISCED 2 level (International Standard Classification of Education: lower secondary education) with 7th grade students becoming familiar with linear equations (e.g., $3x + 2 = x + 8$). This shift also means that mathematics is suddenly becoming more abstract, causing difficulties to many young adults (Witzel et al., 2003). However, despite the process of learning abstract algebra can be effectively supported. One option to support students in inclusive classrooms is to provide them with differentiated materials instead of traditional textbooks, allowing them to work on an appropriate level (Wadlington & Wadlington, 2008). Differentiated materials are supplementary tasks to regular textbooks that vary in terms of difficulty and prompts used (Sullivan

et al., 2009). Differentiation in task design can also be explained as task variations to the original tasks that provide an appropriate level for a diverse group of students, including those experiencing difficulty, while working on similar conceptual content as other classmates (Sullivan et al., 2015). Differentiated materials must take into account the differences in cognitive processes that cause difficulties in learning (Cortiella & Horowitz, 2014). Due to a lack of good quality differentiated materials - especially after primary school – a gap appears between pedagogies and available resources that could influence the instruction and decisions that teachers make (Gueudet & Trouche, 2011).

When developing learning materials that are more suitable for inclusive classroom, the difficulty level of the tasks should increase gradually, one step at a time (Griffith & Ruan, 2005). Breaking down sub-topics into smaller steps allows students to attain knowledge in a slow-paced systematic manner which is in line with cognitive load theory emphasizing the optimal number of concepts and its content learned per page or per one set of learning material, while not oversimplifying it (Paas et al., 2003). The language used in task design must present clarity and the instructions should be written in a short, simple and concrete manner as students with difficulties often face challenges in comprehension (Griffith & Ruan, 2005). In case the instructions have multiple actions, it should be separated into sub-instructions with each step written on a new row. After all, the academic language used in instructions can enhance students' understanding of the mathematical concept (Prediger, 2022). In addition to a step-by-step approach and clear instructions, the appearance and design of tasks should provide visual cues and ease of navigation through the learning materials (Roberts, 2008). It is strongly suggested to use colours as visual cues in algebra task design to support making connections, finding out patterns and memorizing different parts of the conceptual content (Watson & Thompson, 2015). In addition to that, students' attention can be supported with different background effects of learning materials, for example, frames and fill out colours (Cuoco, 2001). The above-mentioned principles can lead to designing learning materials that support students in inclusive learning environments.

Aim and research questions

Taking into account the importance of differentiated materials in inclusive classrooms and the challenges that young adults face when learning algebra, this study is focusing on the development of differentiated worksheets on the topic of polynomials in ISCED 2 level. The aim of the study was to identify design-related factors of differentiated algebra worksheets that can have an effect on students' learning from the perspective of special education teachers. The research questions were as follows:

- 1) Which design related factors support students' learning with differentiated mathematics worksheets in teachers' opinion?
- 2) Which design related factors hinder students' learning with differentiated mathematics worksheets in teachers' opinion?

Methods

In order to answer the research questions, design-based research was used to develop, test and evaluate a set of polynomials worksheets. Design-based research allows researchers to tackle an

educational problem with an intervention carried out in close cooperation with practitioners as stakeholders, followed with an evaluation round (Anderson & Shattuck, 2012; Krainer, 2011). In the current study, the intervention was teachers using the developed worksheets and the evaluation included text-based feedback that was analysed qualitatively.

Sample

A purposive sampling was applied in order to recruit participants who meet the following criteria: 1) the teacher is a certified special education OR learning support teacher; 2) the teacher has an experience of teaching ISCED 2 level mathematics; 3) the teacher was able to use polynomials worksheets in autumn 2022 with their students. A call for participating in the study was forwarded through the e-mail list of the Association of Special Education Teachers in Estonia. The final sample consisted of six teachers who voluntarily tested the worksheets with their students and took part in an evaluation survey.

Design procedure and data collection

A set of 15 worksheets (55 pages) on the topic of polynomials for grade 8 was developed based on the National Curriculum of Estonia and the design principles stated in the introduction of this paper. Prior to this study, no differentiated materials were available for Estonian 8th grade mathematics students. The goal of the worksheets was to provide students the opportunity to practice the abstract nature of algebra in a step-by-step manner, as in regular materials the steps are usually not distinguished. The worksheets were forwarded to six teachers who tested them with 19 students in total during fall 2022. All teachers worked in general education schools that follow inclusive education principles and tested the worksheets with students who have been struggling with learning mathematics and are in need of differentiation. The teachers could decide on their own when, how and for how long they were using the worksheets in their teaching. After the testing period an evaluation survey was filled out by the teachers. In this current paper we are using the part of data that focused on student learning. The teachers were asked in one open-ended question to share their thoughts about their students' learning while using the worksheets. The design principles described in the introduction were not explicitly exposed to the teachers. If the teachers pointed out an aspect that could be improved in the worksheets, changes were applied to the final version of worksheets in order to finish the design-based research cycle.

Data analysis

For 15 worksheets, 48 segments of written feedback were received. An inductive content analysis was carried out in order to analyse the data. Sentences or parts of sentences that carried a meaning in order to answer the research questions were labelled with codes. Then, the codes under both research questions were divided into more general categories, depending on their focus, in order to synthesise the qualitative results. The coding and categorizing were reviewed and adjusted three times in order to ensure the trustworthiness of the results.

Results and discussion

The results and discussion are organized based on the emerged categories in order to emphasize the algebra worksheet design aspects that support students' learning, and the aspects that hinder learning.

Next, the results are presented and discussed together with quotations from teachers' answers and examples from developed worksheets.

Step-by-step difficulty increase

Changing the tasks' difficulty from easier to more difficult step-by-step was one of the aims for the development of the worksheets. The difficulty level was accordingly approached for all tasks on one worksheet, as well as internally for one task. The participating teachers brought up the need for taking slower and smaller steps with students struggling. They noticed that the step-by-step approach was one feature on the worksheets that helped their students to understand algebra better:

“The first tasks are very good. They guide students to talk through and explain where the numbers and variables come from. Also, they guide them to check their answer. It was very pleasant step-by-step practicing.”

Another benefit that was pointed out by teachers was the possibility to choose tasks that match students' level. It must be acknowledged that teacher play an important role in mediating mathematics tasks to students, considering their prerequisite knowledge to decide on the level of differentiation (Sullivan et al., 2015). In the current study, the teachers expressed their professional decision making in choosing appropriate tasks and valued the variations of task difficulty, as suggested in the previously referred literature.

To give an example of step-by-step task difficulty in the developed worksheets, Figure 1 presents the difficulty level change across one worksheet as well as across one task. On example A (worksheet “Multiplication of terms and polynomials”) the number of operations is manipulated throughout all tasks on the worksheet. The first two tasks are mainly aimed to recognize the position of terms and their function when multiplying term with a polynomial. The third task requires to write out the multiplication in correct order. Finally, the fourth task expects the student to directly write an answer of the multiplication. On example B (worksheet “Adding polynomials”) the changing difficulty across one task is achieved with the size and complexity of the numbers used and the number of operations needed to solve the problem.

A	B
Task 1 $5(2x + 1) = 5 \cdot \quad + 5 \cdot \quad$	A. $4a + (2a + 5) = \quad$
Task 2 $4(3x + 2) = \quad \cdot 3x + \quad \cdot 2$	B. $(4n + 3m^2) + (2m^2 + n) = \quad$
Task 3 $a(5 + 3) = \quad + \quad$	C. $(5a - 6b - 9) + (3a + 4b + 4) = \quad$
Task 4 $-x(y + 6) = \quad$	

Figure 1: An example of step-by-step difficulty increase

The teachers also mentioned how the increasing level helped students to stay interested and motivated to learn a certain concept that would otherwise seem too difficult to them. Allowing students to make smaller steps towards the learning goal supports their sense of succeeding and motivation to learn mathematics (Griffith & Ruan, 2005). Teachers in the current study also appreciated an example solution in the beginning of every task, as well as the opportunity to check an answer key after

completing a task. They described how this helped their students to become more independent and get quicker feedback about their own learning, indicating signs of self-monitoring. This is in line with Griffith and Ruan (2005) stating that applying a certain skill thoughtfully in a step-by-step manner supports the development of self-monitoring. As concluded in this study, in order to support students learning, algebra worksheet tasks must be designed in a way that it promotes self-monitoring every small step before reaching the final solution.

Visual cues

Second category of design element that teachers brought up as supporting their students' learning was the use of visual cues. In the developed worksheets different methods of visual cues were administered to support algebra learning: colour cues, highlighting, pointing arrows, and marking different steps with symbols. Next, a quotation from a teacher stating the positive effect of visual cues:

The first task of this worksheet directed students' attention to the signs and from there on students noticed signs much better than they usually do. Also, the colours supported their comprehension.

It is noted in literature that the visual representation of a task should have a cognitive purpose rather than only catching their attention (Sullivan et al., 2015). In the example of developed worksheet "Multiplication of polynomials" (Figure 2), on row C the colours are used on the developed worksheets to direct students' attention to the order of operations. As it was one of the first tasks, the burden of signs is removed. The colours of green and blue on the first two terms were used throughout the whole worksheet of that topic as being colour consistent is suggested in literature (Harper et al., 2009). In the example of E, the student already has to understand the concept of multiplying polynomials, however, extra attention is directed to the negative signs with using yellow highlight. In the example of F, a light-yellow background is used to emphasize the existence of constant that should not be forgotten when solving the problem.

C. $(2 + 3x)(1 - 3x) =$ - + -

E. $(-3 + x)(5 - 2x) =$

F. $(x - 3)(x^2 - 2x + 5) + 3x =$

Figure 2: An example of different visual cues

These are few examples of the visual cues on the worksheets that were brought up by the teachers as supporting factors in students' learning, confirming the existing research on using colour cues, especially in learning algebra (Sullivan et al., 2015). Visual cues are known for reducing time to locate meaningful information, supporting working memory, and reducing the overall cognitive load (Roberts, 2008).

Oversimplifying of tasks

Previously, a step-by-step difficulty was brought up as a benefit of the developed worksheets. The results of the study reveal that designing tasks for the first steps might have a risk of oversimplifying.

The teachers brought up that some tasks were too simple or produced automaticity rather than an independent skill, resulting in having less effect on actual learning of the topic. Looking at the corresponding tasks, a common design factor was constantly re-writing something while this operation could have been avoided (see, for example, step A from a task in worksheet “Organizing polynomials”, Figure 3). Teachers described how being asked to repeatedly write the same thing led students to boredom and loss of interest.

2. Organize the polynomial:

A. Write the terms.

B. Organize the order of the terms.

C. Write an organized polynomial.

Example: $2a + 3a^2$

$2a$

$3a^2$

Organized order:

$3a^2$

$2a$

Organized polynomial: $3a^2 + 2a$

Figure 3: An example of oversimplifying of tasks

In addition to what was already mentioned, the teachers pointed out some other factors that show oversimplifying of tasks: too little number of terms, too many positive numbers and no challenges with negative numbers, and finally avoiding special cases that might appear in algebra. According to Paas et al. (2003), designing a content that is extraneous is a concern in task design research and related to cognitive load theory. In order to specify the perfect content of a task that has a cognitive benefit to students more research on task design and the differences of learners is necessary, as also suggested by Watson and Ohtani (2015).

Overuse of visual cues

The teachers using and evaluating the worksheets brought up a confrontation – from one hand they value visual cues, but on the other hand, they pointed out certain situations where visual cues were overused. This happened mostly when visual cues were used to decrease the cognitive burden from students, e.g., having a part of answer written for them. In teachers’ opinion this does not help students to acquire the algebra topic fully. Next, a quotation from a teacher describing their experience with a task presented on Figure 4 (worksheet “Factoring perfect square trinomials”).

The brackets and the exponent could have been removed earlier. Right now, what happened to my students is that they did all the calculations but did not write the exponent on their own. All students made the same mistake.

Watson and Thompson (2015) emphasize that the aim of visual design should be to support students’ learning in terms of coordinating the eye and the brain in executing actions, as opposed to only making the page or task look more visually attractive. This is coherent with the results of the current study as giving out too many visual hints can hinder students’ own learning, as experienced by the teachers participating in the study.

Original problem:	J. $9x^2 - 18x + 9 = (\text{.....})^2$
Updated problem:	J. $9x^2 - 18x + 9 = \text{.....}$

Figure 4: An example of overusing visual cues

The teachers added that having too much written in the answer of the task causes missing out the concept, unless the completed part is removed earlier in the worksheet. Therefore, it is necessary to find a balance when designing visual cues and allowing the students to become more independent over time.

Conclusion

The current paper focused on developing differentiated algebra worksheets that could be used in an inclusive education setting for the purpose of differentiation. 19 students from grade 8 who have been struggling with learning mathematics had a chance to study with newly developed differentiated algebra worksheets on polynomials and their teachers shared their professional feedback on the effectiveness of the worksheets. The results showed that in teachers' opinion step-by-step difficulty increase can benefit students, as it allows them to study on an appropriate level and speed. Teachers also pointed out the usefulness of visual cues, mainly in terms of directing their attention. However, on the other hand, there is a risk of oversimplifying tasks and overusing visual cues. This result illustrates how sensitive is the balance between effective and non-effective aspects of algebra worksheet design for inclusive classrooms. It is important to point out that the results of the study cannot be generalized to the whole population due to its qualitative nature and small sample size. However, the results can be used in future developments of differentiated learning materials and studied in larger settings to specify the details of previously mentioned aspects and their effect in learning mathematics in inclusive classroom.

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