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On the Challenges of Estimating the Low-wavenumber Wall Pressure Field beneath a Turbulent Boundary Layer using a Microphone Array

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Abstract

The low-wavenumber components of the turbulent boundary layer (TBL) wall pressure field (WPF) are known to be the primary cause of structural vibration in low-Mach number flows, despite the maximal energy of the TBL being at the convective wavenumber. Existing semi-empirical TBL models show good agreement in predicting the WPF levels in convective region but differ significantly in the low-wavenumber domain. This study aims to highlight the challenges of estimating the low-wavenumber WPF in a TBL using a microphone array. A regularized Fourier-based approach is proposed to numerically study the estimation of the low-wavenumber WPF. Performance of the proposed method is initially evaluated by comparing the estimated WPF against a closed-form input TBL model. Effects of sensor spacing, co-array factor, and sensor distribution on the estimation of the low-wavenumber WPF levels are then investigated. To mimic experimental measurements, a virtual acoustic experiment is proposed, involving the synthesis of snapshots of TBL-induced WPF. It is demonstrated that although with relatively small number of snapshots the convective region can be identified, a significant number of snapshots is required to well estimate the TBL low-wavenumber region.

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Keywords: turbulent boundary layer, wall pressure fluctuations, microphone array, low-wavenumber domain, virtual acoustic experiment

1. Introduction

The interaction of a fluid flow with structures generates vibration and noise, which has significant implications in many engineering applications, including the vibration and noise produced in water transport pipelines, the prediction and reduction of aircraft cabin noise, and sound generation in automobiles [1, 2, 3]. Different internal and external forces can cause the vibration and radiated noise of these structures. TBL, as one of the main noise and vibration contributors, generates pressure fluctuations over a surface, subsequently imposing an unsteady load on the structure that leads to noise and vibrations.

Although the maximum energy of the TBL occurs around the convective wavenumber, it is known that the low-wavenumber components of the WPF beneath a TBL is the main cause of structural vibration in low Mach number flows associated with marine applications [4, 5, 6, 7]. This is because the structure filters the convective ridge of the TBL excitation at frequencies well above the aerodynamic coincidence frequency [8]. This has been graphically illustrated in Fig. 1 where the schematic of spatial matching of the wavenumber-frequency spectrum of the TBL WPF, $\phi_{pp}(k, \omega)$, and vibration modal response of the structure, $\Psi_s(k, \omega)$, are plotted under the condition $U_c < c_b < c_0$ where U_c , c_b and c_0 are the fluid convective velocity, bending wave speed of the structure and speed of sound, respectively. The vibration response of the structure can be mathematically determined by integrating the product of the WPF excitation and modal response of the structures in the wavenumber domain [3]. Therefore, accurate estimation of the WPF in this region is of paramount importance to the prediction of TBL-induced vibrations.

A variety of semi-empirical TBL models have been developed and are available in literature such as the Corcos [9], Mellen [10] and Chase [11]. Despite the fact that most models are in good agreement when it comes to predicting the convective region, there is a significant discrepancy at sub-convective region as shown in Fig. 2. Historically, it has been difficult to model and measure the low-wavenumber levels of the TBL WPF due to their relatively low amplitudes compared to the amplitude of the WPF at convective wavenumber. Moreover, most of the existing body of research on

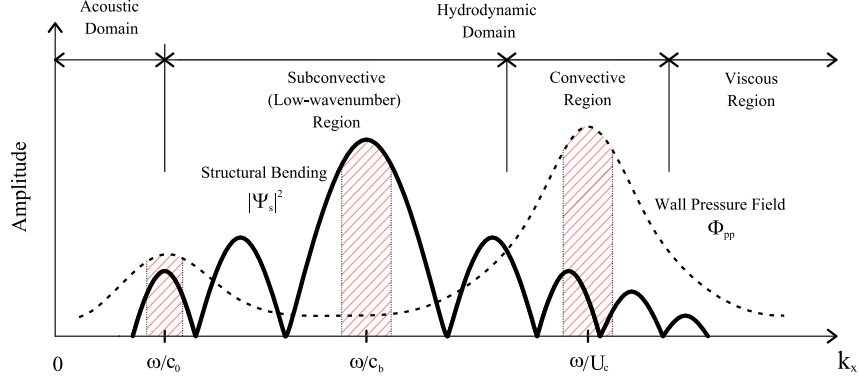


Fig. 1. Schematic of the spatial matching of the wavenumber-frequency spectrum of the TBL WPF and vibration modal function when $U_c < c_b < c_0$ (not in scale). The solid thick line represents the squared magnitude of the structural mode shape, while the dashed line represents the WPF.

35 estimation of the TBL WPF focuses on the convective ridge and not on the
 36 low-wavenumber region despite of its importance. This highlights the need
 37 for further study to better understand the process of identification of the
 38 low-wavenumber WPF. Hence, this work aims to investigate the key param-
 39 eters in the estimation of the low-wavenumber WPF beneath a TBL using
 40 numerical study.

41 Microphone arrays are typically used to measure the WPF. They di-
 42 rectly capture the sound waves generated by the WPF. To estimate the
 43 WPF, microphones are usually placed near a rigid wall exposed to turbu-
 44 lent flow, recording the sound waves produced by the pressure fluctuations.
 45 However, this approach has limitations related to spatial resolution, which
 46 is constrained by microphone spacing and configuration. Reducing the spac-
 47 ing between microphones can enhance resolution but often requires more
 48 sensors, increasing both cost and setup complexity. Conversely, increasing
 49 the distance between sensors may result in the failure to capture all sam-
 50 ples of incoming sound waves, leading to a reduction in the resolution of
 51 high-frequency pressure fluctuations. Furthermore, the data recorded in this
 52 approach can easily be contaminated by background noise and instrument
 53 recording noise.

54 To address these issues, advanced signal processing techniques have been

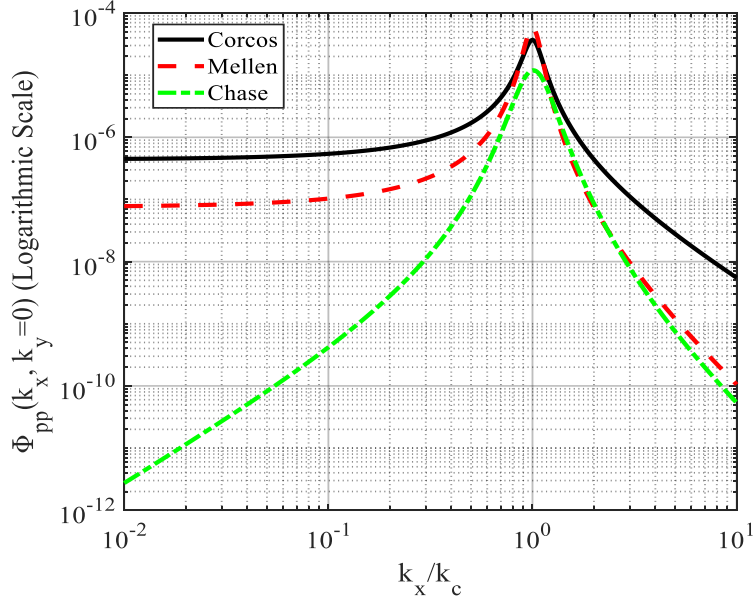


Fig. 2. Wavenumber-frequency spectra for $f = 1000$ Hz as a function of non-dimensional wavenumber for $k_y = 0$ for three semi-empirical TBL models. k_x and k_y represent the streamwise and spanwise wavenumbers, respectively, and k_c is the convective wavenumber.

55 developed to extract the WPF information [3, 12]. This enables the direct
 56 measurement of pressure fluctuations, offering detailed information about
 57 the pressure field's characteristics [13]. One widely used signal processing
 58 technique is beamforming, which combines microphone outputs to form a
 59 directed or focused sensitivity beam. This enhances the desired signal while
 60 suppressing interference and noise from other directions. Various beamform-
 61 ing techniques are discussed in [14, 15, 16]. It should be noted that some
 62 researchers have used vibration measurements of a structure excited by a
 63 TBL to estimate the WPF [17]. However, in this work, we consider only
 64 the use of a microphone array, which is the most commonly used method in
 65 the literature, with a focus on highlighting challenges on estimation of the
 66 low-wavenumber WPF.

67 Panton and Robert made the initial attempt in using microphones to mea-
 68 sure the turbulent wall-pressure spectrum [18]. They utilised only two micro-
 69 phones and increased the distance between them along a line for measuring
 70 two-point cross-spectral pressure. This concept had been further expanded
 71 to the point that an array of sensors was used for the first time by Maidanik

72 [19] and was further developed by Blake and Chase [20] and Farabee and
73 Geib [21]. In this approach, the microphones are spaced regularly at specific
74 intervals in a linear streamwise array. Using a linear streamwise equidistant
75 array with intervals of d , they could recognize pressure fluctuations around
76 $k_x = \pi/d$ by analyzing alternate microphone outputs. These mode arrays
77 were then used to calculate spectral levels in sonic and subsonic regions by
78 selecting suitable frequencies [3]. The large surface area of each individual
79 sensor effectively filters out many of the undesired pressure fluctuations asso-
80 ciated with convective ridge motions. However, some noises were introduced
81 into the measurement through spatial aliasing [22]. This issue arises due to
82 the finite size of practical sensors and the process of signal averaging over a
83 sensor's surface, which inherently limits the precision of the WPF measure-
84 ments at high frequencies [23]. Consequently, using larger sensors results in
85 reduced resolution for high-frequency pressure fluctuations. Corcos [24] was a
86 pioneer in exploring the relationship between sensor size and the correspond-
87 ing spectral attenuation based on theoretical foundations. More recently,
88 Hu [23] introduced a correction model to address high-frequency attenuation
89 associated with sensor size when measuring WPF beneath the TBL.

90 Aliasing occurs in any array where sensor spacing cannot resolve the
91 smallest turbulent scales [3]. The results of using a large number of sensors
92 were presented by Manoha [25] and Bermer [26] to alleviate the aliasing effect.
93 In order to comprehensively capture the spatial characteristics of the two-
94 point cross-spectrum of the WPF, which can subsequently undergo Fourier
95 transformation to generate the wavenumber-frequency spectrum of wall pres-
96 sure, an alternative methodology was introduced [27, 28]. In this technique,
97 transducers were deployed in an array spanning the diameter of a disk, ar-
98 ranged along a line that can be rotated to various angular positions. By
99 conducting measurements repeatedly at different rotational orientations of
100 this array, it becomes possible to extract numerous cross-spectral attributes
101 pertaining to the boundary layer. Subsequently, a wavenumber analysis can
102 be conducted to derive valuable insights from the data. The technique of
103 using large arrays and small sensors has enabled investigators to create clear
104 maps illustrating the convective ridge and acoustic cone [29].

105 It should be noted that a periodic arrangement of microphones is not
106 the best choice since it causes redundant distances between sensors [30].
107 Various techniques have been proposed to optimize array efficiency. One
108 of the most common techniques is using an array with a spiral shape [31].
109 The advantage of using a non-equidistant array was studied by Haxter and

110 Spehr [30] in 2014. They evaluated the efficiency of equidistant and non-
111 equidistant array patterns in detecting a single source in the wavenumber
112 domain. They showed that opposed to the non-equidistant spaced array
113 pattern, the equidistant array has amplitudes on the side lobes identical
114 to that of the main lobe. In other words, the non-equidistant spacing of
115 the transducers has the capability to transfer aliasing effects at a greater
116 wavenumber than in the equidistant array. In addition to Nyquist criterion
117 and array pattern, the co-array size plays an important role in the array
118 performance [3]. Co-array describes the number of different cases where the
119 distance between every pair of sensors is unique [32]. More recently, Schram
120 et al. [33] applied a similar procedure to Ref. [30] and used microphones on
121 a rotatable disk to minimize the number of rotation angles and acquisition
122 time while providing a relatively uniform sampling of the co-array plane.

123 Beamforming is a powerful technique for enhancing the resolution of sensor
124 arrays, enabling more accurate localization and characterization of sound
125 sources in complex or noisy environments. This method is developed to amplify
126 signals arriving from specific directions. In the context of TBL problems,
127 beamforming is particularly useful for identifying and localizing regions
128 with high levels of WPF and provide valuable insights into the characteristics
129 of turbulent flows. Ehrenfield and Koop [34] were among the pioneers
130 who utilized the beamforming method in their analysis. They measured the
131 WPF beneath a compressible TBL at a high subsonic Mach number using
132 a sparse array of pressure transducers in a wind tunnel. They applied the
133 infinite beamforming technique and deconvolution algorithm to deconvolve
134 the wavenumber-frequency spectrum from the surface pressure array data.
135 They only detect the domains associated with convective peak and acoustic
136 peak in their studies and showed that acoustic noise is particularly dominant
137 in the lower frequencies. In 2017, Haxter et al [35], conducted a study that
138 built upon the work of Ehrefried and Koop [34] by using the same microphone
139 array arrangement to obtain the phase velocity of TBL pressure fluctuations
140 at high subsonic Mach number from wind tunnel data affected by strong
141 background noise. They used a method called CLEAN-SC to remove the
142 dominant existing acoustic signals and their coherent parts in the beam-
143 forming map, which improved the accuracy of their results. Additionally,
144 Prigent et al. [36] used beamforming and DAMAS deconvolution techniques
145 to process a synthetic field consisting of a diffuse acoustic field and the Corcos
146 WPF model. To estimate the WPF, they utilized an aligned microphone
147 array with a rotating configuration.

148 This paper aims to highlight the challenges of using an array of micro-
149 phones for estimating the low-wavenumber region of TBL wall pressure fluc-
150 tuations. Most previous studies have primarily focused on identifying the
151 convective ridge and acoustic peak. Moreover, the importance of the low-
152 wavenumber domain in the vibration of structures subjected to turbulent
153 flow as well as the significant discrepancies between different existing TBL
154 models for this region are the main motivations for this work. For this
155 purpose, the study employs a regularized Fourier-based approach (RFBA).
156 This approach relies on the inverse Fourier transform (IFT) expression that
157 links the cross spectrum density (CSD) of the pressure in both physical and
158 wavenumber space. The discretization of the integral in this expression is
159 achieved using the rectangular rule, which results in a linear matrix system.
160 An adapted regularization technique is then used to invert this system and
161 estimate a stable solution. To assess the capability of the RFBA in esti-
162 mating the low-wavenumber components of the WPF, numerical simulations
163 of a TBL excitation are conducted, and the WPF estimated by the RFBA
164 using a microphone array is compared with the reference WPF of the input
165 TBL model. Considering this process, the effect of number of sensors, ar-
166 ray pattern, co-array factor and data averaging on the estimated WPF are
167 examined. Moreover, to mimic experimental measurements, a virtual acous-
168 tic experiment is proposed, involving the synthesis of snapshots of the TBL
169 WPF.

170 The structure of the paper is as follows: Section 2 describes the RFBA,
171 while Section 3 evaluates its effectiveness. Section 3.1 focuses on calculating
172 the CSD matrix of the pressure measured by a virtual microphone array from
173 a closed-form semi-empirical TBL model and particularly studies the effects
174 of three factors of sensor spacing, co-array factor, and sensor distribution on
175 estimation of the WPF in the low-wavenumber domain. In order to simulate
176 real experimental measurements, Section 3.2 estimates the CSD matrix of
177 the pressure measured by the virtual microphone array by averaging different
178 snapshots of the WPF induced by the TBL. These snapshots are generated by
179 employing so-called uncorrelated wall plane wave (UWPW) technique [37].
180 Performance of the RFBA on estimating the WPF in the low-wavenumber
181 domain is evaluated based on this virtual experiment. The paper concludes
182 with a discussion in Section 4 and a summary of the findings.

183 **2. The Regularized Fourier-based Approach**

184 This section covers the theoretical formulation of the regularized Fourier-
 185 based approach to estimate the WPF in the wavenumber domain using pres-
 186 sure measurements obtained from microphones. Fig. 3 shows a network of
 187 N_s flush-mounted microphones that are installed on a rigid surface. They
 188 are distributed within a rectangular area measuring $L_x \times L_y$. The position
 189 of each microphone is determined by the coordinates \mathbf{x}_i , denoted as (x_i, y_i)
 190 for $i \in \{1, N_s\}$. The sensors are used for recording the WPF beneath a TBL.
 191 The TBL is assumed to be homogeneous, stationary and fully developed over
 192 the surface. The x -axis is considered parallel to fluid flow with a constant
 193 free stream velocity of U_∞ .

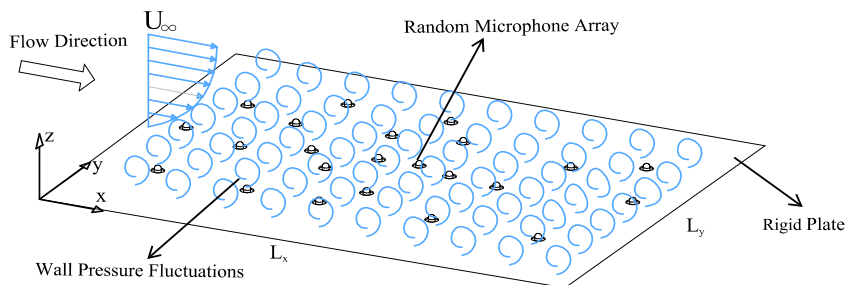


Fig. 3. Schematic representation of a microphone array mounted within a rectangular area with dimensions L_x in length and L_y in width to measure wall pressure fluctuations from the TBL.

194 The wavenumber-frequency spectrum $\phi_{pp}(k_x, k_y, \omega)$ of the wall pressure
 195 $p(x, y, t)$ can be expressed as follows [34]

$$\phi_{pp}(k_x, k_y, \omega) = \frac{1}{4\pi^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} R_{pp}(\xi, \eta, \omega) e^{-j(k_x\xi + k_y\eta)} d\xi d\eta, \quad (1)$$

196 where (ξ, η) are the distances between two points in the (x, y) plane, ω
 197 is the angular frequency, $j = \sqrt{-1}$ is the imaginary unit, and k_x, k_y are
 198 wavenumber components in the streamwise and spanwise direction, respec-
 199 tively. $R_{pp}(\xi, \eta, \omega)$ is the temporal Fourier transform of the space-time cor-
 200 relation function of wall pressure given by [34]

$$R_{pp}(\xi, \eta, \omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} G_{pp}(\xi, \eta, \tau) e^{j\omega\tau} dt, \quad (2)$$

201

$$G_{pp}(\xi, \eta, \tau) = \langle p(x, y, t) p(x + \xi, y + \eta, t + \tau) \rangle, \quad (3)$$

202 where the angle bracket $\langle \dots \rangle$ denotes the mathematical expectation. The
 203 IFT of the Eq. (1) can be used to determine how the wavenumber spectrum
 204 $\phi_{pp}(k_x, k_y, \omega)$ relates to $R_{pp}(\xi, \eta, \omega)$

$$R_{pp}(\xi, \eta, \omega) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \phi_{pp}(k_x, k_y, \omega) e^{j(k_x \xi + k_y \eta)} dk_x dk_y. \quad (4)$$

205 By employing a rectangular integration method over a truncated wavenumber
 206 domain, one can approximate $R_{pp}(\omega)$ between two points \mathbf{x}_i and \mathbf{x}_j as follows

$$R_{pp}(\xi_{i,j}, \eta_{i,j}, \omega) \approx \sum_{l=1}^{N_k} \phi_{pp}(k_{x,l}, k_{y,l}, \omega) e^{j(k_{x,l} \xi_{i,j} + k_{y,l} \eta_{i,j})} \delta k_x \delta k_y, \quad (5)$$

207 where $(\xi_{i,j}, \eta_{i,j}) = (x_i - x_j, y_i - y_j)$ with $i, j = 1, 2, \dots, N_s$ and $\delta k_x, \delta k_y$
 208 are the wavenumber resolutions in the streamwise and spanwise directions,
 209 respectively, and $N_k = N_{k_x} \times N_{k_y}$ corresponds to the total number of grid
 210 points in the truncated wavenumber space, and each vector index l is assigned
 211 uniquely to a grid point $(k_{x,l}, k_{y,l})$. A cut-off wavenumber is defined to take
 212 into account the convective contributions of the TBL WPF (see Section 3).
 213 Eq. (5) can be represented in matrix notation as follows

$$\mathbf{S}_{pp} = \mathbf{Q} \Phi_{pp}, \quad (6)$$

214 where \mathbf{S}_{pp} is a vector consisting of the cross-spectrum elements and Φ_{pp}
 215 is a vector consisting of the unknown WPF components in the truncated
 216 wavenumber space as follows

$$\mathbf{S}_{pp} = \begin{bmatrix} R_{pp}(\xi_{1,1}, \eta_{1,1}, \omega) \\ R_{pp}(\xi_{1,2}, \eta_{1,2}, \omega) \\ \vdots \\ R_{pp}(\xi_{i,j}, \eta_{i,j}, \omega) \\ \vdots \\ R_{pp}(\xi_{N_s, N_s-1}, \eta_{N_s, N_s-1}, \omega) \\ R_{pp}(\xi_{N_s, N_s}, \eta_{N_s, N_s}, \omega) \end{bmatrix}_{N_s^2 \times 1}, \quad \Phi_{pp} = \begin{bmatrix} \phi_{pp}(k_{x,1}, k_{y,1}, \omega) \\ \phi_{pp}(k_{x,1}, k_{y,2}, \omega) \\ \vdots \\ \phi_{pp}(k_{x,l}, k_{y,l}, \omega) \\ \vdots \\ \phi_{pp}(k_{x, N_{k_x}}, k_{y, N_{k_y}-1}, \omega) \\ \phi_{pp}(k_{x, N_{k_x}}, k_{y, N_{k_y}}, \omega) \end{bmatrix}_{N_k \times 1} \quad (7)$$

217 The components of Φ_{pp} are organized such that the first N_{k_y} components
 218 correspond to $\phi_{pp}(k_{x,1}, k_{y,l}, \omega)$ with $l \in \{1, N_{k_y}\}$, the next N_{k_y} components

219 correspond to $\phi_{pp}(k_{x,2}, k_{y,l}, \omega)$ with $l \in \{1, N_{k_y}\}$, and so on. Also, for a
 220 microphone array with N_s sensors, the elements of \mathbf{S}_{pp} can be measured for
 221 the discrete separations of sensors, $(\xi_{i,j}, \eta_{i,j})$, which the first N_s components
 222 correspond to $R_{pp}(\xi_{1,j}, \eta_{1,j}, \omega)$ with $j \in \{1, N_s\}$, the next N_s components
 223 correspond to $(\xi_{2,j}, \eta_{2,j}, \omega)$ with $j \in \{1, N_s\}$, and so on. Besides, \mathbf{Q} is the
 224 matrix with the following elements

$$\mathbf{Q} = \delta k_x \delta k_y \begin{bmatrix} e^{j(k_x, 1\xi_{1,1} + k_y, 1\eta_{1,1})} & e^{j(k_x, 1\xi_{1,1} + k_y, 2\eta_{1,1})} & \dots & \dots & e^{j(k_x, N_{k_x}\xi_{1,1} + k_y, N_{k_y}\eta_{1,1})} \\ e^{j(k_x, 1\xi_{1,2} + k_y, 1\eta_{1,2})} & \ddots & & & \vdots \\ \vdots & & e^{j(k_x, l\xi_{i,j} + k_y, l\eta_{i,j})} & & \vdots \\ \vdots & & & \ddots & e^{j(k_x, N_{k_x}\xi_{N_s, N_s-1} + k_y, N_{k_y}\eta_{N_s, N_s-1})} \\ e^{j(k_x, 1\xi_{N_s, N_s} + k_y, 1\eta_{N_s, N_s})} & \dots & \dots & \dots & e^{j(k_x, N_{k_x}\xi_{N_s, N_s} + k_y, N_{k_y}\eta_{N_s, N_s})} \end{bmatrix} \cdot \quad (8)$$

225 Considering Eq. (6), we arrive at N_s^2 equations for the N_k unknown
 226 coefficients. In most cases, the number of unknowns N_k exceeds the num-
 227 ber of equations N_s^2 . Eq. (6) is therefore an under-determined system and
 228 the system of equations has no unique solution. Using the Moore-Penrose
 229 inverse of matrix \mathbf{Q} can yield a solution with minimal 2-norm, but the prob-
 230 lem posed by Eq. (4) is equivalent to a first-kind Fredholm integral equation
 231 that is known to be ill-conditioned [38]. Hence, the inversion method derived
 232 from the discretization of the Riemann integral formula leads to a severely
 233 ill-conditioned linear system (i.e., Eq. (6)) with many tiny singular values.
 234 This means applying the Moore-Penrose inversion using singular value de-
 235 composition (SVD) generates inadequate results. Since matrix \mathbf{Q} can contain
 236 small rounding errors due to computer calculation and vector \mathbf{S}_{pp} can con-
 237 tain errors induced by measurement in practice, the ill-conditioning of \mathbf{Q} can
 238 greatly amplify these errors, resulting in erroneous results. However, adapted
 239 regularization techniques can produce useful stabilized solutions [38, 39, 40].
 240 The goal of regularization theory is to provide proper side constraints with
 241 optimal weights so that the regularized solution is a good approximation of
 242 the unknown solution. Different regularization techniques described in [38]
 243 were applied to Eq. (6) to evaluate the WPF in the low-wavenumber domain.
 244 The truncated generalized singular value decomposition (TGSVD) method
 245 with minimising the first derivative 2-norm of the solution was found to be
 246 the most appropriate [38, 41]. The regularization parameter is determined
 247 from the corner of the discrete L-curve produced by the TGSVD method
 248 [42]. For the numerical applications presented herein, the Matlab package
 249 developed by C. Hansen for the analysis and solution of discrete ill-posed
 250 problems [38] was utilized (See Appendix A).

251 **3. Results and Discussion**

252 To evaluate the WPF in the low-wavenumber domain, the procedure de-
 253 scribed in Section 2 is employed, and the results obtained by the RFBA are
 254 examined.

255 According to the Graham formulation [43, 44], the CSD of the WPF can
 256 be computed using various models for auto-spectral density (ASD) of the
 257 pressure field, $\Psi_{pp}(\omega)$, and the normalized CSD of the pressure field, $\tilde{\phi}_{pp}(k, \omega)$,
 258 independently from each other as follows

$$\phi_{pp}(\mathbf{k}, \omega) = \Psi_{pp}(\omega) \left(\frac{U_c}{\omega} \right)^2 \tilde{\phi}_{pp}(\mathbf{k}, \omega). \quad (9)$$

259 In this work, the ASD function of the WPF is evaluated using the Goody
 260 model described in Appendix B (Eq. (B.1)). It should be noted that $\Psi_{pp}(\omega)$
 261 is a one-sided radial frequency spectrum. Therefore, to convert it into cyclic
 262 frequency spectrum density $\Psi_{pp}(f)$, $\Psi_{pp}(\omega)$ was multiplied by 2π . For the
 263 normalized CSD function, various semi-empirical models have been devel-
 264 oped [45]. The Corcos model is by far the most popular model since it con-
 265 siders homogeneity across the surface, and this assumption leads to a cross
 266 spectrum model dependent only on the separation distances [9]. Thus, the
 267 Corcos model has two separate relationships for representing the in-flow and
 268 cross-flow directions of the WPF [9]. Even though separability is convenient
 269 analytically, it is not a realistic assumption. Other researchers recognized
 270 this issue and proposed a simple change to the Corcos model. For example,
 271 Mellen proposed an elliptical coherence zone, which is different from the Cor-
 272 cos model with the rhombic coherence zone [3]. It is well known that Corcos
 273 model overpredicts the amplitude of the low-wavenumber domain, whereas
 274 the Mellen model provides more realistic predictions of the low-wavenumber
 275 levels. This has been confirmed by comparing the vibration responses of a
 276 plate excited by a TBL modelled by the Corcos/Mellen models with exper-
 277 imental data [46]. Hence, the Mellen model is used here as the normalized
 278 CSD function (see Appendix B, Eq. (B.2)). For all the subsequent numerical
 279 analyses, a wavenumber resolution of $\delta k_x = \delta k_y = 4 \text{ m}^{-1}$ is considered and
 280 the results are presented at frequency of 1000 Hz.

281 Furthermore, in order to assess the effectiveness of the proposed method
 282 for estimating the WPF in the low-wavenumber domain, we have defined
 283 the low-wavenumber domain as the region within the flexural wavenumber
 284 $(-k_b \leq k_x, k_y \leq k_b)$ of a steel plate with a 1 mm thickness. The plate's

285 properties include a Young’s modulus of 210 Gpa, a density of 7800 (kg m⁻³),
 286 and a Poisson’s ratio of 0.3, resulting in a flexural wavenumber of $k_b = 63.26$
 287 m⁻¹. The low-wavenumber region is indicated with the square area in Fig. 4
 288 where the CSD function of the reference TBL using the Goody and truncated
 289 Mellen models is plotted. In Section 3.1 and 3.2 we employ the RFBA
 290 to estimate the WPF in the low-wavenumber domain and the results are
 291 compared with those simulated using the theoretical WPF formula based on
 292 the Goody and truncated Mellen models as shown in Fig. 4.

293 A turbulent flow with an air flow speed of $U_\infty = 50$ m s⁻¹ is assumed
 294 flowing over the rigid surface, see Fig. 3. The values of air density and
 295 the kinematic viscosity are set to 1.225 kg m⁻³ and 1.5111×10^{-5} m² s⁻¹,
 296 respectively. It is assumed that the TBL is homogeneous, stationary and fully
 297 developed over the panel surface. The TBL parameters used for this analysis
 298 are given in Table 1. Moreover, the convective velocity U_c is approximated
 299 using Bull’s model [47, 5] as follows

$$U_c \approx U_\infty \left(0.59 + 0.3e^{-0.89\delta^*\omega/U_\infty} \right), \quad (10)$$

300 where δ^* is displacement thickness.

301 The simulations are performed in Matlab on a desktop computer with
 302 32 GB of RAM and four physical cores. To employ Eq. (5), one needs to
 303 truncate the wavenumber domain. It is necessary to note that the range
 304 of the considered wavenumber domain should be large enough to be able
 305 to include the significant contribution of the CSD function. Hence, a cut-off
 306 wavenumber of $1.2k_c$ was used in both the streamwise and spanwise directions
 307 to take into account the convective contributions of the TBL WPF, where
 308 $k_c = \omega/U_c$ is the convective wavenumber. It is noteworthy to mention that
 309 converge studies have been done for the selection of the cut-off wavenum-
 310 ber and wavenumber resolution to ensure that the input TBL is accurately
 311 modelled.

Table 1. TBL parameters for a air flow with speed of 50 m/s.

Parameter	Value
TBL thickness δ (mm)	5.77
TBL displacement thickness δ^* (mm)	0.729
Wall shear stress τ (pa)	5.989

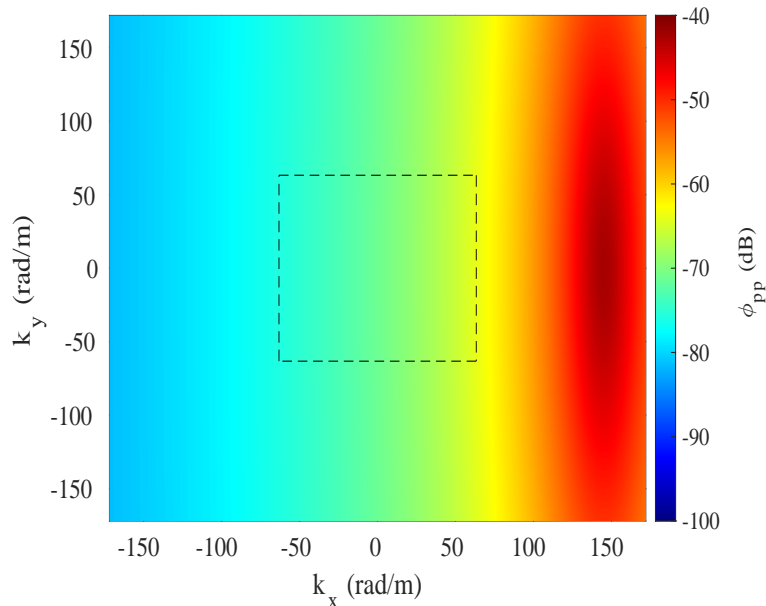


Fig. 4. Contour plots of the Goody+Mellen wavenumber-frequency model for a flow speed of 50 m/s at 1000 Hz.

312 ***3.1. Effect of Microphone Array Parameters on the Estimated***
 313 ***TBL Wall Pressure Field***

314 In this section, the effects of array parameters namely number of sensors,
 315 co-array factor and sensor distribution on the performance of the RFBA are
 316 examined. The array size is kept constant in all subsequent calculations.

317 ***3.1.1. Effect of Sensor Spacing***

318 Before processing the array data for signal analysis, the initial step in-
 319 volves establishing the relative position of sensors, which is crucial in array
 320 creation. In this process, special attention must be given to avoid spatial
 321 aliasing. Spatial aliasing arises as a result of spatially under-sampling the
 322 aperture of the array. To avoid aliasing in time domain signal processing, it is
 323 essential to sample the signal at a rate of at least twice the highest frequency.
 324 This sampling rate, known as the Nyquist rate [48], can also be applied in
 325 spatial domain signal processing by ensuring that the sampling interval does
 326 not exceed one-half wavelength [32]

$$k_{\text{sample}} = 2k_{\text{max}} = \frac{2\pi}{\Delta x}. \quad (11)$$

327 This equation indicates a direct relationship between the Nyquist wave-
328 form frequency and the sampling interval Δx . This interval, as determined
329 by the Nyquist principle, sets a limit on the maximum distance that can
330 exist between microphone positions. As mentioned above, a criterion of 1.2
331 times of the convective wavenumber is considered for the highest waveform
332 frequency in measuring the pressure with a microphone array. This criterion
333 is taken into account in Eq. (11), so the minimum distance between the
334 microphone positions is obtained as $\Delta x = \pi/k_{\max}$. If this criterion is not
335 fulfilled, then the aliasing can be observed in the low-wavenumber domain,
336 which is the region of interest.

337 In this section, an equidistant cross-array pattern with the fixed size of
338 $L_x = 455$ mm and $L_y = 375$ mm is assumed to demonstrate the aliasing
339 phenomenon and the effects of sensor spacing on the estimated TBL WPF.
340 The study considers a minimum of 16 sensors, with the number of sensors
341 increased by 4 until the maximum of 68 sensors is reached (as shown in the
342 supplementary document). Fig. 5 presents the results for only four selected
343 cases, namely those with 16, 32, 48, and 68 sensors. As an additional feature,
344 Figs. 5 (c), (g), (k), and (o) demonstrate all possible vector spacings between
345 all pairs of sensors, along with the co-array factors corresponding to each case
346 study. In the upcoming section (Section 3.1.2), the impact of this parameter
347 will be discussed. Fig. 5 (b), (f), (j), and (n) show the color map and
348 Figs. 5 (d), (h), (l), and (p) show the corresponding cross-section view of
349 the estimated WPF obtained by RFBA, respectively, for different number
350 of sensors of cross-array pattern. The color maps in the Fig. 5 include a
351 rectangular area which is surrounded by flexural wavenumber of the assumed
352 plate and denoting the range of low-wavenumber domain which needs to be
353 evaluated ($-k_b \leq k_x, k_y \leq k_b$). This range is shown in the cross-section view
354 of the results with the red dashed-line.

355 To quantify the performance of the proposed method in the estimation
356 of the TBL WPF, the mean absolute error (MAE) of the estimated WPF in
357 the low-wavenumber domain is calculated for each case with respect to the
358 reference input TBL model based on the Goody and Mellen Models [49, 10]
359 in the corresponding low-wavenumber domain. The following formula is used
360 to compute the MAE of the estimated WPF in the low-wavenumber domain

$$\text{MAE} = \frac{1}{N_{k_{LW}}} \sum_{l=1}^{N_{k_{LW}}} |10\log_{10}\phi_{pp}^e(k_{x,l}, k_{y,l}, \omega) - 10\log_{10}\phi_{pp}^r(k_{x,l}, k_{y,l}, \omega)|, \quad (12)$$

361 where $N_{k_{LW}}$ corresponds to the total number of grid points in the low-
 362 wavenumber domain. $\phi_{pp}^e(k_{x,l}, k_{y,l}, \omega)$ and $\phi_{pp}^r(k_{x,l}, k_{y,l}, \omega)$ are the estimated
 363 and reference wavenumber-frequency spectrum of the WPF, respectively. It
 364 can be observed from Fig. 5 that by increasing the number of sensors, the es-
 365 timated WPF is improved, and it gradually converges towards the reference
 366 input TBL model (see, Fig. 4).

367 It is clear from Figs. 5 (f) and (h) that RFBA struggle to provide rea-
 368 sonable estimation of the WPF due to the presence of aliasing phenomenon
 369 when $N_s < 48$. For the given array size, the aliasing effect is mitigated by
 370 increasing the number of sensors to 48. This is consistent with Nyquist crite-
 371 rion, as for the considered array size, according to Eq. (11) at least 24 sensors
 372 are required along the x -axis and 20 sensors along the y -axis to satisfy the
 373 Nyquist criterion and avoid aliasing effect. Assuming that the number of
 374 sensors is the same in both the streamwise and spanwise directions, the min-
 375 imum number of sensors required to satisfy the criterion is $N_s = 48$ which is
 376 what we observed in Figs. 5 (j) and (i). An interactive plot demonstrating
 377 the impact of increasing the number of sensors on reducing the aliasing effect
 378 is shown in Fig. S1 of the supplementary document.

379 As illustrated by Figs. 5 (j) and (n) and their corresponding MAEs, the
 380 accuracy of estimated results is improved by increasing the number of sensors
 381 from 16 to 48, but adding more sensors does not significantly enhance the
 382 estimated WPF in the low-wavenumber domain (see Figs. 5 (j) and (n)).
 383 This suggests that respecting the Nyquist criterion alone is not sufficient for
 384 obtaining accurate estimation of the low-wavenumber WPF. However, this
 385 does not hold true for the convective region. Fig. 5 (h) shows that estimation
 386 of the convective region is much easier than the low-wavenumber domain as it
 387 has the highest amplitude in the domain. Moreover, a good estimation of this
 388 region is achieved using only 24 sensors which does not satisfy the Nyquist
 389 criterion (see Fig. S1) and the estimated result in this region is quite accurate
 390 when the Nyquist criterion is fulfilled. Therefore, unlike the low-wavenumber
 391 region, accurate estimation of the convective region is possible by fulfilling
 392 only the Nyquist criterion.

393 Figs. 5 (c), (g), (k), and (o) also show that the co-array factor F is always
 394 below 0.4 for all the sensor spacing using the equidistant cross array. The
 395 effect of this parameter is examined in the following section.

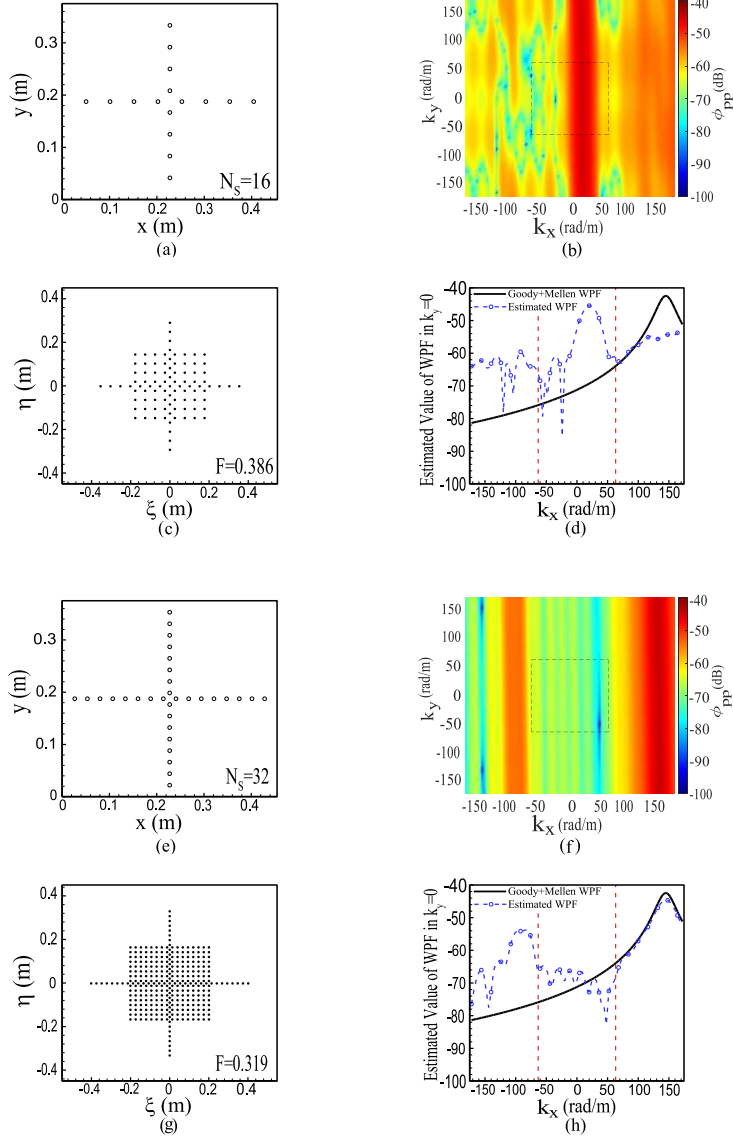


Fig. 5 - Part 1. Comparison of the estimated WPF using RFBA for equidistant cross-array pattern with 16 (a-d) and 32 (e-h) sensors, respectively. Equidistant cross arrays for each case are shown in (a) and (e) and associated set of distinct vector spacings between sensors are presented in (c) and (g). Co-array factor (F) are displayed for each case and MAEs calculated between the reference input TBL model and the estimated low-wavenumber WPF shown in (b) and (f) are 13.48 dB and 5.67 dB, respectively. 2D wavenumber-frequency spectra for $k_y = 0$ are plotted against longitudinal wavenumber in (d) and (h).

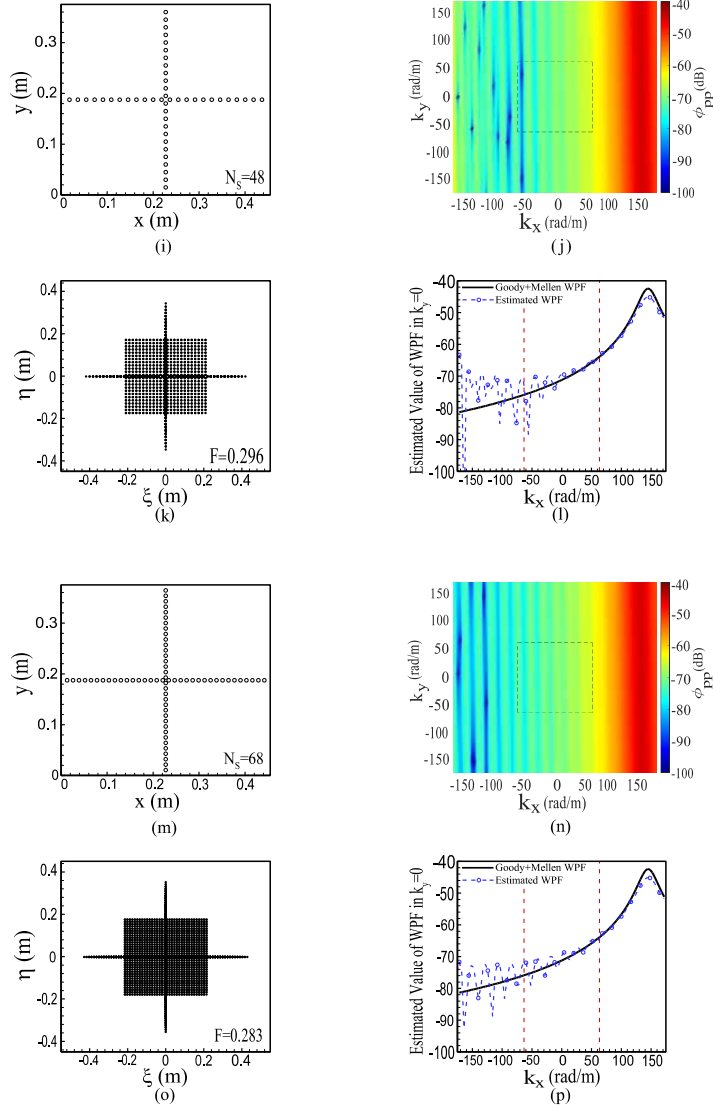


Fig. 5 - Part 2. Comparison of the estimated WPF using RFBA for equidistant cross-array pattern with 48 (i-l) and 68 (m-p) sensors, respectively. Equidistant cross arrays for each case are shown in (i) and (m) and associated set of distinct vector spacings between sensors are presented in (k) and (o). Co-array factor (F) are displayed for each case and MAEs calculated between the reference input TBL model and the estimated low-wavenumber WPF shown in (j) and (n) are 1.81 dB and 1.49 dB, respectively. 2D wavenumber-frequency spectra for $k_y = 0$ are plotted against longitudinal wavenumber in (l) and (p).

396 **3.1.2. Effect of Co-array Factor**

397 In terms of array performance, in addition to the minimum distance be-
 398 tween sensors, the size of co-array is an important factor to be considered.
 399 Co-array describes the number of different distances between every pair of
 400 sensors in the array [3]. Given an array of N_s sensors whose locations are
 401 given by

$$\mathbf{x}_m, \quad m = 1, 2, \dots, N_s. \quad (13)$$

402 The associated set of vector spacing between all pairs of elements in the array
 403 can be expressed as

$$\mathbf{X}_p = \mathbf{x}_m - \mathbf{x}_n, \quad m = 1, 2, \dots, N_s, \quad n = 1, 2, \dots, N_s. \quad (14)$$

404 The set of points \mathbf{X}_p is called the co-array of the array \mathbf{x}_m [50]. To evaluate
 405 the efficiency of the different periodic array pattern with respect to aperi-
 406 odic ones, the F factor is introduced below, which show the ratio of the
 407 actual number of unique vector spacings of an array, P , to the corresponding
 408 maximum number of spacings, P_{\max}

$$F = \frac{P}{P_{\max}} \leq 1. \quad (15)$$

409 Since there are N_s^2 vectors, and N_s of these are zero, the maximum possible
 410 unique vector spacings in an array consisting of N_s sensors can be calculated
 411 as follow

$$P_{\max} = N_s^2 - (N_s - 1). \quad (16)$$

412 An optimal array maximizes the number of unique vector spacings, re-
 413 sulting in $F = 1$. The low value of F means that there will be a large number
 414 of duplicate distances between the sensors, which usually can be seen in the
 415 periodic pattern. For instance, in Figs. 5 (c), (g), (k) and (o), the F fac-
 416 tor decreases with an increase in the number of sensors. This indicates that
 417 when more sensors are added in a cross-array pattern at equal distances, the
 418 size of P will not increase as much as P_{\max} (Eq. (16)) due to the repetitive
 419 occurrence of the same distances. Thus, it can be inferred that improper
 420 sensor positioning can lower the F factor. This is one of the reasons why an
 421 accurate WPF estimate in the low-wavenumber domain cannot be obtained
 422 by equidistant cross array pattern even with 68 sensors (see Figs. 5 (n) and
 423 (p)).

424 To maximize the co-array size while using a fixed number of sensors, it
425 is generally preferable to opt for a non-equidistant arrangement of the array.
426 This will result in a relatively low level of secondary lobes on the estimated
427 WPF, which appear due to the aliasing effect [32]. In the following, the
428 effect of the non-equidistant cross-array pattern on estimation of the WPF
429 is studied.

430 To maximize F factor in the cross-array pattern, the position of sensors
431 are arranged non-equidistantly on the two cross lines, and the same study as
432 above (Section 3.1.1) has been carried out again. Fig. 6 presents the results
433 with the same number of sensors as studied in Section 3.1.1. Similar to
434 the equidistant-array pattern, the convective region is the first region where
435 the estimated WPF converges to the reference model. However, in the non-
436 equidistant array pattern, only 16 sensors are required to identify this region
437 (Figs. 6 (b) and (d)), whereas in the equidistant array pattern, it takes at
438 least 24 sensors (see Fig. S1). Moreover, the estimated WPF obtained by
439 the RFBA in each case (Figs. 6 (b), (f), (j), and (n)) is more accurate than
440 corresponding case in the equidistant cross-array pattern (Figs. 5 (b), (f), (j),
441 and (n)), which is evident by the lower MAE for the non-equidistant array.
442 As it can be seen from Figs. 6 (f) and (h), in this case the aliasing effect is less
443 profound for the array with 32 sensors when compared to the corresponding
444 case shown in Figs. 5 (f) and (h). Consequently, this improvement results in
445 a decrease in the MAE from 5.07 dB to 3.03 dB.

446 In the supplementary document, readers can access an interactive plot
447 (Fig. S2) that showcases how WPF estimation in the low-wavenumber do-
448 main is affected by 14 arrays of non-equidistant cross-array patterns. The
449 plot includes the results for different arrays from 16 to 68 sensors, with in-
450 crements of 4.

451 Comparing the results presented in Figs. 5 and 6, it can be concluded that
452 respecting the Nyquist criterion and the maximum co-array factor can result
453 in a better estimation of the WPF. In the next section, it is demonstrated
454 that in addition to sensor spacing and co-array factor, sensor distribution
455 plays a key role in accurate estimation of the TBL WPF. Effect of this factor
456 has been investigated using a random array pattern in the following section.

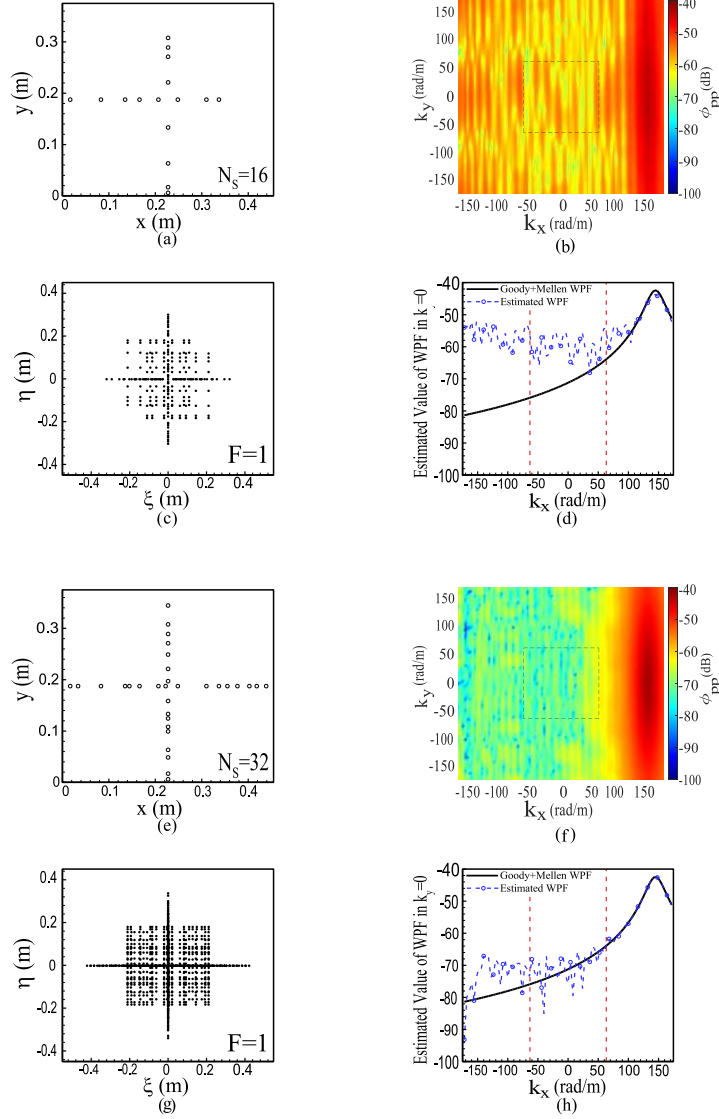


Fig. 6 - Part 1. Comparison of the estimated WPF using RFBA for non-equidistant cross-array pattern with 16 (a-d) and 32 (e-h) sensors, respectively. Non-equidistant cross arrays for each case are shown in (a) and (e) and associated set of distinct vector spacings between sensors are presented in (c) and (g). Co-array factor (F) are displayed for each case and MAEs calculated between the reference input TBL model and the estimated low-wavenumber WPF shown in (b) and (f) are 10.63 dB and 3.03 dB, respectively. 2D wavenumber-frequency spectra for $k_y = 0$ are plotted against longitudinal wavenumber in (d) and (h).

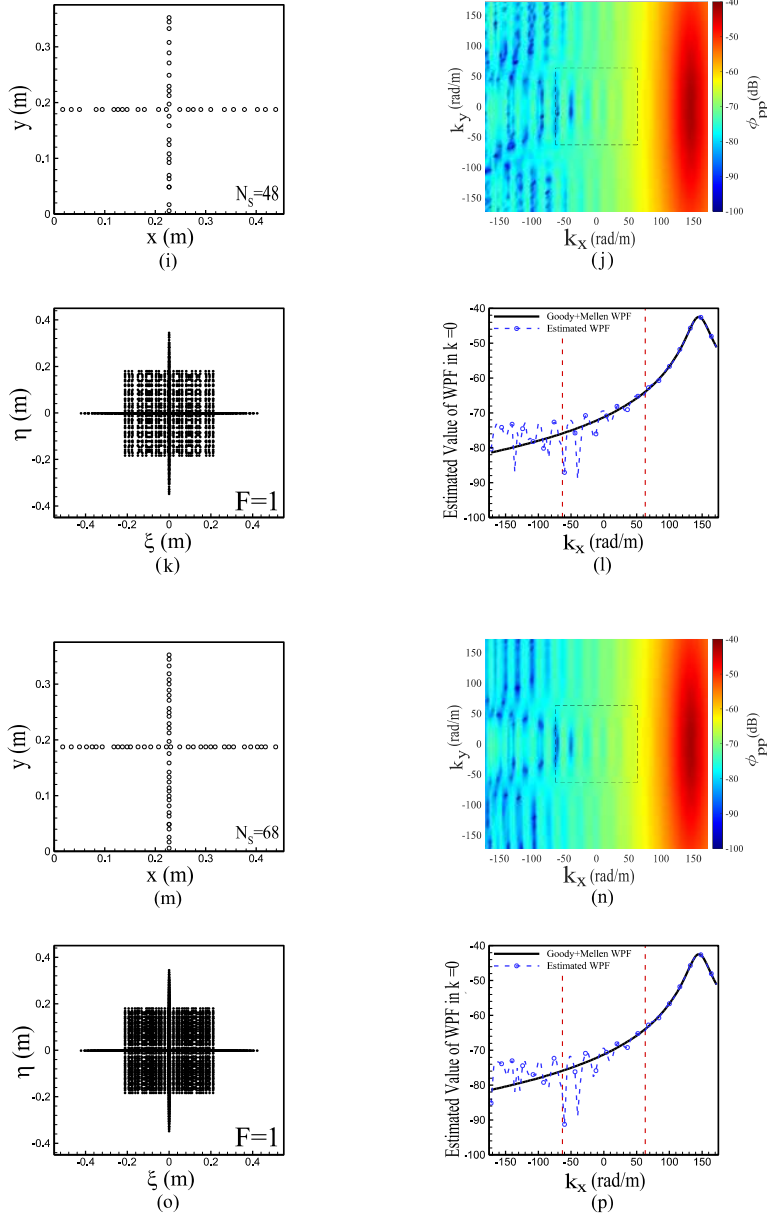


Fig. 6 - Part 2. Comparison of the estimated WPF using RFBA for non-equidistant cross-array pattern with 48 (i-l) and 68 (m-p) sensors, respectively. Non-equidistant cross arrays for each case are shown in (i) and (m) and associated set of distinct vector spacings between sensors are presented in (k) and (o). Co-array factor (F) are displayed for each case and MAEs calculated between the reference input TBL model and the estimated low-wavenumber WPF shown in (j) and (n) are 1.45 dB and 1.31 dB, respectively. 2D wavenumber-frequency spectra for $k_y = 0$ are plotted against longitudinal wavenumber in (l) and (p).

457 3.1.3. *Effect of Sensor Distribution*

458 As discussed above, the half-wavelength criterion is the main constraint
459 of regular array patterns. Failure to meet this criterion, results in spatial
460 aliasing, which produces secondary lobes on the estimated WPF as it was
461 illustrated in Figs. 5 (b) and (f). It is possible to diminish these secondary
462 lobes by removing all periodicities from the microphone array. This results
463 in a class of arrays known as irregular or aperiodic arrays [32].

464 In order to design an irregular array, a random process can be used to
465 determine sensor locations. Another option would be to use an algorithm
466 that ensures a certain degree of irregularity in sensor positions. The latter
467 should be used whenever a sensor location can be specified and controlled
468 because a knowledge-based sensor location approach outperforms a random
469 algorithm [32]. In this work, the second approach is employed for distributing
470 the sensors and creating a random-array pattern. Fig. 7 shows the estimated
471 WPF using the RFBA for four random array patterns with the number of
472 sensors of 16, 32, 48, and 68. For additional visualization, an interactive plot
473 (Fig. S3) containing 14 random array patterns with sensors ranging from 16
474 to 68 in increments of 4 is available in the supplementary document. All the
475 configurations meet the Nyquist criterion and have the maximum possible
476 co-array factor (i.e. $F = 1$). For example, for the first irregular array of 16
477 sensors, the sensor arrangement was designed such that at least one pair of
478 sensors satisfied the Nyquist criterion in both the streamwise and spanwise
479 directions. Following this, in each subsequent step, four new sensors were
480 added to the previous arrangement in such a way that at least one existing
481 sensor could meet the minimum distance required by the Nyquist criterion
482 for each new sensor. This process was repeated up to the fourteenth array
483 of 68 sensors. Also, the position of sensors was chosen in a manner that the
484 F factor was always maximized and equal to 1. The obtained results in Fig. 7
485 show that using the irregular array with above conditions will avoid spatial
486 aliasing and also generate a more coherent vector spacing separation of ξ and
487 η which lead to the better estimation of the WPF compared with the regular
488 array patterns.

489 Fig. 7 shows that using the irregular array the estimated WPF converges
490 to the reference WPF much faster than that using the regular array. For
491 example, Figs. 7 (j) and (l) show that applying RFBA to a random array with
492 48 sensors provides excellent estimations of the WPF in the low-wavenumber
493 domain with a mean absolute error of less than 1 dB. Moreover, RFBA

494 provides an accurate result in the entire considered wavenumber domain using
495 68 sensors (shown in Figs. 7 (n) and (p)). Therefore, employing a random
496 array pattern while adhering to the Nyquist criterion and optimizing the
497 co-array factor yields improved WPF estimations in comparison to the other
498 array patterns investigated in Sections 3.1.1 and 3.1.2. In Section 3.1, we used
499 a closed-form semi-empirical TBL model for computing the CSM. However, in
500 practice, only limited number of samples/snapshots of the WPF is available.
501 To investigate the impact of this factor on the proposed RFBA method, we
502 introduce a virtual acoustic experiment, which we examine in detail in the
503 subsequent section.

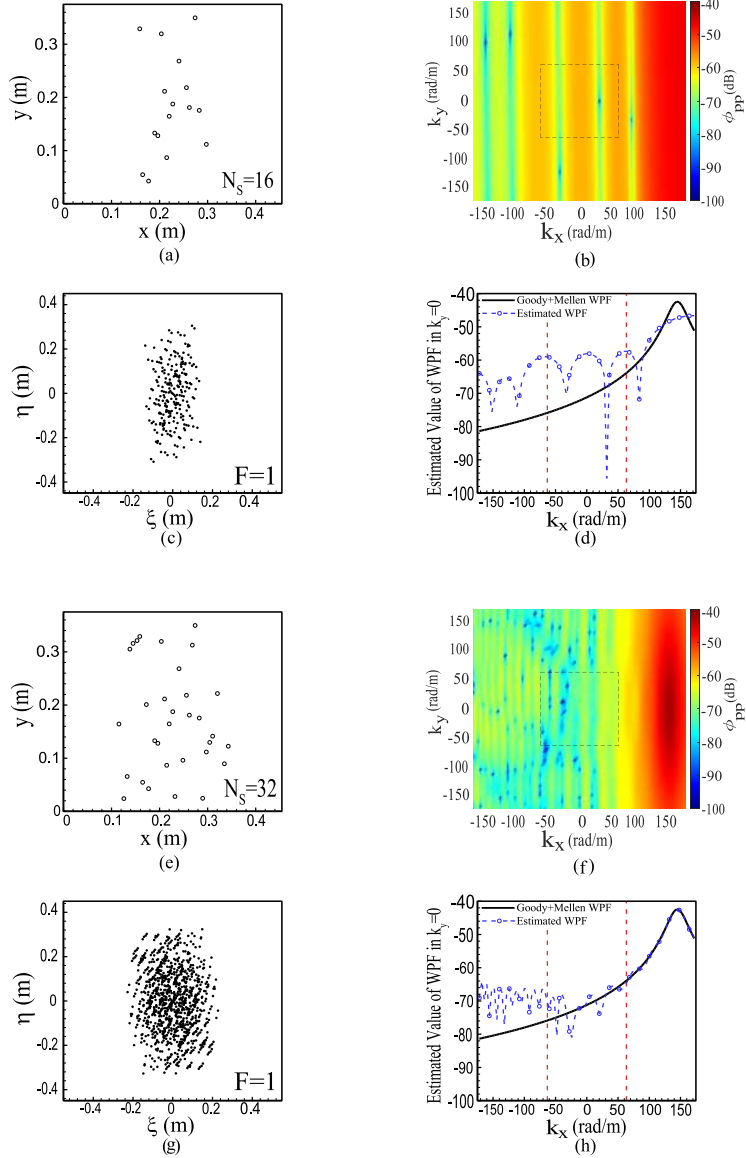


Fig. 7 - Part 1. Comparison of the estimated WPF using RFBA for irregular-array pattern with 16 (a-d) and 32 (e-h) sensors, respectively. Irregular arrays for each case are shown in (a) and (e) and associated set of distinct vector spacings between sensors are presented in (c) and (g). Co-array factor (F) are displayed for each case and MAEs calculated between the reference input TBL model and the estimated low-wavenumber WPF shown in (b) and (f) are 10.08 dB and 2.68 dB, respectively. 2D wavenumber-frequency spectra for $k_y = 0$ are plotted against longitudinal wavenumber in (d) and (h).

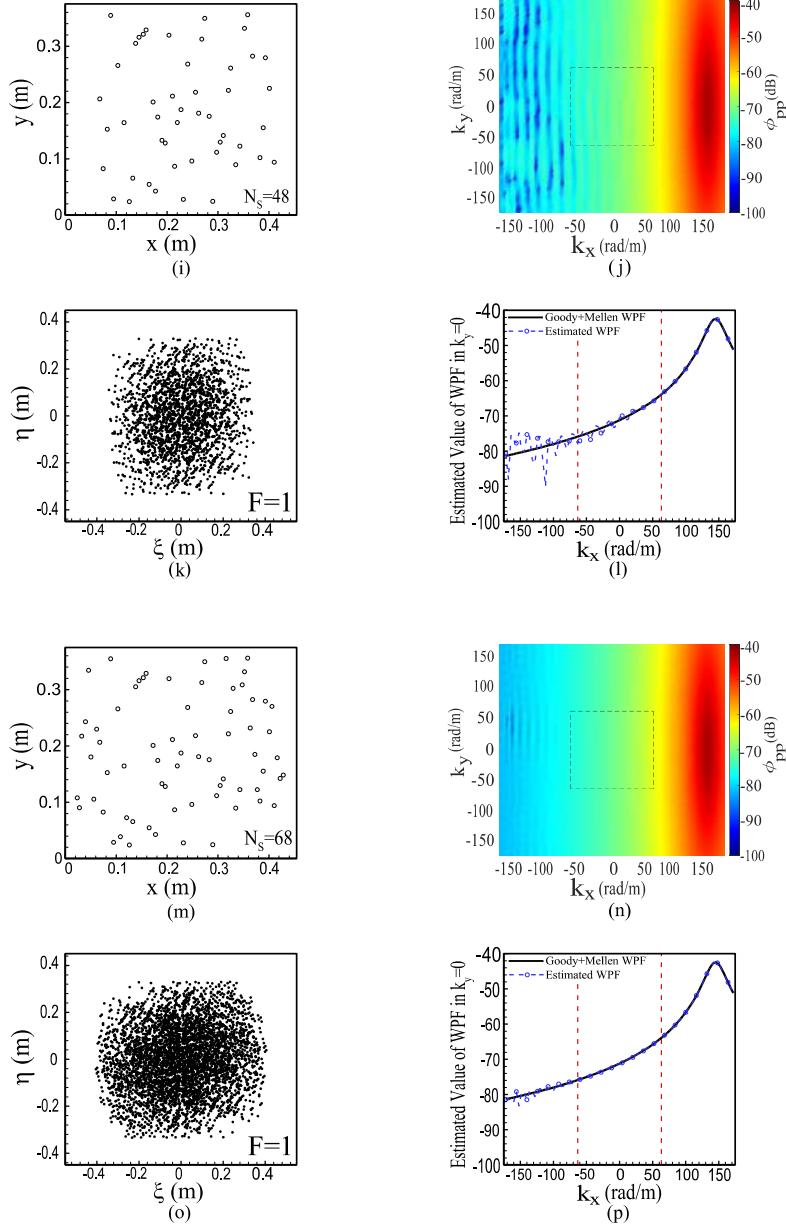


Fig. 7 - Part 2. Comparison of the estimated WPF using RFBA for irregular-array pattern with 48 (i-l) and 68 (m-p) sensors, respectively. Irregular arrays for each case are shown in (i) and (m) and associated set of distinct vector spacings between sensors are presented in (k) and (o). Co-array factor (F) are displayed for each case and MAEs calculated between the reference input TBL model and the estimated low-wavenumber WPF shown in (j) and (n) are 0.61 dB and 0.09 dB, respectively. 2D wavenumber-frequency spectra for $k_y = 0$ are plotted against longitudinal wavenumber in (l) and (p).

504 **3.2. Virtual Acoustic Experiments**

505 In the previous sections, the CSM was calculated from a closed-form semi-
 506 empirical TBL model. It was then utilized to investigate the effects of sensor
 507 spacing, co-array factor and sensor distribution on the performance of the
 508 RFBA in estimation of the WPF in the low-wavenumber domain. However,
 509 the theoretical TBL models cannot realistically simulate an experimental
 510 situation. Since the TBL pressure fluctuation is a random process, if several
 511 records of these pressure fluctuations are taken under the same experimental
 512 conditions, they would not be identical due to the random nature of the
 513 excitation. Each outcome of an experiment, in the case of a random process,
 514 is called a sample function. If n experiments are conducted, all the n possible
 515 outcomes of a random process constitute what is known as the ensemble of
 516 the process.

517 In this section, this process is simulated using a virtual acoustic experi-
 518 ment where different deterministic realizations of the TBL pressure fluctua-
 519 tions are computed, and the CSM is then estimated from ensemble average
 520 of these realizations.

521 **3.2.1. Wall Pressure Field Snapshots using the UWPW Technique**

522 Simulation of random TBL with deterministic loading is the main concept
 523 of the UWPW technique [37]. This approach mimics experimental conditions
 524 and calculates the WPF underneath of a TBL by ensemble averaging of the
 525 different realization of wall pressure at each frequency. The pressure beneath
 526 the TBL for the r th realization can be represented by a set of UWPWs at
 527 the q th sensor of the array pattern as follows [46, 37, 51]

$$p^r(\mathbf{x}^q, \omega) = \sum_{l=1}^{N_k} \sqrt{\frac{\phi_{pp}(k_{x,l}, k_{y,l}, \omega) \delta k_x \delta k_y}{4\pi^2}} e^{j(k_{x,l}x^q + k_{y,l}y^q + \theta_l^r)}, \quad (17)$$

528 where θ is a random phase uniformly distributed in $[0, 2\pi]$. Similar to Eq.
 529 (5), N_k corresponds to the total number of grid points in the truncated
 530 wavenumber space. It is important to note that a cut-off wavenumber of $1.2k_c$
 531 was employed in both the streamwise and spanwise directions to consider
 532 the convective contributions of the TBL WPF. As an illustration, Fig. 8
 533 displays the representation of four realizations of the surface pressure filed
 534 at a frequency of 1000 Hz and a flow velocity of 50 m/s. These realizations
 535 are employed in ensemble averaging of different realizations to compute the
 536 CSM of the WPF. Fig. 9 shows a flowchart describing the implementation

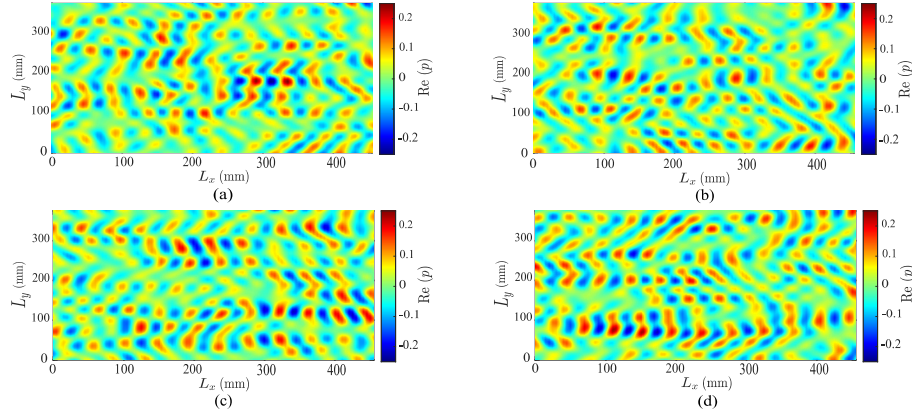


Fig. 8. (a)-(d): Four different realizations of the WPF synthesized by the UWPW technique using the Goody and truncated Mellen models for a flow speed of 50 m/s at 1000 Hz.

537 of the UWPW technique in the virtual acoustic experiment for estimation of
 538 the WPF in the low-wavenumber domain.

539 Herein, three patterns (equidistant-cross array, non-equidistant-cross ar-
 540 ray, and irregular array) with 68 sensors are analyzed, and the impact of vary-
 541 ing the number of realizations on estimating the WPF in the low-wavenumber
 542 domain using the RFBA is evaluated. The supplementary document contains
 543 an interactive plot showcasing 19 case studies for the three array patterns,
 544 highlighting the impact of varying numbers of realizations on estimation of
 545 the WPF in the low-wavenumber domain (see Fig. S4). Besides, for each
 546 array pattern, the MAE is shown to help quantifying the accuracy of the
 547 estimated WPF for different realizations. Fig. 10 only shows some selected
 548 results for four different number of realizations. The obtained results shown

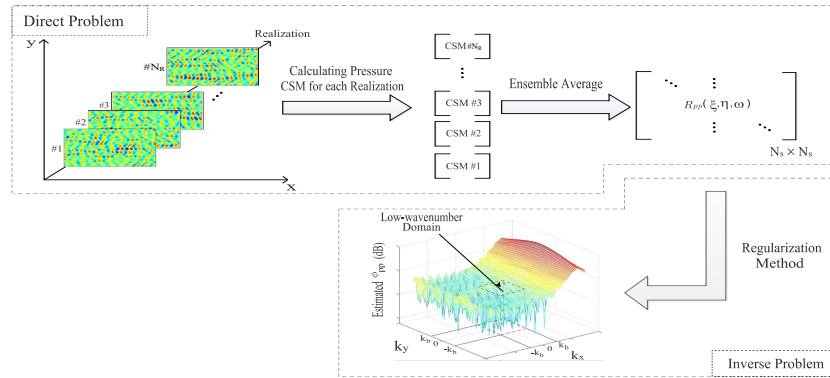


Fig. 9. Simulation process in the virtual acoustic experiments using the UWPW technique.

549 in Figs. 10 (f1) and (i1), indicate that a relatively small number of realiza-
550 tions is sufficient to identify the convective zone of the WPF in this virtual
551 experiment and the random array pattern exhibits a better performance com-
552 pared with the other two arrays shown in Figs. 10 (d1), (g1), (e1) and (h1).
553 However, for the estimation of the WPF in the low-wavenumber domain, a
554 considerable number of realizations is necessary. For example, Fig. 10 (f2)
555 shows that after 50000 realizations, the estimated results has a MAE of ap-
556 proximately 4.5 dB. Moreover, increasing the number of realizations from
557 50000 to 200000 reduces the MAE by only 1 dB (see Fig. 10 (l2)). This
558 can be attributed to the fact that in the virtual experiment an approximate
559 CSM is used which struggles to realise the pressure fluctuations in this region
560 due to their low amplitudes compared to the convective region. Moreover,
561 the MAE values for three different patterns indicate that the irregular-array
562 pattern performs better than the equidistant and non-equidistant cross array
563 patterns when evaluating the WPF in the low-wavenumber domain.

564 To analyze the WPF synthesized with Eq. (17), the coherence obtained
565 from the WPF of N_R realizations are compared with the coherence obtained
566 from the Mellen+Goody model's analytical formula in Fig. 11. By using
567 N_R realizations, the coherence between point \mathbf{x} and \mathbf{x}' can be estimated as
568 follows

$$\Gamma(\mathbf{x}, \mathbf{x}', \omega) = \frac{\left| E \left[p^r(\mathbf{x}, \omega) \overline{p^r(\mathbf{x}', \omega)} \right]_{r \in \{1, \dots, N_R\}} \right|}{\sqrt{E \left[|p^r(\mathbf{x}, \omega)|^2 \right]_{r \in \{1, \dots, N_R\}} E \left[|p^r(\mathbf{x}', \omega)|^2 \right]_{r \in \{1, \dots, N_R\}}}}, \quad (18)$$

569 where $p^r(\mathbf{x}, \omega)$ is given by Eq. (17).

570 Fig. 11 shows the results of Eq. (18) for $N_R = 50, 500, 5000$, and
571 $N_R = 50000$ when applied to the 34 equidistant sensors positioned in the
572 streamwise direction. It can be observed that a relatively small number of
573 realizations is sufficient to estimate the coherence of the WPF between closely
574 spaced sensors, which plays a vital role in calculating the convective peak in
575 the WPF. However, there are significant discrepancies between the estimated
576 coherence and the analytical one for sensors that are spaced far apart. By in-
577 creasing the number of realizations, the estimated coherence for sensors with
578 larger spatial separation approaches the analytically calculated coherence,
579 which is crucial for accurate WPF estimation in the low-wavenumber range.
580 This behaviour clarifies why a large number of realizations is necessary to
581 estimate the WPF in the low-wavenumber range.

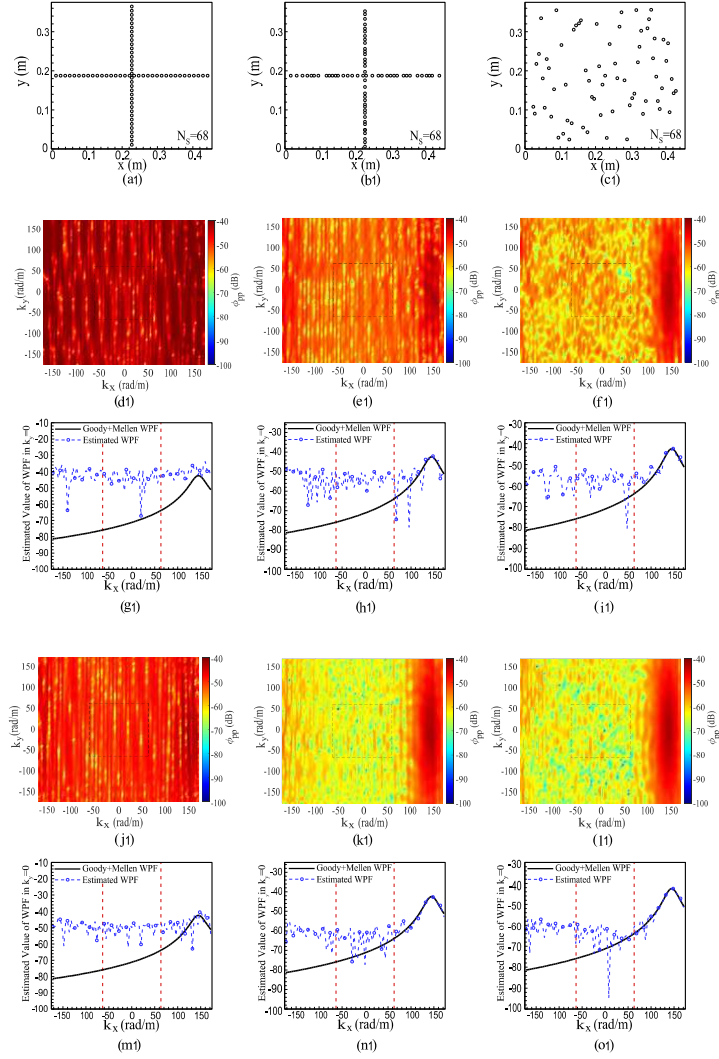


Fig. 10 - Part 1. Comparison of the estimated WPF using the UWPW technique for three different array patterns, each comprising 68 sensors shown in (a1)-(c1) for 500 realizations (d1-i1) and 5000 realizations (j1-o1). The color maps depicting the estimated WPF are presented in (d1), (e1), (f1), (j1), (k1), and (l1), with respective MAEs between the reference input TBL model and the estimated low-wavenumber WPF of 26.86 dB, 16.20 dB, 12.90 dB, 20.12 dB, 8.01 dB, and 7.48 dB. The cross-section view of the estimated low-wavenumber WPF are illustrated in (g1), (h1), (i1), (m1), (n1), and (o1).

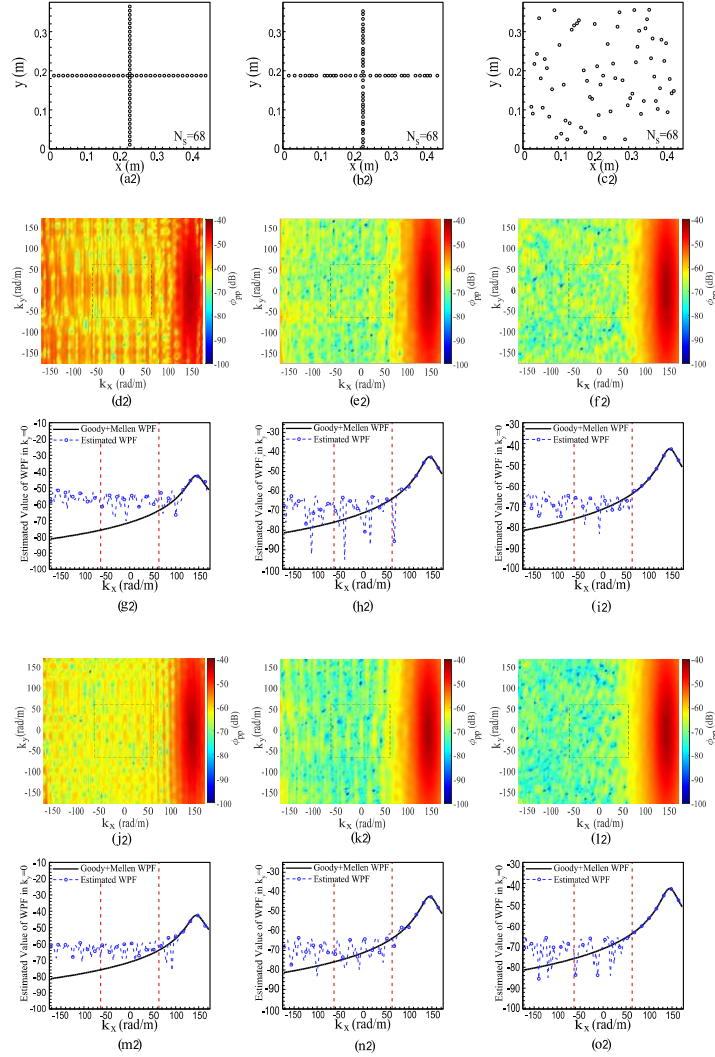


Fig. 10 - Part 2. Comparison of the estimated WPF using the UWPW technique for three different array patterns, each comprising 68 sensors shown in (a2)-(c2) for 50000 realizations (d2-i2) and 200000 realizations (j2-o2). The color maps depicting the estimated WPF are presented in (d2), (e2), (f2), (j2), (k2), and (l2), with respective MAEs between the reference input TBL model and the estimated low-wavenumber WPF of 10.80 dB, 4.65 dB, 4.55 dB, 8.37 dB, 4.22 dB, and 3.39 dB. The cross-section view of the estimated low-wavenumber WPF are illustrated in (g2), (h2), (i2), (m2), (n2), and (o2).

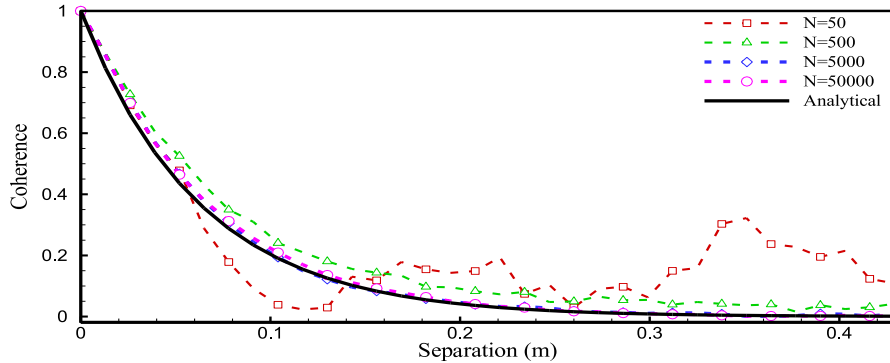


Fig. 11. The TBL pressure field coherence as a function of the spatial separation in the streamwise direction. Solid line, analytical formula of the Mellen+Goody model; dashed lines, numerical estimation considering 50, 500, 5000 and 50000 realizations.

583 **3.2.2. Effect of the Convective Ridge on the Estimation of the**
 584 **Low-wavenumber WPF**

585 In previous Sections, we exclusively utilized the Mellen model as the
 586 input TBL model. Since the levels of the WPF between the convective peak
 587 and the low-wavenumber domain are different for different semi-empirical
 588 models (see Fig. 2), in this section we investigate how this difference will
 589 affect the estimated low-wavenumber WPF. Hence, we have implemented two
 590 additional models: the Chase model [11, 44] and the Corcos model [9, 44], as
 591 input TBL models. As can be seen from Fig. 2, among these three models,
 592 the levels of low-wavenumber WPF are the highest for Corcos model and the
 593 lowest for Chase model while they are somewhere in between for the Mellen
 594 model. The disparities between the convective peak level and the mean value
 595 of the WPF within the low-wavenumber domain are approximately 19 dB,
 596 27 dB and 33 dB for the Corcos, Mellen and Chase models, respectively. In
 597 both the Corcos and Chase models, the convective peak occurs at a similar
 598 level as observed in the Mellen model [44].

599 For the estimation of the WPF, we employed a random array pattern with
 600 68 sensors, as shown in Fig. 10 (c1). We calculated the MAE for the three
 601 TBL models with different numbers of realizations, and the results are sum-
 602 marized in Table 2. The findings indicate that when using the Corcos model,
 603 fewer realizations are required for an accurate estimation of the WPF within
 604 the low-wavenumber domain. In fact, with just 50,000 realizations, we can
 605 achieve WPF estimation with a MAE of approximately 2 dB. This number

Table 2. MAE of the low-wavenumber WPF for multiple numbers of realizations using three closed-form semi-empirical models as reference input TBL models.

Semi-empirical models	Number of Realizations							
	500	5,000	50,000	200,000	500,000	700,000	1,000,000	2,000,000
MAE (dB) - Corcos Model	6.19	3.39	2.18	1.36	0.60	0.51	0.48	0.32
MAE (dB) - Mellen Model	12.90	7.48	4.55	3.39	2.72	2.61	2.19	2.03
MAE (dB) - Chase Model	26.06	20.31	15.97	13.87	11.27	11.14	10.59	9.34

606 of realizations is significantly fewer than what is needed for the Mellen model
 607 (nearly 1,000,000 realizations) to reach the same level of accuracy. This can
 608 be attributed to the fact that the difference between the convective peak and
 609 the low-wavenumber levels in the Corcos model is smaller than that in the
 610 Mellen model. Therefore, the low-wavenumber components of the WPF are
 611 less contaminated by the convective ridge. Consequently, a lower number of
 612 realizations is necessary to attain an accurate estimation of the WPF within
 613 the low-wavenumber domain.

614 This has been further confirmed by the results for the Chase model where
 615 its MAE exceeds that of the Mellen model. For example, when using the
 616 Chase model, to achieve WPF estimation with approximately 9 dB error, al-
 617 most 2,000,000 realizations are required. Since the disparity in the WPF lev-
 618 els between the convective peak and the low-wavenumber domain is the high-
 619 est for the Chase model among the considered models, the low-wavenumber
 620 components of the WPF are mostly masked by the large-amplitude compo-
 621 nents of the WPF in the convective ridge. This is one of the main challenges
 622 of measuring the low-wavenumber pressure fluctuations using a microphone
 623 array in real-world scenarios as the difference between the convective peak
 624 and low-wavenumber levels of the WPF is not known. This means it is not
 625 clear how many snapshots of the measured signal is required to achieve an
 626 accurate estimation of the low-wavenumber region.

627 It should be noted that in this virtual experiment the effect of data sam-
 628 pling and using an approximate CSM on the estimation of the TBL pressure
 629 field is demonstrated which is only one aspect of a real experiment. How-
 630 ever, other common sources of error including instrumental, environmental,
 631 procedural, and human errors exist in practice. These errors can be either
 632 random or systematic, impeding the accurate estimation of the WPF in the
 633 low-wavenumber domain.

634 4. Conclusions

635 In this work, the efficacy of using a microphone array on the estimation
636 of the TBL WPF in the low-wavenumber domain was studied. A regularized
637 Fourier-based approach was proposed to identify the low-wavenumber levels
638 of the WPF. Effects of three array parameters, namely sensor spacing, co-
639 array factor and sensor distribution on the performance of each method were
640 examined. It was shown that to achieve accurate estimation of the WPF all
641 the three factors should be considered. It was found that to obtain accurate
642 results, in addition to the Nyquist criterion, one needs to use an irregular
643 array pattern with the maximum possible co-array factor ($F = 1$). It was
644 also observed that reasonable estimation of the WPF in the convective region
645 is much easier than that in the low-wavenumber domain and can be achieved
646 with relatively small number of sensors.

647 Moreover, the effectiveness of using a microphone array to estimate the
648 WPF in an experimental condition was evaluated using a virtual experiment
649 where the CSM was approximated by an ensemble average of different re-
650 alization of the WPF generated by the UWPW technique. This mimics an
651 experimental measurement where many samples are collected from the ran-
652 dom TBL pressure fluctuations. It was illustrated that increasing the number
653 of realizations results in more accurate estimation of the wall pressure spec-
654 trum. Although, with relatively small number of realizations the convective
655 region can be identified, a significant number of realizations is required to
656 well estimate the low-wavenumber levels in the TBL pressure field.

657 To investigate the effect of the convective ridge on the identification of
658 the low-wavenumber domain WPF, three different TBL models were used
659 individually as input reference models. It was observed that the difference
660 between the convective peak and the low-wavenumber levels significantly af-
661 fects the accuracy of low-wavenumber WPF estimation. In other words, the
662 greater this difference, the higher the number of realizations. This happens
663 because the convective ridge obscures the low-wavenumber components of
664 the TBL WPF. This underscores a key issue when trying to capture the low-
665 wavenumber pressure fluctuations using a microphone array in real-world
666 scenarios, as the exact difference between the convective peak and the low-
667 wavenumber levels is unknown in practice. As a result, it remains unclear
668 how many snapshots of the recorded signal are required to achieve an ac-
669 curate estimation of the low-wavenumber region. Moreover, this highlights
670 the challenges in estimation of this region in the experiments where not only

671 a limited number of data samples can be recorded but also there are many
672 different sources of error and uncertainties such as background noise, instru-
673 mental and human error.

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677 **Appendix A. TGSVD method**

678 Using Eqs. (5) and (8), one can calculate the vector \mathbf{S}_{pp} and matrix \mathbf{Q} .
679 To obtain the best estimation of the WPF components, the following steps
680 from Ref. [38] are employed:

681 **Step 1:** compute discrete first derivative operators;

682 `L=get_1(size(Q,2),1);`

683 **Step 2:** Compute the compact generalized SVD of a matrix pair;

684 `[UU,sm,XX]=cgsvd(Q,L)`

685 **Step 3:** Compute all TGSVD solutions;

686 `k_tgsvd=1:size(sm,1)`

687 `[X_tgsvd,Rho,Eta]=tgsvd(UU,sm,XX,Sp,k_tgsvd);`

688 **Step 4:** Find the corner of a discrete L-curve via an adaptive pruning algo-
689 rithm;

690 `k_corner=corner(Rho,Eta)`

691 **Step 5:** Find the estimated WPF components for the optimal regularization
692 parameter obtained from the corner method;

693 `Phipp=X_tgsvd(:,k_corner)`

694 **Appendix B. ASD function and normalized CSD function**

695 The Goody model: The empirical model of the ASD function of the
 696 pressure field is given by [49]

$$\Psi_{pp}(\omega) = \frac{3\tau_\omega^2 \delta \left(\frac{\omega\delta}{U_e}\right)^2}{U_e \left(0.5 + \left(\frac{\omega\delta}{U_e}\right)^{0.75}\right)^{3.7} \left(1.1R_T^{-0.57} \left(\frac{\omega\delta}{U_e}\right)\right)^7} \quad (\text{B.1})$$

697 where $R_T = U_\tau^2 \delta / U_e \nu$ and U_e is the boundary layer edge velocity.

698 The Mellen model: The Mellen normalized wavenumber-frequency model
 699 is given by [10]

$$\tilde{\phi}_{pp}(k_x, k_y, \omega) = \frac{2\pi (\alpha_x \alpha_y)^2 k_c^3}{(\alpha_x \alpha_y k_c)^2 + (\alpha_x k_y)^2 + (\alpha_y^2 (k_x - k_c)^2)^{3/2}} \quad (\text{B.2})$$

700 where $\alpha_x = 0.1$ and $\alpha_y = 0.77$.

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