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# Cooperation processes in inclusive learning settings with a special focus on mathematical potential

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*In inclusive education, all learners should be enabled to develop their individual potential. With respect to mathematical difficulties, there are already numerous concepts for inclusive classrooms. In contrast, the promotion of mathematical potential often takes place outside the classroom in exclusive learning situations. This allows only few opportunities for cooperation and negotiation between all learners in classroom. In the research reported here, learning environments for inclusive education had been designed to promote mathematical potential as well as the participation of all learners. The paper focuses on children's cooperative negotiation processes in inclusive learning settings with special focus on mathematical potentials.*

*Keywords: Mathematical learning processes, inclusive education, students with special needs, mathematical potential, cooperation.*

## **Inclusion**

The introduction of the term *inclusion* into school practice (UN Convention on the Rights of Persons with Disabilities, United Nations, 2006) emphasizes an important wider perspective, which is fundamental for our research orientation: in an inclusive education system, all persons should have the chance to develop their individual potential and should be perceived, accepted, and valued in their diversity and individuality. Thus, the goal of inclusive teaching is to support all students in the best possible way (Ainscow et al., 2006). Numerous efforts in the context of inclusion focus primarily on mathematical difficulties. But inclusion means the *promotion of mathematical potential*, too. However, international comparative studies like TIMSS 2019 (Mullis et al., 2020) indicate that the promotion of high-achieving students in Germany is less successful than the promotion of low-achieving learners. It also points to a decline in high-performing primary school children, that reach or exceed the defined standards in mathematics. That's why the potential of *all* pupils (not only those with special talents) is increasingly focused in academic and educational policy efforts (Dexel et al., 2019). The present study refers to a perspective of inclusion as an interplay of joint and individual learning processes (Moser Opitz et al., 2018). Every learner can develop his or her individual mathematical potential through adaptive mathematical learning opportunities as well as social participation (Krähenmann et al., 2019). The paper focuses on this interplay of individual support and joint learning situations in inclusive learning environments with special focus on mathematical potential. In this paper we understand a *learning environment* as a framework for several tasks based on a common mathematical structure (Hirt & Wälti, 2022).

## **Promotion of mathematical potential**

In different research orientations, there is a wide range of concepts related to the construct of special mathematical potential. There is no consistent and sometimes even contradictory understanding of this term (Leikin, 2018). While aptitude in a psychological context is often understood in the sense of giftedness measured by tests, research in German didactics of mathematics tends to work with systems of characteristics/properties to identify and describe giftedness (e.g. Kämpnick, 1998). The

study presented in this paper joins a broader view on mathematical potential. From this point of view potential-oriented support in inclusive learning environments enables the promotion of each child's individual abilities. This dynamic view contains that individual facets of mathematical potential can develop independently of the current level of performance and thus emphasizes the aspect of development of mathematical potential (Leikin, 2018). In Germany, concepts to support mathematical potential are often located outside the classroom or school (e.g. children's university). In concepts of enrichment, selected children receive additional, enriching learning opportunities (Wieczerkowski et al., 2000). Even these concepts of enrichment, in which all students work spatially together, make it difficult to initiate processes of joint negotiation about mathematical insights, because not all children work on the same task. Consequently, many concepts to support mathematical potential are designed rather *exclusively* and allow few possibilities to cooperate, communicate, and learn with and from each other in the sense of inclusion as explained above.

### **Cooperative learning settings**

In various research traditions, learning mathematics is no longer seen as a purely individual, mental process of the individual child, but as a learning process in which social interaction and communication with others is an essential basis for the development of new mathematical concepts. To promote mathematical potential as well as the participation of all learners, it is important to understand what characterizes fruitful processes of communication and cooperation. These aspects have to be considered to develop and investigate learning environments for inclusive education. Numerous studies emphasize that adequate mathematical communication makes a decisive contribution to the success of learning processes and has a beneficial effect on the growth of learning (Gersten et al., 2009; Stacey & Gooding, 1998). For this reason, it is important, that all learners have opportunities to participate in these communication processes. Therefore, learners have to be given access to the mathematical subject. Such processes of cooperation and communication are not only important for the construction of new mathematical knowledge. In the context of inclusive education, they also offer a rich opportunity for real participation for *all* learners. It is assumed that precisely the exchange of learners' experiences and concepts on different levels can initiate learning processes: The diversity and heterogeneity of learners can benefit the insights of all so that the joint engagement with a common subject can be fruitful (Feuser, 2005). Therefore, inclusive education should interplay between individual and joint learning processes (Wocken, 1998). In order to reach this, various cooperative learning situations have to be created, which enable children to achieve different learning goals but which are related to a common subject, so that communication and cooperation becomes possible. A well-explored concept in the German didactics of mathematics is the concept of *natural differentiation*. In this concept, all children *receive the same task*, which is holistic and sufficiently complex, so that it opens up learning opportunities for children on different levels. By working on a common task, a meaningful discussion about individual strategies and insights can take place. This initiates productive situations of learning *with* and *from* each other (Scherer & Krauthausen, 2010). Thus, this concept offers many advantages for dealing with the diversity of learners. However, looking at inclusive education from a wider perspective that includes particular mathematical difficulties as well as mathematical potential, *natural differentiation* is not always sufficient to create learning situations for *all* children: Therefore, all learners have to be enabled to get access to the subject matter at their individual level of difficulty as well as to participate in joint processes of

cooperation and communication (Häsel-Weide & Nührenbörger, 2021). The concept of parallelization builds on the idea of natural differentiation, but it can offer broader possibilities to realize the demands mentioned above. It is often associated with mixed-grade teaching but is equally relevant to accommodate diversity in inclusive learning settings. In the present parallelized learning environment, the tasks which children work on, follow the design of *natural differentiation*, but also allow to explore analogous mathematical relations at different levels. Due to these target-differentiated levels, all learners can be given individual access to the subject of learning. Likewise, this opens up a variety of possibilities for a fruitful exchange between the learners. They can contribute their discoveries, which they have obtained at the respective levels, so that social participation is guaranteed for all (Nührenbörger & Pust, 2016). Because of these advantages, the present research project uses parallelization in order to initiate participation for all learners as well as learning processes with a special focus on mathematical potential. The study explores if the concept of parallelization realizes these requirements and how children's communication and cooperation processes can be characterized.

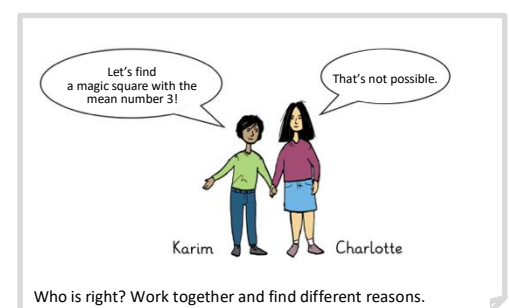
## Methods

The study develops a parallelized learning environment built on the concept of natural differentiation and focusing on Magic Squares (MS). MS offer opportunities to discover and explore rich mathematical structures at different levels, which make them suitable for both natural differentiation and parallelization. In a MS the sum of each row, column, and diagonal is equal (Figure 1), and is called the magic number. In MS of odd-order the magic number is related to the odd-order and the mean number (e.g. in a 3x3 MS the magic number is three times the mean number). Furthermore, there are several possibilities to find new MS, for example by systematically varying the numbers (Hirt & Wälti, 2022). The current study is an ongoing PhD project, which explores two different parallelized learning environments. In this paper we refer to the learning environment MS, in which so far 31 children (aged 9 to 10 years, Grade 4) were videographed working on the different tasks. The researcher also takes the role of the interviewer. The learning environment is divided into three interview sequences. In the first sequence, children work individually in a naturally differentiating learning environment to become familiar with MS, and to explore first structures. The researcher then forms pairs of students on the basis of the student's outcome from the first sequence. The pairs work together at the same subject level. According to the concept of parallelization, some teams work at level A and other teams at (the more complex) level B to offer target-differentiated learning opportunities in Sequence 2. So individual access to the subject of learning at different levels is offered. Children working on level A explore relations between the mean and magic numbers as well as systematic variations of numbers in 3x3 squares. Children working at level B focus on mathematical structures of different MS of size  $n \times n$ , which is more complex than the task of level A. The challenge is to find MS of different sizes (3x3, 4x4, ...,  $n \times n$ ) with the magic number 90. Especially the comparison of odd- and even-order squares offers substantial discoveries. In Sequence 3, the students work together in new teams composed

4	3	8
9	5	1
2	7	6

Magic Number: 15

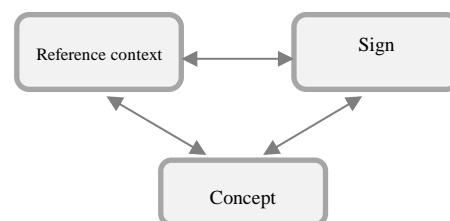
**Figure 1: Magic square (3x3)**



**Figure 2: Task of Sequence 3**

of one learner each from level A and B. Now they work on a task of natural differentiation (Figure 2). Both learners can participate in working on this task by applying insights they made working on the task of level A or B in Sequence 2, e.g. arguments about the relation between the mean and magic number as well as systematic variations (A) or structures of different MS (size  $n \times n$ ) (B).

For the research project, the epistemological triangle (Steinbring, 2009; Figure 3) represents a substantial analytical tool for reconstructing conceptual interpretations as well as their change in children's cooperation processes. At this point, essential features of the epistemological triangle will be briefly presented: All mathematical knowledge requires certain *signs* to encode it (Figure 3). These *signs* do not have an automatic meaning in themselves, the meaning has to be produced by the learners. This requires appropriate *reference contexts*, to develop an interpretation of *signs*. On this basis, Steinbring understands the development of new mathematical knowledge (*concept*) as the active production of interpretations between the *signs* and the *reference contexts* (for detailed description see Steinbring, 2009). The analytical tool is used to reconstruct in three steps how children interpret the task (*sign*), which *reference contexts* they consult for this purpose, and which mathematical *concepts* develop. That helps to describe how children's individual interpretations can be characterized in processes of communication and cooperation.



**Figure 3: The epistemological triangle (based on Steinbring, 2009)**

## First results

The following scenes present first insights in the results of the project with regard to three different focuses:

1. Applying individual insights from the previous sequence into the discovery phase
2. Accessibility to the subject of learning and social participation
3. Promotion of mathematical potential

In Sequence 2, the children Lenny and Lotta initially worked separately from each other on different levels. Lotta worked with her partner on level A. In doing so, she made discoveries and explorations about the relation between the mean and the magic number. Lenny worked with his partner on the more complex level B. He examined various MS that showed fundamental structural differences and did not make the relation between the mean and the magic number immediately visible in all squares. With these previous experiences, Lotta and Lenny come together in a new working group to deal with the task from Sequence 3 (Figure 2).

### Lotta brings an important impulse into the interaction so that participation arises

- Lotta Write down the three.  
 Lenny Mmh [*notes the 3*]. I believe we can  
 Lotta And we have to write a nine here. [*points to the field "magic number"*]  
 Lenny [*looks surprised*] Hmhm [*thinks; writes down the 9*] I didn't know that this has to be a nine [*looks sad*].  
 Interv. Maybe Lotta can explain it to you.  
 Lotta Because three times three is nine and you always have to calculate the number in the middle always three times.  
 Lenny Ahh [*5 sec.; amazed*].

	3	

Magic Number: \_\_

The children have to argue whether a MS with the mean number 3 is possible. This requirement represents the *sign* that they have to interpret. After they have written the 3 in the middle field of the MS, Lotta initiates the problem-solving process. She refers to her insights from Sequence 2, where she explored the relationship between the mean and the magic number (*reference context*), and determines the magic number “9”. This finding represents an important *concept* for the further working process in the third sequence. When the interviewer asks Lenny if he understands why the magic number is “9”, Lenny denies. Lotta explains how to work out the magic number. Through her discovery, which is new to Lenny, Lotta gives an important insight into the communication, which is taken up productively by both children in the further process of interaction. The scene leads to the assumption that the individual support by the previous target-differentiated approach in Sequence 2 is fruitful. In this way, both pupils (especially Lotta, who worked at the less complex level A) can apparently participate in a profitable way while working together on the task of Sequence 3.

### Despite different conceptual interpretations, cooperation emerges

Lenny But that doesn't work. Maybe we need one, maybe that could work but only maybe two, and four, [*notes 2 in F4 and 4 in F6*]<sup>1</sup> that gives

F1	F2	F3
F4	F5	F6
F7	F8	F9

Lotta two and three are five [*taps on F4, F5, F6*] yes that's nine.

Lenny That doesn't work.

Lotta Writes down a five [*taps on F2*].

Lenny Because we need a five, then it makes nine. So, if we make a five, we have to put the numbers two or three. That doesn't work. Or one or four. Four we have already. That doesn't work.

2	3	4

Lotta Because then you have to use a number twice.

Lenny And here we have to put the seven [*taps F1*], but that doesn't work because then we have to take four and one, but we already have a four, that doesn't work at all.

Lotta Yes, it only works if we add a five [*notes 5 in F2*] and a one [*notes 1 in F8*], then we also have nine.

	5	
2	3	4
	1	

Lenny And then everything else doesn't work.

Lotta But here you can put a six [*taps F1*], then we have nine, but you can't use a zero. Because if you put a six here [*taps F1*], a three there [*taps F5*], and a zero there [*taps F9*], it makes nine.

Interv. And why are there no more?

Lotta Because you can't use eleven and three, then it's already higher.

Lenny Wait, maybe like this, seven [*writes 7 in F1*] and a zero [*writes 0 in F7*] now that's nine [*points to the left column*] and then six [*writes 6 in F9*].

7	5	
2	3	4
0	1	6

Lotta But seven plus three [*taps 7 and 3 in succession*] is already ten.

Lenny Ohh [*thinks for 5 sec.*] Yes. That doesn't work. There is not a single magic square with the three.

Lotta Uh. You can't use a number above nine, because then the results are higher.

Interv. Mhm [*confirming*], and what's the problem with that?

Lenny We have nine cells.

Lotta But you can't use nine because nine plus three is twelve, you can't use eight either, that's eleven.

<sup>1</sup> For a better understanding of the transcript, the cells of the MS are numbered consecutively.

Lenny Uh, you can't use nine [taps on empty F3], eight [taps on F5], seven [taps on empty F3], six [taps on F5].

Lotta But you can use the six.

Lenny We can yes. We can use only six, five numbers because the three is already there [taps the mean number in F5]. But we still have eight cells to fill [circles finger counterclockwise over the outer 8 cells].

In the further process of interaction Lotta and Lenny try to find a MS with the mean number 3 and the magic number 9 (*sign*). To find the missing numbers in the cells of the MS, both learners use the insight that the sum in the rows, columns, and diagonals has to be "9" (*reference context*). By first entering the numbers 2 and 4 in the middle row of the MS and trying out a possible solution together, Lenny already develops the assumption that the MS cannot be solved. It seems that Lotta and Lenny are looking at different relationships in the MS: Lotta tries to construct single rows, columns, and diagonals with the sum 9 without noting the overall structure of the MS. Whereas Lenny's statements suggest that he is looking at those relations between cells of the different rows, columns, and diagonals on the overall structure of the MS. Although the approaches of the children partly suggest different concepts of the MS, both children do not work separately from each other, but continuously take up ideas of the other and refer to each other. For example, Lotta explains with reference to Lenny's statements that no numbers can be used twice. In the course of the further interaction, both jointly develop various reasons why the MS cannot be solved with the mean number 3. In this context, they state, (1) that numbers would have to be used twice to solve it, (2) that the MS consists of nine cells, but that the number 9 cannot be used in a cell because the sum of the mean number, and 9 would be greater than the magic number 9 as well as (3) that only five or six numbers can be used to create the magic number, but that eight cells have to be filled. It is possible that the children's different interpretations also reflect the different perspectives and insights from the previous work on different levels in Sequence 2. That scene shows that both learners can participate in the communication and cooperate although they bring different discoveries. They use them for argumentation in a fruitful way for both learners. This can be seen as another indication, that the concept of parallelization opens up rich possibilities for social participation and fruitful cooperation.

**Lenny uses insights from the collaborative working process to construct solutions in an extended number range.**

Lenny I'll try other magic squares and see if I can find more reasons.  
 [writes in the empty MS] two point five, two (..) three point five, one  
 [notes the numbers], maybe that works but only with decimals.  
 [15 sec. silence] two point three [notes 2.3 in F7] and there has to be  
 three (.) three point seven, three point seven [notes 3.7 in F3]. Yes.

	3.5	3.7
2.5	3	2
2.3	1	

Lenny then explains that he wants to check whether there are other reasons or solutions for this task. Again, finding a MS with the mean number 3 and the magic number 9 represents the *sign*, that has to be interpreted. To find the missing numbers in the cells of the MS, Lenny refers to the common insights (e.g. sum in the rows, columns, and diagonals equals 9) (*reference context*). Against the background that Lenny and Lotta have previously realized together that the number of digits (0 to 6) are insufficient to fill all cells of the MS, Lenny now develops a new idea. He puts different decimals into the MS and adds them up. At this point, Lenny fundamentally transcends his previous conceptual interpretations by going beyond the previous number range of natural numbers and developing

conceptual interpretations in the context of rational numbers (*concept*). That can lead to the assumption, that MS (as a common subject in an inclusive learning environment) can help to develop mathematical potential, too. In addition to the target-differentiated approaches in Sequence 2, children can make a variety of challenging discoveries that go beyond the usual number range.

## Conclusion

The analysis on the basis of the epistemological triangle makes it possible to describe how children's individual interpretations can be characterized in processes of communication and cooperation. The following initial findings emerge from the data analysis: It seems, that the special setting of the learning environment can open up a well-founded accessibility for children, potentially caused by the combination of parallelization and naturally differentiating learning tasks. There are indications that learners generate complementary insights at different levels in Sequence 2, which can create a fruitful basis for processes of mathematical negotiation and participation in the third sequence. The discussion with a partner seems to be productive in generating interpretations in order to adopt new perspectives (Gersten et al., 2009; Stacey & Gooding, 1998). The example presented in the analysis shows, that Lotta gains insights into a more overall structural view of the MS by cooperating with Lenny. Although Lotta worked at the less complex level A in Sequence 2, she also introduces discoveries about the relation between mean and magic numbers into the cooperation, from which Lenny benefits. It can be assumed, that on this basis, the children succeed in developing joint solution ideas and justifications, and therefore everyone has the possibility to participate (Krähenmann et al., 2019). Beyond this, the mathematically rich learning environment and the process of cooperation apparently enable children with mathematical potential as well to benefit not only socially but also in developing their individual mathematical potential (Leikin, 2018). Rather, the MS in Sequence 3 (that cannot be solved in the number range of natural numbers) also opens up challenging new mathematical explorations to develop interpretations in other number ranges. In general, such a learning environment cannot automatically guarantee cooperation between both children; there has to be always a certain degree of willingness to cooperate. Nevertheless, the first data analysis suggests that it is precisely the combination of parallelization and naturally differentiated learning that opens up a well-founded accessibility to the subject of learning for all learners and enables rich insights, which are needed to participate in the processes of negotiation in the subsequent sequence. In which way these initial findings can be generalized, has to be investigated in further studies.

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