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► **To cite this version:**

Fiene Bredow, Christine Knipping. Teacher actions framing argumentation in the mathematics class. Thirteenth Congress of the European Society for Research in Mathematics Education (CERME13), Alfréd Rényi Institute of Mathematics; Eötvös Loránd University of Budapest, Jul 2023, Budapest, Hungary. hal-04408283

**HAL Id: hal-04408283**

**<https://hal.science/hal-04408283>**

Submitted on 21 Jan 2024

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# Teacher actions framing argumentation in the mathematics class

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*The focus of this paper are teacher actions which frame mathematical argumentation in secondary, eight-grade classrooms. Identifying and characterising teacher actions in mathematical argumentations provides an empirical ground to decide whether these actions can create or forfeit opportunities for students to learn to argue mathematically and to contribute to argumentations. Of particular interest in our research is how teachers frame students' mathematical argumentations in the transition from arithmetic to algebra. In our study we found six categories of teacher actions during mathematical argumentation processes in eight-grade classrooms. Three categories were generated inductively from our data, the other three were deduced from a framework of Conner et al. (2014). Based on these empirical descriptions a discussion which teacher actions are supportive for students to participate in mathematical argumentation and to generate arguments is possible.*

*Keywords: Teacher role, mathematical argumentation, teacher guidance, classroom research.*

## Introduction

Teachers play an important role in the mathematics class in general and in particular, when it comes to developing mathematical arguments and engaging students in successful argumentation processes (Yackel, 2002; Conner et al., 2014). Students' participation in mathematical argumentation is important for their learning of mathematics (Krummheuer, 1995). Research on mathematical argumentation in whole class settings and on the role of the teacher in these argumentations is rare (e.g., Conner et al., 2014; Solar et al., 2021). But for understanding what makes the participation and autonomy of learners in mathematical argumentation possible, it is of interest how teachers lead or moderate classroom discussions. In our research we provide insights into the role of the teacher when students develop mathematical argumentations in classroom practice and discuss which and how teacher actions guide students and their argumentation processes. Our focus is on mathematical argumentation processes in eight-grade classrooms, when the transition from arithmetic to algebra occurs. The following research question guides our analyses: Which actions (and non-actions) of teachers frame mathematical argumentation processes in the mathematics class?

## Theoretical Framework

### Mathematical arguments and argumentation in the classroom

Mathematical argumentation is a discursive activity that is related to a social context (Krummheuer, 1995). In an argumentative context a rational justification for a claim is developed, and its validity is negotiated. In our understanding *mathematical argumentation* is the process whereby *mathematical arguments* are created (the product; Bredow, 2023). What counts as a correct and valid mathematical argument depends on the social community. Typical for mathematical argumentations in classrooms is that several participants together establish or attempt to establish a claim, whereby the participants can have different roles (Krummheuer, 1995). For example, the teacher has a special role as he or she regularly provides the argumentation tasks and moderates discussions. Overall, the context of

mathematical argumentation in class is complex and all participants need to be considered to understand the interactions in this context.

### **The role of the teacher in mathematical argumentation**

In mathematics education, the role of the teacher in the context of mathematical argumentation is discussed in various ways: from a normative point of way, how teachers could or should act (e.g., Cusi & Malara, 2013) and in a descriptive way, based on empirical research (e.g., Conner et al., 2014). Some of the empirical research analyses the mathematical norms and classroom cultures in the context of argumentation (e.g., Schwarz et al., 2006). Overall, there is only little research that analyses specifically the role of the teacher while interacting with one student or student pairs (e.g., Pedemonte, 2018). Also, research on (collective) mathematical argumentations in classroom discussions and the role of the teacher in these argumentations is rare (e.g., Conner et al., 2014).

Schwarz et al. (2006) identified two patterns for how teachers guide classroom discussions and support explanations. In their study, they analyse the teaching of two eighth-grade teachers on the topic of probability. While teacher A took the role of a mediator asking learners to give explanations and to elaborate, teacher B only used local, closed questions, to which learners gave short answers without any explanations. The authors conclude that these differences between the teachers are based on different mathematical norms and classroom cultures. The argumentation culture in the class strongly influences the richness of learners' argumentations and the role of the teacher as a mediator supports students in coming up with arguments. These considerations are connected to Krummheuer's idea of *argumentation formats*: "A format of argumentation is a patterned interaction in which the core of an argument emerges." (Krummheuer, 1995, p. 253). In the mathematics class, teachers influence and establish such argumentation formats.

Pedemonte (2018) investigated how interventions of teachers affect argumentations of students in the construction of geometric proofs. She looked at discussions between a teacher and a student, when the learners' arguments were incorrect. Using Toulmin's model, Pedemonte compared the learners' arguments before and after the intervention and classified the teachers' interventions. One of her findings is that teachers' interventions often not helped learners effectively to construct a proof. However, if the ideas of the learner were incorrect and the intervention of the teacher were an external refutation to the learner's argument, this intervention was helpful to the learner and enabled them to construct a correct proof. Choosing a good intervention is a challenge for teachers because it must be very specific to the argument. In class, this can be very challenging for teachers because different students in general do not develop the same arguments. Also, Conner et al. (2014) demonstrated that teachers' support in terms of content is often necessary for successful argumentation.

Teachers support their students in diverse ways to create successful argumentation in classes. Solar et al. (2021) understand the role of the teacher "as an orchestration of the affordances and constraints of situations and the interaction as they present themselves." (p. 980). This includes three different argumentation support strategies: i) creating "mathematical tasks open to discussion", ii) "communicative strategies", and iii) "strategies to recognize students' thinking" (Solar et al., 2021, p. 977). This means that not only teachers' behaviour during an argumentation, but also mathematical tasks and their engagement with students' thinking are crucial.

As mathematical argumentation is connected to a social context, teachers create opportunities in the mathematics class for student to participate in argumentation or in contrast hinder their students from participating. For example, teachers can support students to develop and present individual ideas and advance mathematical argumentation by themselves. This is especially important for argumentation: “generating a context where students play a leading role and the teacher is receptive to emergent student ideas and is willing to adapt the lesson in response to them [...] is essential for the development of argumentation” (Solar et al., 2021, p. 978). So, it is crucial to look at the whole situation and interaction in the classroom to reconstruct the role of the teacher.

In the framework “teacher support for collective argumentation” Conner et al. (2014) introduce three different types of teacher support, which will be described in the following. Conner and colleagues understand *support* as „[...] any teacher move that elicited or responded to an argument component.” (Conner et al., 2014, p. 409). They continue in a note: “a move did not have to be productive, nor did it have to be part of a productive or mathematically correct argument to be supportive. We interpreted a move as supporting collective argumentation if it elicited or responded to a component of an argument.” (Conner et al., 2014, p. 409). In our understanding this description of support is better categorized more generally as *teacher actions*. As also Pedemonte (2018) shows, teacher interventions often do not help learners effectively to construct a proof, even though their actions are connected in some way to their students’ arguments. We theoretically agree with Conner et al. (2014) that the as “teacher support” categorized actions are potentially supportive actions, but as Pedemonte we found in our empirical data regular occasions were this was not the case. In this paper, we focus on teacher actions as they occur in real classroom data and reconstruct these actions.

The first category from Conner and her colleagues’ framework, called “Direct contributions to arguments” (Conner et al., 2014, p. 417), covers teacher contributions in which teachers directly introduce components of an argument into a discourse to enrich or complete an argumentation. The other two categories contain teacher actions that guide students in contributing to an argumentation or in completing an argument. “Asking questions“ is a key category of teacher actions (Conner et al., 2014, p. 417) and is subdivided into five subcategories: questioning a fact, an idea, a method, an elaboration, or an evaluation. Beyond these ways of support, a teacher may perform “Other supportive actions” (Conner et al., 2014, p. 420). For example, a teacher might direct conversations, evaluate, or repeat statements. In the framework of Conner et al. (2014), all teacher actions are related to a specific argument in a collective argumentation.

All in all, these studies show that there are different support possibilities and teacher actions. In our research, we examine teacher actions framing mathematical argumentations in class. In addition to the theoretically supportive teacher actions, we will reconstruct other teacher actions that create or forfeit opportunities for students to learn to argue and to participate in mathematical argumentations.

## **Methods**

In our study, argumentation processes of three eight-grade classes (13-14 years old, 25-30 students in each class) from Bremen (Germany) are analysed. A learning environment was designed for the transition from arithmetic to algebra, with a focus on mathematical argumentation (four lessons à 90 min; Bredow, 2023). The lessons were taught by the regular mathematics teachers, who knew about

the research focus on teacher actions in the context of mathematical argumentation. The three teachers have different school experiences: One has been a teacher for ten years, another one about five years and the third has been a teacher since half a year. One participant is male, the other two are females. The lessons were videotaped, students' documents copied and transcripts of the discussions prepared. To analyse the teacher actions during mathematical argumentations, the framework of Conner et al. (2014) is used. Based on transcripts of the whole-class discussions the verbal utterances of the teacher, the writings on the board, the tasks, and other things the teacher brought into the discourse were coded and teacher actions were reconstructed. As indicated above, a particular focus of our analyses were: Direct contributions, Questions, Other supportive actions by the teacher (Conner et al., 2014). Based on our empirical data, we generated three inductive categories for characterising teacher actions, which complement Conner et al.'s framework, and discuss these related to existing research.

## Results

All teacher actions that Conner et al. (2014) describe in their framework could also be found in our data: Direct contributions, Questions, Other supportive actions (and corresponding subcategories). But in our analyses, it became evident that not all teacher actions that occurred, could be classified by the "teacher support for collective argumentation" framework of Conner et al. (2014). Teacher actions framing mathematical argumentations seemed to have different intentions and effects on mathematical argumentation. When teachers address mathematical content, discuss methods to improve students' previous knowledge, or try to stimulate them connecting their knowledge, these teacher actions support students in generating mathematical arguments, in a meta-mathematical way. In this context, the addressed content or methods are often not restricted to a concrete mathematical argument but provide general information to students, which helps them to construct a mathematical argument. At the same time, teachers' actions are not always supportive for students, but also sometimes hinder them in participating in mathematical argumentation, when for example false statements are not corrected, teacher questions are misleading, or student contributions overheard. Teachers can also encourage their students to support each other. Analysing these classroom occurrences led us to generate the following three inductive categories: „Meta actions“, „Dishevelling actions“ and „Student actions“, which we describe and illustrate in the next paragraphs.

**Table 1: Overview of teacher actions framing mathematical argumentations in class**

Teacher actions	Meta actions:
	<ul style="list-style-type: none"> <li>- <i>Method knowledge</i></li> <li>- <i>Content knowledge</i></li> </ul>
	Dishevelling actions:
	<ul style="list-style-type: none"> <li>- <i>Mathematical mistakes</i></li> <li>- <i>Asking several questions (more than three in a sequence)</i></li> <li>- <i>Ignoring false statements</i></li> <li>- <i>No explicit link to the claim, given in the task</i></li> </ul>

	<ul style="list-style-type: none"> <li>- <i>Interrupting students</i></li> <li>- <i>Answering their own questions</i></li> </ul>
	<p>Supportive actions (Conner et al., 2014, p. 418)</p> <ul style="list-style-type: none"> <li>- Direct contributions: <i>Claims, Data, Warrants, Rebuttals, Qualifiers, Backings</i></li> <li>- Asking questions: <i>Requesting a factual answer, a method, an idea, elaboration or evaluation</i></li> <li>- Other supportive actions: <i>Directing, Promoting, Evaluating, Informing or Repeating</i></li> </ul>
Student actions (modeled by the teacher)	Asking questions: <i>Requesting a factual answer, a method or elaboration</i>
	Other supportive actions: <i>Evaluating, Informing or Repeating</i>
	Direct contributions: <i>Rebuttals</i>

### Meta actions

The category *Meta Actions* covers teacher actions, which enrich classroom discussions with content knowledge or method knowledge. This provided knowledge is generally not restricted to a specific argumentation but provides additional knowledge to students. Heinze (2004) points out that students often struggle with generating mathematical arguments due to their lack of *content knowledge* (“Faktenwissen”) or *method knowledge* (“Methodenwissen”). Even if teachers are not aware of the specific problems of their learners, they often intuitively or consciously address content knowledge and method knowledge to enable their learners to participate in argumentation. The following examples illustrate aspects of the inductive subcategories: method knowledge and content knowledge.

Meta teacher actions are assigned to the subcategory *Method knowledge*, when general rules and procedures for deriving mathematical arguments are addressed or a new, alternative strategy to argue mathematically is presented. This *Method knowledge* enables students to argue mathematically. If method knowledge is discussed after an accomplished argumentation, the establishment or reflection of a strategy is usually intended. The following example illustrates such an incidence.

- Teacher: Okay, and thank you. Uhm, this means we have found several examples for when this is valid. But why is this not sufficient? Shush. We had a handful of examples. We could accept this and decide this is sufficient, it's verified. But why is this not a correct way of proceeding? Uhm, Johann.
- Johann: There are many more tasks, where we could find a counter example.

Following up on this exchange, the teacher initiates a discussion about how many examples would be needed to be convinced of a statement. Pointing finally out that trying all possible number combinations is impossible, she concludes that a general mathematical argument is required instead. With her meta comments, the teacher establishes a status for examples in mathematical argumentations and clarifies that example-based arguments are not accepted as valid arguments.

Moreover, teachers sometimes address mathematical knowledge or concepts that go beyond the scope of the argument and can be helpful for further argumentations or tasks. We assign these actions to the subcategory *Content knowledge*. For example, when a teacher uses a mathematical argumentation as

an opportunity to not only talk about the distributive law, which was directly relevant for the argument, but also to revise other theorems (e.g., commutative law). Content knowledge is generally addressed by the teacher before or during mathematical argumentations in class. When content knowledge is introduced before the start of a mathematical argumentation, the intention usually is to provide the mathematical knowledge required for the argumentation, which increases students' opportunities for participation, but also directs and restricts their potential contributions. When content knowledge is addressed during argumentations, the teacher generally provides mathematical facts to close gaps in students' argumentations or to connect mathematical concepts in general.

### **Dishevelling actions**

Teacher actions not only support students in participating in mathematical argumentations (Conner et al., 2014), but can also dissuade students from mathematical argumentations. In our data, we regularly found instances of this type and labelled these as *Dishevelling actions*. Such *Dishevelling actions* include activities of teachers that do not advance mathematical argumentations, but inhibit or hinder argumentations, and sometimes even prevent students' own independent argumentations. The category *Dishevelling actions* includes teachers' mathematical mistakes or violations of conversational rules. The following example shows a classroom situation, in which a teacher interrupts the students' argumentation and hinders the students to argue independently.

Hanna: Yes, these [dots] were put together. (Teacher interrupts Hanna.)  
Teacher: Together, right. And the result was again a ... ?  
Tabea: And one [dot] remains alone (incomprehensible)/  
Hanna: And that one still remains alone, because (..)/ (Teacher interrupts Hanna.)  
Teacher: Can one illustrate this with all numbers?  
Tabea: Yes. (Hanna nods her head.)  
Teacher: Yes, right. Yes. Johannes.

Another example of a dishevelling teacher action is a question by a teacher about an elaboration or a fact, which then is answered directly by the teacher. In this case, the mathematical argumentation is enriched by answering the question, but learners are denied the opportunity to argue independently. In addition, teachers can also influence the mathematical argumentation by not acting. For example, ignoring incorrect statements can lead to the development of mathematically incorrect arguments. In our data, we reconstructed six different dishevelling teacher actions (Table 1). But fortunately, teachers not only prevent opportunities for students' involvement in argumentation, but also enable students to independently participate in mathematical argumentation and to learn argumentation.

### **Student actions**

In our data, we reconstructed argumentation processes in which students supported each other, because the teacher had introduced a format of argumentation, where it is necessary for students to interact and question each other. In these situations, teacher actions are regularly implicit. To understand interactions during mathematical argumentations and especially teacher actions framing argumentation processes, it is necessary to analyse these student actions. To acknowledge the complexity of mathematical argumentation in classes and to capture all teacher influences, the category *Student actions* is created. In the analysed argumentation processes, learners occasionally imitated teachers' behaviours and asked specific questions or supported their classmates'

argumentation. Therefore, the category “Student Actions” is linked to the three categories presented in Conner et al.’s framework. Since not all subcategories of this framework were also enacted by the students in the analysed argumentations, this category is limited to seven subcategories (Table 1).

If teachers act as a role model for students (e.g., Cusi & Malara, 2013), teachers encourage students to mutually promote themselves and improve the richness or correctness of mathematical argumentation. Students are encouraged to imitate the teacher’s questions or impulses and thus contribute independently to enhance the mathematical argumentation. Learners’ actions or utterances can thus substitute possible supportive teacher actions. For example, the teacher no longer needs to question the generality of a statement if it has already been addressed by a student. These student actions are usually found during (collective) mathematical argumentations, when learners respond to mathematical arguments presented by classmates and thus become part of the interaction.

The subcategory “Student Actions - Questions: Methods” is coded when students ask their classmates to describe procedures or strategies. Either because they have a different result, do not know a procedure by themselves or want to discuss the procedure afterwards. The following excerpt illustrates such a situation in a mathematics lesson. The task was to give feedback to Paul (a fictional student) about his claim: “The product of two even numbers is divisible by eight”. The students were asked to check this statement with different numbers. Tobias tested divisibility by eight not only for the product of two even numbers (“two times two”), but also for the sum (“two plus two”).

- Tobias: Uhm, please give Paul a feedback. I would say to him (*reads*): Hey Paul, I’m sorry, but this does not fully work, if we only calculate with natural numbers. Eh, two plus two is four and two times two is also four, if you divide this [four] by eight, you get zero point five, but this is not a natural number. (18 sec). Pascal.
- Pascal: I’ve got a question. So, I believe you did this right. But I have a question. Here it [the task] says you, eh ... (*thinking*), you’ve got a product of two even numbers, and how can you check this with different examples of numbers. Did you do this?

In this situation, the teacher allows the students to respond to Tobias’ statements and initially does not respond himself. Therefore, Pascal reacts and evaluates Tobias answer as correct. At the same time, however, he questions Tobias’ approach. This situation also illustrates the relevance of tasks for productive mathematical argumentation in classrooms.

## Conclusion

Teacher actions direct mathematical argumentations in classrooms in various ways (Conner et al., 2014), often in indirect and complex ways as reconstructed in our study of eight-grade mathematics classes. Our analyses illustrate that teachers do not only support specific arguments in class, but also provide support for mathematical argumentation in general. Teachers’ “Meta actions” foster methods for generating mathematical argumentation (method knowledge) and offer relevant content knowledge (Heinze, 2004). In real classroom situations teacher actions are not always supportive. In contrast, they can also distract students from their participation in mathematical argumentations and prevent students from developing their own (correct) arguments by “Dishevelling actions”. Fortunately, teachers often also act as positive role models and establish a format of argumentation in the class, in which all learners are encouraged to participate in mathematical argumentation and to support each other by “Student actions”. In these situations, teachers are usually invisible, but are still



crucial, as they have made such student actions possible and allow them to emerge during argumentations. Furthermore, teachers still control the correctness of the argumentation and thus are participants in the argumentation. But further research in this area is needed. For example, the category “Student actions” does not capture what leads to such actions, as teacher actions are implicit in these situations. Therefore, in following research projects argumentation formats, socio-mathematical norms, and how teachers influence and construct them in classroom are especially interesting: How does a teacher establish argumentation formats that initiate student actions?

## References

- Bredow, F. (2023). *Mathematisches Argumentieren im Übergang von der Arithmetik zur Algebra: Eine qualitative Studie von Lehrkraft-handlungen im Mathematikunterricht* [Mathematical argumentation in the transition from arithmetic to algebra: A qualitative study of teacher actions in the mathematics class]. Springer Spektrum. <https://doi.org/10.1007/978-3-658-42462-6>
- Conner, A., Singletary, L. M., Smith, R. C., Wagner, P. A., & Francisco, R. T. (2014). Teacher support for collective argumentation: A framework for examining how teachers support students' engagement in mathematical activities. *Educational Studies in Mathematics*, 86(3), 401–429. <https://doi.org/10.1007/s10649-014-9532-8>
- Cusi, A., & Malara, N. (2013). A theoretical construct to analyse the teacher's role during introductory activities to algebraic modelling. In M. A. Mariotti, B. Ubuz, & Ç. Haser (Eds.), *Proceedings of the 8th Congress of the European Society for Research in Mathematics Education* (pp. 3015–3024). Middle East Technical University and ERME.
- Heinze, A. (2004). Schülerprobleme beim Lösen von geometrischen Beweisaufgaben – Eine Interviewstudie [Students' problems in solving geometric proof tasks – An interview study]. *ZDM*, 36(5), 150–161. <https://doi.org/10.1007/BF02655667>
- Krummheuer, G. (1995). The ethnography of argumentation. In P. Cobb & H. Bauersfeld (Eds.), *The emergence of mathematical meaning: Interaction in classroom cultures* (pp. 229–269). Erlbaum.
- Pedemonte, B. (2018). How Can a Teacher Support Students in Constructing a Proof? In A. J. Stylianides & G. Harel (Eds.), *Advances in Mathematics Education Research on Proof and Proving* (pp. 115–129). Springer. [https://doi.org/10.1007/978-3-319-70996-3\\_8](https://doi.org/10.1007/978-3-319-70996-3_8)
- Schwarz, B., Hershkowitz, R., & Azmon, S. (2006). The role of the teacher in turning claims to arguments. In J. Novotná, H. Moraová, M. Krátká, & N. Stehlíková (Eds.), *Proceedings 30th Conference of the International Group for the Psychology of Mathematics Education, Vol. 5* (pp. 65–72). PME.
- Solar, H. S., Ortiz, A., Deulofeu, J., & Ulloa, R. (2021). Teacher support for argumentation and the incorporation of contingencies in mathematics classrooms. *International Journal of Mathematical Education in Science and Technology*, 52(7), 977–1005. <https://doi.org/10.1080/0020739X.2020.1733686>
- Yackel, E. (2002). What we can learn from analyzing the teacher's role in collective argumentation. *The Journal of Mathematical Behavior*, 21(4), 423–440.