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Matching a graph with an image representing the situational context: Students' approaches identified by using eye tracking

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Data from real-world contexts are often represented in graphs. The interpretation of these contextual graphs is a major challenge for students. A common misconception that, for instance, reveals students' difficulties is the iconic interpretation of graphs. It occurs when students match a graph with an image representing the situational context. The aim of our study is to trace students' approaches when matching a graph with an image of the situational context. A promising method to investigate students' approaches when interpreting graphs is eye tracking. In our study, we used eye tracking in combination with stimulated recall interviews to investigate how 19 students in grade nine approached matching graphs with images representing the situational contexts. We illustrate our findings through four student examples: The results indicate that the approaches differ considerably, but have in common that they mainly rely on two strategy elements.

Keywords: Eye tracking, functions, graphs, iconic interpretation, situational context.

Introduction

Being able to interpret graphs, especially contextual graphs that contain data from real-world situations (Thomaneck et al., 2022a), is fundamental to meet the demands of today's world (Friel et al., 2001). Data from various areas of life are represented in graphs, but their meaning must be inferred from the graph since it is not accessible directly (Freedman & Shah, 2002). Previous studies revealed that students may have particular difficulties interpreting graphs that represent real-world contexts. These difficulties are due, for example, to the additional requirement to switch between mathematics and the everyday world (Bossé et al., 2011), and they are particularly evident in one of the most well-known and widespread misconceptions when interpreting contextual graphs: iconic interpretation. Iconic interpretation occurs when the graphical representation of a functional context has to be matched with a pictorial representation of the aligning situational context and the graph is then interpreted as a direct image of the situational context. Although this misconception has been repeatedly detected in studies, exact causes have not yet been determined in detail (Nitsch, 2015). This misconception indicates how difficult it is for students to interpret graphs with regard to situational contexts. In our study, we aim to trace students' approaches when matching a graph with an image of the situational context and examine what constitutes successful and unsuccessful approaches. A promising tool to get insights into students' approaches is eye tracking (ET). ET has already been used in many fields in mathematics education and beyond to draw conclusions about learners' cognitive processes from their eye movements, thus providing access to their thinking processes and approaches when working on such tasks. Prior studies have shown that the potential is high in the area of functions and graphs (Andrà et al., 2015; Goldberg & Helfman, 2011; Thomaneck et al., 2022b).

In this paper, we examine how students approach tasks asking them to match two representations of a function: a contextual graph with related images of objects (seemingly) representing the situational context. We ask the following research question: *What approaches do students use when they match a contextual graph with images representing the situational context?* For pursuing this question, we use ET together with Stimulated Recall Interviews (SRIs).

Theoretical framework

Contextual graphs and iconic interpretations

We encounter functional relationships everywhere in daily life and, especially in the media, they are often depicted in graphs (e.g., stock price developments, infection patterns of diseases, forecasts of climate developments). We call these graphs, whose data originate from real-world contexts, contextual graphs. They are characterised by the fact that they are closely linked to the real-world situation and their course is often irregular and cannot be described by an algebraic function term (Thomaneck et al., 2022a). The data presented in the graph cannot be taken directly from the graphical representation, but the person reading the graph must interpret it to extract its meaning. Three types of tasks that increase in complexity can be distinguished for reading data from graphs: locating, integrating and generating tasks (Friel et al., 2001). This has led to three levels of graph comprehension: elementary - extract information from data; intermediate - find relationships in data; overall - move beyond data (Friel et al., 2001). Moreover, graph readers need to relate the data to the situational context by using their situational knowledge (Freedman & Shah, 2002; Friel et al., 2001). A central activity when working with functions, and, thus, therefore in interpreting graphical representations of functional relationships, is the translation into another form of representation (Andrà et al., 2015; Nitsch, 2015). In the case of contextual graphs, it is particularly important to interpret them in terms of their situational context, i.e., to understand what the data presented mean in relation to the real situation. This is particularly challenging for students because of the additional requirement to switch between mathematics and the everyday world (Bossé et al., 2011). For instance, a very widespread misconception reveals these challenges: the graph is interpreted as a direct image of the situational context. This misconception has been repeatedly found in studies for decades (e.g., Bell & Janvier, 1981; Li, 2006). It is referred to by various terms (e.g., pictorial distraction, graph as a picture, iconic translation/interpretation). In this paper we will use the term iconic interpretation.

To gain access to how students understand the graph and interpret it in terms of situational context it is important to study their approaches in detail. A method that seems promising in this respect is ET.

Eye tracking in the domain of functions and for the analysis of comparison strategies

When using ET, eye movements are recorded, and the participant's gazes are usually visualised in videos that show the students' field of view. This method is increasingly used in mathematics education research (Strohmaier et al., 2020). From a psychological perspective, which we also adopt for our study, the aim of using ET is to draw conclusions about students' cognitive processes from their eye movements (König et al., 2016). Strohmaier et al. (2020) emphasise that ET is particularly beneficial to study the use of visualisations of mathematical objects because the processing of visualised information involves complex mental processes that are challenging to study but could be approached by ET. Previous studies have shown ET to be a promising method in the area of functions

and especially graph interpretation. In these studies, ET allowed detailed insights into students' cognitive processes when working on tasks in this domain. For instance, ET has helped to show that the way students grasp formulas and graphs differs considerably due to the characteristics of the representation, but also to how familiar students are with different types of representations (Andrà et al., 2015). Studies that specifically address the interpretation of graphs have also been conducted with ET. Goldberg and Helfman (2011) were able to reconstruct a three-stage processing model when reading values on graphs using ET and compared the expressions of the three stages for linear and radial graphs. In combination with SRIs, ET can also be beneficially used to identify students' perceptions of change in graphs and their covariational reasoning (Thomanek et al., 2022b).

We aim to study how two different elements (here: representations of functional contexts) are matched with each other. To the best of our knowledge, there are no comparable studies in the domain of functions, but in that of fractions. Obersteiner and Tumpek (2016), among others, compared saccades between corresponding fraction components (numerator and denominator) of different fractions and between the fraction components of the same fraction to analyse students' comparison strategies. Thus, it can be assumed that students' eye movements also provide useful insights into how students approach tasks that ask them to match two representations of a function, namely a contextual graph with images representing the situational context.

Methods

Sample, task design, and setting

The study was conducted with 19 ninth grade students ($d = 15;8$) of which 13 indicated that they were female (68 %) and six that they were male. Students from every school type of the still widespread tripartite German school system participated. The study presented here is part of a larger study in which the participants worked with two different contextual graphs based on task designs from the Shell Centre (1985). The first graph shows the ride of a racing car in the second lap of an even racetrack. The second graph shows the filling process of a vessel with a constant jet of water. For each of the two graphs there is an introduction describing the situational context and subsequently six tasks of different questioning levels (Friel et al., 2001). The focus of this paper is on the final task of

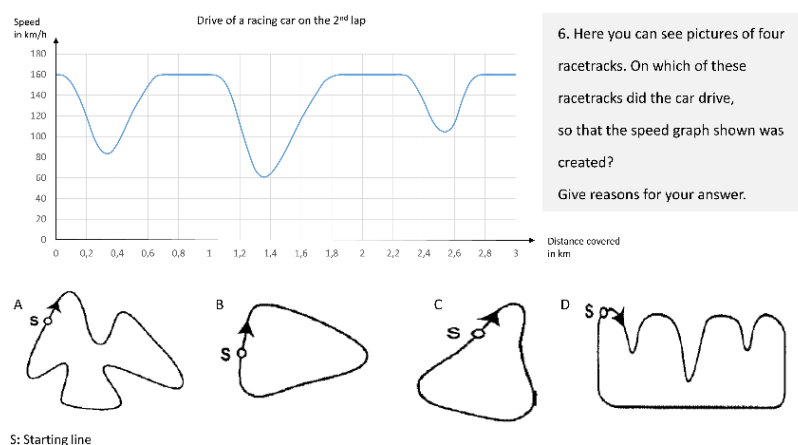


Figure 1: Ride of a racing car, task 6 (translated to English)

the graph showing the ride of a racing car (Figure 1). In this task, the participants are presented with the graph they already know from the previous tasks and four schematic images of racetracks. The task is to decide on which of the racetracks the car drove, so that the graph shown matches (overall level; see above). Participants were asked to give reasons for their answer. At the beginning of each the data collections, the instructor (first author of this paper) described the procedure in detail to the participants. They were asked for demographic data and then worked individually on the tasks presented on a screen (27", viewing distance: 62-75 cm). They wore ET glasses when working on the tasks and reported their answers verbally. Afterwards, an SRI was conducted.

Eye tracking and stimulated recall interviews

For data collection, we used the head-mounted ET system Tobii Pro Glasses 2 (50 Hz binocular, infrared, 45 g). The built-in microphone recorded the verbal responses so that they were automatically included in the respective gaze-overlaid video produced by the Tobii software. These videos show the students' gazes as a wandering red dot, and they include their utterances. After conducting the essential one-point calibration, we performed a nine-point verification to calculate the ET accuracy. We repeated this procedure after the first six tasks and after finishing all tasks. In our study, the average accuracy was 1.2° (under ideal conditions: 0.62°, Tobii Pro AB, 2017), which corresponds to 1.3 – 1.55 cm on the screen depending on the viewing distance.

Subsequent to the completion of the tasks, the instructor conducted an SRI with the participants. The stimulus for these interviews were the gaze-overlaid videos (similar to Schindler & Lilienthal, 2019). Gaze-overlaid videos are a very powerful stimulus to recall memories of one's own thoughts as they make eye movements visible, which are often also unconscious (Stickler & Shi, 2017). The procedure of the SRI is that the participant and the interviewer watch the video together and the participant pauses the video to explain what his/her thoughts were at that particular moment. Additionally, the interviewer can pause the video at appropriate points and ask about the thoughts. The interviews were conducted about 15 minutes after completion of the tasks and lasted between 20 and 50 minutes. The SRIs were recorded with a video camera positioned diagonally behind the participants.

Data analysis

The data analysis for the entire study comprises a multi-step procedure. We will only present the steps that are relevant to the research question posed in this paper. We transcribed students' utterances while working on the tasks and the SRIs. In order to analyse the eye movements, we described them qualitatively, e.g., *the gaze fixates on the first minimal turning point of the graph and then jumps to the second curve of picture A*. Moreover, we created gaze plots using the Tobii software (see Figures 2-4). Gaze plots show the time sequence of fixations and the time spent looking at that certain point on the stimulus by the diameter of the circle, while the order of fixations is numbered in this representation and subsequent fixations are connected by a line. Gaze plots therefore provide detailed insights into students' approaches. As we are interested in the approaches taken for the matching of graph and image, we analysed gaze plots covering the period of time from the beginning of the sixth task to the time when the answer to the task was given. The description of the eye movements and the gaze plots were then used to analyse the individual approaches. For each person, we then used the identified phases to infer which strategy elements make up the individual approach. Finally, we

consulted the utterances during the work on the tasks and the SRIs as a source of information and examined to what extent the identified approach can be verified by the utterances of the students.

Results

Analysing the gaze plots and the qualitative descriptions of students' eye movements revealed that students mainly use two recurring gaze patterns when working on the task. The gazes either jump between the graph and an image of the racetrack (a), or between two or more images of racetracks (b) (see Figure 2). We have therefore identified these gaze patterns as two different strategy elements: *between* (a) and *within* (b). For both strategy elements, one can distinguish whether or not the gaze jumps occur between corresponding elements of two representations. In the following, we will illustrate how four students took different approaches for the matching of a graph with an image representing the situational context and how the identified strategy elements occur in them.

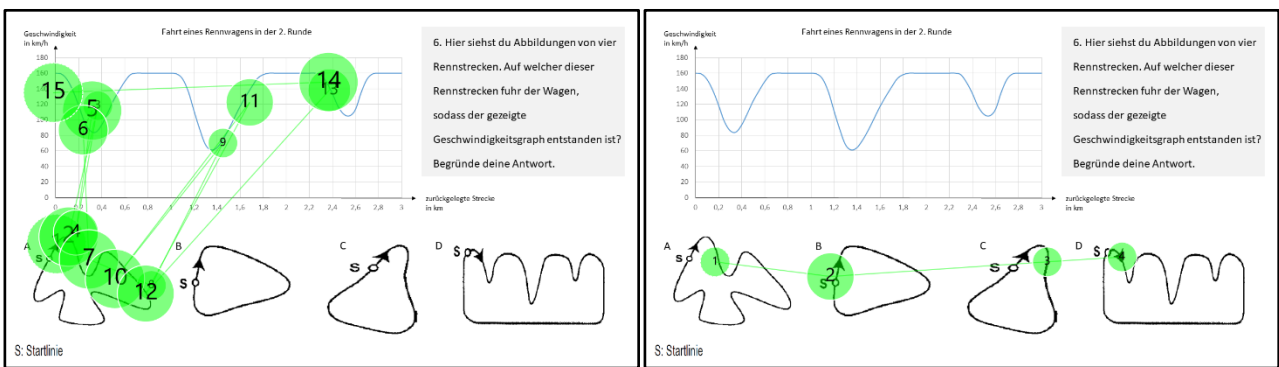


Figure 2: Exemplary gaze plots for the strategy elements a (left, *between*) and b (right, *within*)

The gazes of the first student, Ben, jump back and forth almost exclusively between the images of the four racetracks, suggesting that he mainly uses strategy element *within*, which is also visible in the horizontal lines between the images in the gaze plot of his eye movements (see Figure 3). Only twice does the gaze jump from one of the images to the graph. He decides on the correct answer choice, racetrack B, but can hardly verbalise his cognitive processes when working on the task in the SRI; he only recalls that this was the most logical option for him.

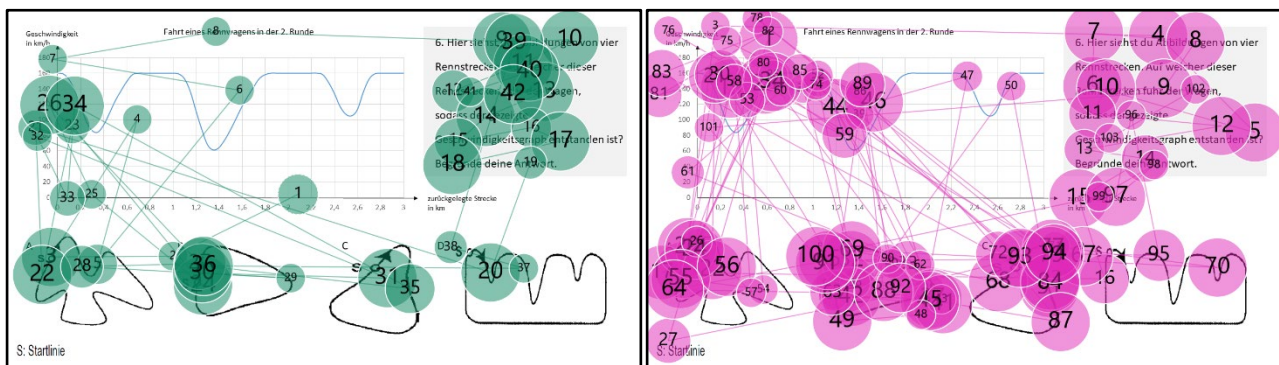


Figure 3: Gaze plots of Ben (left) and Charlotte (right) who correctly solved the task

Charlotte also gives the correct answer B. In contrast to Ben, she uses both strategy elements, with a stronger focus on *between* (see Figure 3). Her eye movements show that she first compares the first section of racetrack A piece by piece with the graph, then successively all sections of racetrack B.

Racetrack C is not compared as systematically with the graph. Her visual focus here is mainly on the indentation between the first and second curve and the corresponding location of the graph. Racetrack D is not compared with the graph, but only with the other images of the racetracks. In the SRI, Charlotte verifies the identified approach. She explains that she quickly ruled out racetrack A because of too many curves and that comparing racetrack B and the graph showed that it was a good fit.

Jacob, who also mainly uses strategy element *between*, decides for the wrong racetrack D. His eye movements reveal that he initially compares racetrack A and later on racetrack D with the graph (see Figure 4). Only one short phase was identified, in which he uses strategy element *within*. In the SRI, he explains that he “wanted to see where it would go inwards and if it would fit from the lengths.” He says that both racetracks A and D are possible and theoretically correct, whereas B and C are too straight. He finally decides for D because “it looks the same and fits perfectly”.

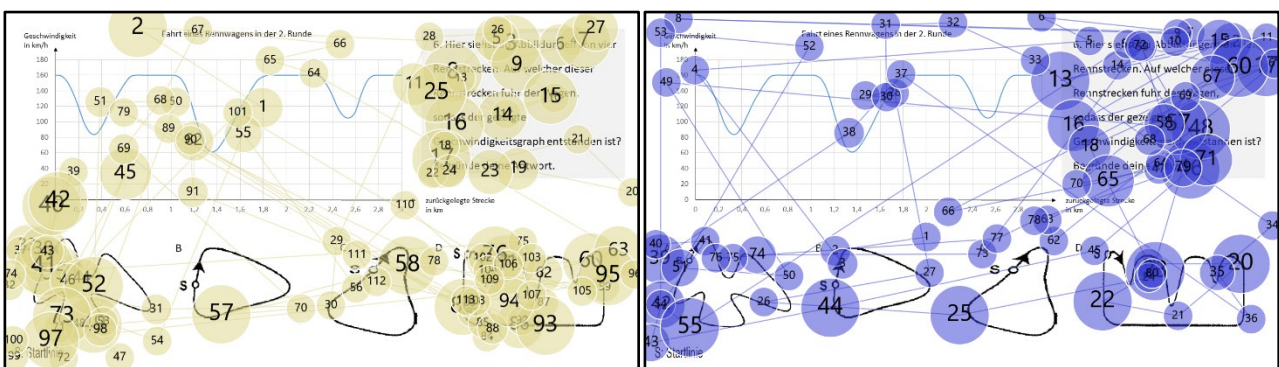


Figure 4: Gaze plots of Jacob (left) and Daniel (right) who did not solve the task correctly

The last student, Daniel, uses both strategy elements *between* and *within* to approximately the same amount. However, his eye movements show long phases in which neither strategy element is used, but there are many fixations on the text, the axes, or non-meaningful points that contain no relevant information to solve the task (see Figure 4). In the SRI, he explains his thoughts on the exclusion of racetracks B and C, claiming that the curves are not sharp enough. In addition, he was unsure if more than one racetrack could fit and be selected. This also explains why he names both racetracks A and D as the answer.

Discussion and outlook

In this paper, we used four cases to illustrate students’ approaches when matching a contextual graph with images (seemingly) representing the situational context. In line with Obersteiner and Tumpek (2016) we identified two strategy elements based on recurring gaze patterns that seem to be helpful from a normative point of view to solve the task: a) the search for corresponding components *between* graph and image and 2) *within* different images. We have chosen two exemplary students who succeed in matching graph and image and two students showing iconic interpretations—a typical misconception that frequently appears in this type of task.

Comparing the two students who succeed in matching the graph with the image, it is noticeable that the approaches differ significantly. Ben, on the one hand, predominantly uses strategy element *within* and compares elements between the different images of the racetracks. His eye movements suggest that he already had an idea of what the racetrack should look like from the previous tasks, since he

has very few fixations on the graph. Charlotte, on the other hand, predominantly compares *between* one image and the graph. Her entire approach seems much more systematic, which is also reflected by the fact that she can recall her approach for racetracks A and B afterwards in the SRI. She does not go into as much detail about the exclusion of the racetracks C and D in the interview. Based on her eye movements and her statements about the other racetracks, it is likely that she was able to exclude C because of the indentation and D using the information obtained from the comparison of the racetrack A with the graph, that there are too many curves.

Both Jacob and Daniel choose images that look visually the same or similar to the graph (iconic interpretation). On a superficial level, Jacob's approach looks similar to Charlotte's. He also primarily uses comparisons between graph and images, but restricts this to the two images that look similar to the graph. Thus, the use of the strategy element and the extent to which they are used does not seem to be the sole determining factor. More detailed analyses are needed of which parts of the graph and the images are considered, why which images are included in the selection, and what the students' thoughts are while comparing. In addition, other strategy elements that we have not yet identified may play a role in matching a contextual graph with an image that (seemingly) represents the situational context. It is remarkable that Daniel makes little use of the identified strategy elements. His eye movements consist mainly of fixations on the text, on diagram elements such as the axes, and on non-meaningful points. On the one hand, this can be an indication of a lack of a strategic approach, but on the other hand, it can also indicate uncertainty concerning mathematical aspects or emotional arousal (Thomanek et al., 2022a). What supports the latter assumption is Daniel's statement in the SRI that he does not know whether two racetracks can be matched to the graph.

To conclude, it was possible to identify two main strategy elements and to distinguish individual approaches that emerged from them with the help of students' eye movements. In order to identify factors that promote or prevent the occurrence of iconic interpretations, however, more detailed analyses are necessary, for which the eye movements can be very helpful data, as they are well suited to reveal information on students' individual approaches when interpreting (contextual) graphs.

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