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From *weight* to *value*: Focusing on verbal signs during a digital artifact-mediated algebraic activities

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Dealing with unknowns for solving linear equations can be particularly challenging for students with MLD, but research findings suggest that the use of digital artifacts supports these students in constructing algebraic meanings. Using the Theory of Semiotic Mediation, we analyse a case study of a high school student with MLD, focusing on the evolution of signs that were developed during tutoring sessions with the use of Dynamic Digital Artifacts which embeds a balance model of linear equations. A combined use of quantitative and qualitative analytical tools leads us to describing the features of the student and researcher's semiotic activity that support or impede the development of mathematical signs for the notions of unknown and solution of a linear equation.

Keywords: Equation, unknown, solution of a linear equation, balance model, semiotic mediation.

Introduction

Research in mathematics education has highlighted many difficulties in the domain of school algebra teaching and learning, that include giving meaning to indeterminate quantities and to the solution of a linear equation (e.g., Kieran, 2022). Tackling unknowns for solving linear equations is a substantial part of many mathematics curricula, but for students with Mathematical Learning Disability (MLD) dealing with these algebraic notions can be particularly challenging and daunting. However, research findings suggest that the use of digital artifacts has the potential to support these students in constructing mathematical meanings (e.g., Robotti 2017). Based on this scenario, in this study we look at the construction of meaning for the mentioned algebraic notions through the evolution of signs, exploiting the use of dynamic digital artifacts. The study is part of an Italian larger funded research project, *DynaMat*, that, through a design-based methodology, is conducting case studies of 10th grade students with a history of low achievement in mathematics. In this paper, we adopted the lens provided by the *Theory of Semiotic Mediation* (Bartolini Bussi & Mariotti, 2008) in order to explore the role of the teacher and of the artifact in supporting the process of the semiotic mediation through which the signs linked to the artifact move towards mathematical signs.

Theoretical framework, research aim, and Dynamic Digital Artifact

We built on Bartolini Bussi and Mariotti's (2008) *Theory of Semiotic Mediation (TSM)* since it provides a suitable theoretical lens to study the process of turning activities with artifacts into mathematical activities. According to this theoretical lens, the teacher manages two main processes: the design and the implementation of activities. In the former the teacher makes appropriate choices about the artifacts to be used, the tasks to be proposed and the mathematics knowledge at stake (in this report we take into account some of the tasks designed by the team of researchers within the *DynaMat* project). In the latter, the teacher monitors and manages the students' observable processes

(semiotic traces), to decide how to interact with the students in order to transform signs linked to the use of artifacts into mathematical signs. The students are in charge of the resolution of the task through the use of the artifact proposed by the teacher; doing this, they produce signs (drawings, words, gestures, bodily movements, and so on), which are linked to the artifact but that are not yet mathematical signs. The teacher collects all these signs in order to organize a path for their evolution towards mathematical signs, that is, standard shared signs for the chosen mathematical knowledge. The teaching-learning process is conceived as a process which involves the production and elaboration of three main categories of signs: *artifact signs*, which refer to the context of the use of the artifact, *mathematical signs*, which refer to mathematical context, and *pivot signs*, which act as bridges between the artifact signs and the mathematical signs. In this study we specifically focus on the evolution of signs: from artifact signs to mathematical signs. Moreover, the TSM also offers a description of possible teacher’s “communicative acts” in order to support this evolution (Maracci & Mariotti, 2012). However, the tutoring setting, which characterizes the experimental setting of this study, is quite different from the usual teacher-class setting, where after the activity with the artifact there is “the possibility of negotiating a shared meaning within the class community” (Bartolini Bussi & Mariotti, 2008, p. 756). This community dimension is restricted to a one-to-one exchange.

The balance model is a common metaphor used in teaching linear equations, for conceptualizing the equal sign and promoting strategies to deal with unknowns and equality (Otten et al., 2019), in order to move towards the conceptual knowledge of algebraic equations. Building on these previous studies, within the DynaMat project we designed a *dynamic digital artifact (DDA)* which consists of a two-pan balance and a box where the student has at disposal some “known” weights, represented as colored shapes with a number inside which refers to their value, and some “unknown” weights, represented by empty shapes (Figure 1). Colors are used to mediate the idea of the numerical values: the same color is a sign for the *same value*. The user can move any weights (known and unknown) from the blue box to the pans (and vice-versa). Weights can be moved one at a time, and when a weight is placed or removed from a pan, the artifact produces dynamic feedback (e.g., Figure 1b). All the weights have been set up previously by the designer and the user cannot create new weights.

This DDA has been designed to mediate several mathematical meanings. In this paper we focus only on the *unknown*. The unknown weight’s value mathematically corresponds to the specific number, not yet known, that makes the equality between the two algebraic expressions represented by the set of weights on each pan true.

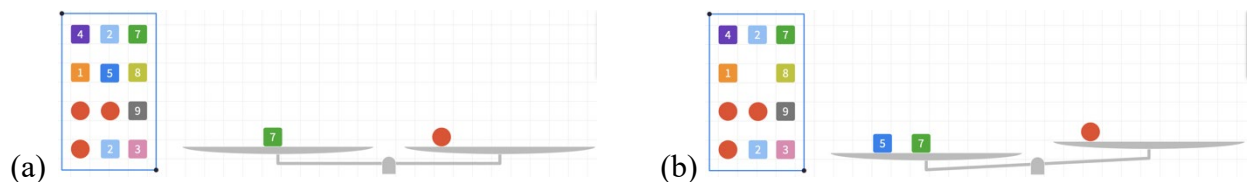


Figure 1: Dynamic Digital Artefact with known and unknown quantities

In this paper we are interested in investigating the case study of a student with MLD. Although there is not a shared definition of MLD (Lewis, 2016), for our purposes in this paper we consider students “having MLD” as students with persistent low achievement specifically in mathematics who are at risk of being labelled by clinicians as “having a mathematical learning disability” or who have been

diagnosed clinically with such a condition. In tune with the TSM, we focus on the evolution (in time) of signs by mapping and comparing the student and the researcher's signs to reveal possible synergies or misalignments. Therefore, *we try to understand to what extent do the artifact signs or pivot signs – be they produced by the researcher or by the student – evolve into mathematical signs and thus, to what extent the student with MLD constructs meanings for algebraic notions.*

Methods

The student, fictionally named Ilenia, age 15 at the time of the intervention, has demonstrated low performance in working memory tasks, inefficient and unsuccessful counting strategies mainly rooted in counting-on processes, few arithmetical facts to be retrieved, several difficulties in dealing with different senses of numbers. The sequence consisted in 5 meetings of 2 hours each (2 sessions of 45 minutes each, with coffee-break between), which took place in an out-of-school center. Ilenia worked under the guidance of a researcher, she was provided with touch-screen tablets for the activities, and she was engaged in activities with DDA. Data consists of audio-video recordings, Ilenia's written productions, and screen recordings of the tablets. In tune with the TSM, the analysis will focus on the use of signs (words, drawings, gestures) related to dealing with indeterminate quantities.

The teaching sequence starts with a free exploration of the artifact; then there are several tasks which involve the request to gain (or maintain) the equilibrium of the balance by adding or removing items or to find out the weight of unlabeled objects; the student is asked to explain her reasoning. Before acting, the researcher asks to explain possible actions on the artifact and predict its feedback. Only after claiming her predictions, Ilenia has the possibility to verify them by moving the chosen weights. In order to stimulate the production of written signs, we propose tasks where Ilenia is asked to represent the givens or the resolution. According to the analysis of the semiotic potential of the DDA with respect to the notions of equation and unknown quantity, we expected a certain evolution of signs: during the exploratory phase students can use the terms "circle", "shape", "weight", "red item" to talk about the unknown. These artifact signs could evolve into pivot signs like "right number" or just "value". At the same time, the use of verbs could have an evolution. Starting from "the circle weights", through "the circle is worth" and "the weight is" to "the circle is equivalent to", "the value is", we presumed that students' ways to talk about the unknown could evolve through these stages.

Data analyses

Despite our assumptions, Ilenia's most relevant productions were verbal. For this reason, data were processed using *MAXQDA*, a software designed for computer-assisted mixed methods research. Data consists of 8 transcripts of Ilenia's working sessions. In order to identify artifact signs or pivot signs in Ilenia and the researcher's semiotic activity, we have defined labels linked to the artifact and its use. The a priori analysis of the semiotic potential of the artifact suggests certain keywords. Moreover, exploiting the *MAXQDA* system and in particular its *Word Cloud* functionality, we identified in the verbal production the most recurring keywords linked to the artifact (e.g., weight, pans) and its use (e.g., to raise or to lower, to put or to leave) and to the algebraic meanings of *equality*, *unknown*, *equation* and *solution of an equation* (e.g., value, worth (of a weight), number). These keywords inspired the design of specific labels to be used in the coding of the transcripts. Thanks to the *Automatic Labeling* and manual semantic adjustments we were able to code all the transcripts. We

used quantitative tools of *MAXQDA* for detecting the most used labels and their co-occurrences; such an analysis allows us to capture the evolution of signs. Due to the richness of data and for space constraints, in this paper we will focus only on the labels linked to the notion of *unknown* and *solution of a linear equation*: Worth/Value, Number, Weight, Measure, Find out, Shape¹. In particular, “value” or “to be worth” are used by the researcher for moving from known value toward unknown quantities. We assume that these labels are connected with specific mathematical notions according to two criteria: the connection is suggested both by the a priori analyses of the semiotic potential of the artifact and of the tasks, the qualitative analyses of the word use in context. The co-occurrences of labels developed both by Ilenia and by the researcher, allow us to analyze to what extent signs evolve to construct the algebraic meanings of *unknown* and *solution of a linear equation*. In order to outline such an evolution, we exploit the two MAXQDA visual tools – *Code map* and *Codeline* – for observing the relationship between labels and, therefore, signs. Finally, since we are interested in gaining insight into the possible interplay between student and researcher’s signs, the labels we have developed are speaker-sensitive. Therefore, each label name contains an (S) for the Student and a (R) for the Researcher.

Results of the analyses

The *Code map* tool allows us to visually represent the number of label occurrences (i.e., the use of a certain group of words) in a selected set of transcripts by the dots dimension (Figure 2). Moreover, the more two labels co-occur in the data, the closer they are placed together on the map. Figure 2 visualises the occurrences of labels in the same utterances within the 8 sessions.

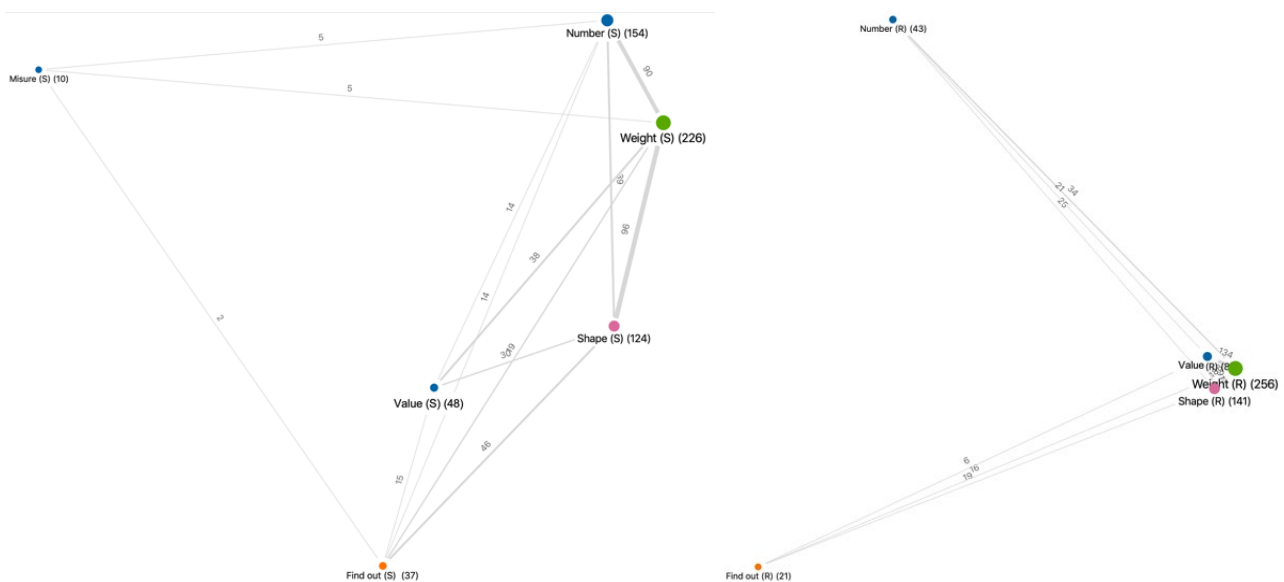


Figure 2: Code map of selected labels in Ilenia (on the left) and Researcher’s (on the right) verbal productions during the 8 sessions

¹ Each label maps all the nuances of a certain word. For example, *Worth/Value* includes: worth, value, to be worth, should be worth, to have value, and so on. The label *Shape* maps all the specific shapes of the weight: little rectangle, star, little cube, triangle, and so on.

We interpret the closeness of labels as an attempt to promote a certain semantic connection, more than others. For example, the pair Weight-Worth/Value maps a sentence like “*the weight of value 8*” and the triplet Weight-Shape-Worth/Value maps a sentence like “*Let’s try to assign to the weight of the square another value*”. Therefore, this visual tool shows at a glance the researcher’s attempt (Figure 2, on the right) to foster the use of the verbal sign “*value*” when she talks about specific objects (labeled as *shape*) or *weights* upon the scale. Indeed, we can notice the close-up position of these labels. On the other hand, the labels that map Ilenia’s utterances are more scattered (Figure 2, on the left), demonstrating the slow appropriation of the words prompted by the researcher. This is evident in the distance of *number* from *value* and the proximity with the *weight* and *shape* labels.

The output of this visual tool coupled with the analyses of the semiotic potential of the digital scales suggests a deeper investigation on the label *value*, the use of which is heavily promoted by the researcher, but less integrated into Ilenia’s verbal production. To this aim, a different visual tool was set up: the *Codeline* (Figure 3). It is a case-based visual function that displays a sequential view of a single transcript’s coded segments. Each column represents a range of paragraphs of the transcript, the rows display the labels; the cells are filled with a colored symbol, if a certain label is in the paragraph. It gives us a visual overview of a single transcript, so you can quickly identify which labels appear and co-occur. In particular, since the labels we have developed are speaker-sensitive, we use this tool for observing the introduction of specific words (which correspond to a certain label) and their use by one of the speakers.



Figure 3: Codeline of tutoring session 1 where the occurrences of the word “value” as R-label or S-label are highlighted

Following the lines corresponding to *Value* (Figure 3, row 4 and row 5), we can notice that the word “value” is introduced by the researcher, and its use is not mirrored by the student. Indeed, although the researcher uses this word repeatedly (see the several colored instances within row 4 in Figure 3), the student avoids its use up to the utterances range 315–335, where we find the first (and last) instance of this label. The sequence of R-label and S-label in this range suggests that Ilenia does not spontaneously use “value”, rather she seems to borrow the researcher’s use for answering a specific question. This can be seen through and confirmed by a qualitative analysis of the utterances in this range. Therefore, the use of this second visual tool allows us to select specific moments to be analyzed in greater depth. This is possible by combining a qualitative analysis of specific transcript excerpts.

Introduction, use, and evolution of worth: Focus on excerpts

Looking at the semantic context of the first instance of *Value(S)*, we can notice that the researcher tries to build a bridge between two signs: the situated sign “weight”, as introduced and extensively used by Ilenia (see Figure 3, row 2), and the verbal sign “value” in the context of numerical equivalence. Here is the corresponding excerpt:

Researcher: [...] if we choose that on the left we put the weight of value [8], so I mean let's say the object that has weight 8.

[...]

Ilenia: The same weight as the 8.

Researcher: Ok, weight of value 8.

The word is used as a noun and related (see “I mean” and “let’s say”) to the situated sign “weight” that was developed so far. Further ahead in this and following sessions, the researcher uses “value” also as a verb “one weight is worth 3, the other weight is worth 1” in order to promote the building of the aforementioned bridge. Therefore, the excerpt provides a prototypical example of the twofold ways in which the researcher intervenes: she introduces a certain word and its use (see line 1) in term of verbal sign, she tries rephrasing the student’s situated sentences (see line 3) with this word to promote the transition of the situated sign towards a verbal sign which has the potential to become a pivot sign. We note that it is an implicit negotiation of the use of a word. As demonstrated by the lack of *Value(S)*-label within the Codeline, these actions seem to be useless in promoting the development of desired signs. Nevertheless, at a certain point the quantitative data concerning the total occurrence number of the label *Value(S)* reveals that Ilenia adheres to the use of this word. Moreover, the qualitative analysis suggests that she develops a personal meaning of “value”, i.e. it becomes a personal sign for Ilenia. Indeed, pursuing the educational goals of the activities, the researcher uses “(the weight is) worth” (in Italian “valore” is a noun and “valere” is the corresponding verb) for referring to the numerical quantity behind the items on the scale pans, be they reported on the item for explicit or implicitly as in the case of unknown quantities. This means that the researcher refers to the value of the weight almost unchangeably in the context of numerical equivalence and linear equations. However, Ilenia seems to prefer referring to “the weight of the weight” rather than “the value of the weight” when the numerical quantity is reported on the item. Ilenia’s anchoring in situated signs is demonstrated by all the comparative claims which involve the weight: “same weight”, “more weight”, “less weight”, “total weight”. Therefore, it seems that for Ilenia all the items are “weights labeled with a certain number” and, in this case, the reference to a value that is elsewhere does not make sense. In other words, the presence of the iconic sign (i.e., the number-label) makes the reference to its value useless. The Codelines of sessions 2 and 4 shows an increasing use of “value” in Ilenia’s verbal production (9 and 15 instances, respectively). The qualitative analysis of the transcript reveals that, when she needs talking about weights which are iconically represented as shapes without a number label, she resorts to “value”. The first instance occurs when Ilenia is facing the following task on the scale (see Figure 1a): *The balance you see is in equilibrium. In your opinion, why is it so?*

So far Ilenia has faced the problem of moving only numbered items in order to have a balanced scale. For the very first time, Ilenia faces a new iconic representation, that is a circular shape without a number-label inside (Figure 1a), and she seems to be aware that this shape has a weight. Here is the corresponding excerpt reporting on the first claims about the new item.

Ilenia: That we always have the **numbers**, ehm, and we have a **little ball**.

Researcher: Ok, what has this ball got?

Ilenia: This ball has **the weight that is worth 7** (it: “peso che vale”)

[...]

Ilenia: Because if it were less weight than the 7, the scale would be different, because if it were worth less, then the 7 would drop and the ball would rise. So, knowing that it is balanced, I know **that here there is the 7** [on the left] **and this is worth 7** [on the right] (it: “questo vale 7”).

We can notice that, in order to describe the situation (Figure 1a), Ilenia chooses to talk about familiar iconic signs (see the adverb “always”) as “numbers” using a less situated lexicon, and the new item as “little ball” in a very situated manner but referring to the value (see line 3). Ilenia puts aside her reluctance to use “value” or “(to be) worth” for talking about the numerical values behind the weights, when she needs to distinguish between two kinds of weights that are represented using two different iconic signs. The selection of a different word – from *weight* to *value* – is Ilenia’s feedback to a specific need that arises inside the activity with the digital artifact and was triggered by the task design. From now on and thereafter, Ilenia keeps talking about “weight” for known quantities, while she starts using “value” for unknown quantities. In other words, this sign seems to have the potential to function as a pivot sign toward the construction of the mathematical meaning of *the unknown* and possibly *solution of a linear equation*.

Conclusions

The case study we report on concerns the semiotic activity involving two subjects: the researcher and a high school student with MLD, Ilenia. Under the lens provided by the TSM, we focus on the development of verbal signs as observable evidence of the construction of new meanings.

We have interpreted the results of the quantitative analyses by assuming that the co-occurrence of labels and their closeness in the transcripts reveal a semantic connection between the corresponding words. In this way, we have observed that while the researcher tries to promote the blended use of *weight*, *(to be) worth*, *value* and *shape* in the context of solving an equation or maintaining a numerical equivalence building on the balance metaphor, Ilenia prefers other semantic connections between *shape*, *weight*, and *number*. The observation of this general trend drove the qualitative analyses, which show in greater depth that Ilenia's verbal signs remain narrowly anchored in the experience with the DDA never evolving towards mathematical sign. We can say that the interaction with the artifact alone does not support the evolution of signs and therefore the meaning construction. A careful design of the DDA and the sequence of the tasks need to be coupled with specific teacher’s interventions to promote the process of signification. In this sense, a telling example is provided by the case of “value”, which becomes a verbal sign to be used when there are items that correspond to an unknown quantity. The use of this new sign is essentially prompted by the need of choosing a different word for talking about a new object (the little ball) but coupled with the past researcher’s offer of a specific word (i.e., *worth*) for talking about weight values. In this sense, offering a word in another context made that word potentially at the student’s disposal, so that Ilenia has used it when a specific need is triggered by the activity with the DDA. Other fruitful situations occur when a word that was proposed by the student is mirrored in the researcher’s verbal production and when the use and the meaning of a certain word or iconic sign are established as the result of an explicit agreement. For example, Ilenia is willing to use a certain word when it is asked by the researcher and she is able to provide a reason (see “yes, because”).

Broadly speaking, more than other students, Ilenia needs that the process of signification is made explicit as much as possible, otherwise she does not adhere to suggested expressions or she uses certain words occasionally only for harmonizing her answer with the researcher’s verbal style. In retrospect, the study suggests that in a tutoring situation with MLD students, the teacher’s

interventions are crucial and possibly different to those that are effective in a classroom setting. These considerations open a new research line in the grove of the TSM, in order to investigate to what extent the teacher's communicational acts, as they were modeled within this theory, can be translated into and effectively used within tutoring sessions. Moreover, in the case of Ilenia, time plays a crucial role. Indeed, data suggests that Ilenia's development of meaning occurs over very long periods of time. This is consistent both with the general resistance to change of the features of MLD profiles, and with the personal dimension of the semiosis process. The case of the use of "value" allows us to advance the hypothesis that, if a longer intervention took place, Ilenia might have developed mathematical signs, and therefore meanings.

Finally, this study provides an example of a fruitful methodological approach to the analyses of a huge amount of data. The possibility of visually displaying quantitative data allows us to look at general trends (see the Code map) and critical moments (see the Codeline) to be qualitatively analyzed in greater depth. MAXQDA visual tools are used as a kind of sieve for detecting and focusing on both macro and micro emergent aspects. In this sense, these are new artifacts at the researchers' disposal that turn out to be *tools for thinking* even before being *tools for communicating* within our research community.

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