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Becoming diagram: Experimenting with diagrams to explore geometrical and topological concepts in tapestry weaving

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Interpreting and generating diagrams are key skills in both mathematics and crafting. This paper examines the ways in which diagrams are used by a novice weaver in learning to work on a tapestry loom. Highlighting the active and experimental qualities of mathematical diagrams in the writings of Charles Sanders Peirce, the paper explores how sketches, gestures, and experimental weaving 'become diagrammatic', generating ideas that are mathematical and material all at once.

Keywords: Diagrammatic activity, fibre craft, topology, materiality, gesture.

Introduction: Exploring diagrammatic thinking through tapestry weaving

Interpreting and generating diagrams are key skills in both mathematics and crafting. Ancient Greek mathematicians relied on diagrams to prove both geometric and arithmetic propositions (Netz, 1999) and diagrammatic reasoning continues to hold justificatory significance in topological fields like knot theory (De Toffoli & Giardino, 2014). Educational researchers have long stressed the importance of supporting mathematics students to use diagrams in problem solving (Chen & Herbst, 2013; Nunokawa, 2006; Poyla, 1945; Yancey et al., 1989). But mathematics learners are not the only ones engaged in diagrammatic inquiry. Crafters – from knitters to carpenters – also rely on diagrams to develop and design their work, learn new techniques and patterns, and collaborate across fields.

Studies working at the intersections of mathematics and fibre crafts (e.g., tapestry weaving) help us to (re)consider what mathematics is, and can be, in diverse contexts (Thompson, 2022). Harris (1988) argues that the mathematical concepts embedded in textile production can combat the “depressingly narrow” (p. 24) view of ‘practical’ mathematics offered by many mainstream mathematics resources. Saxe and Gearheart (1990) demonstrate, for example, that training in textile crafts supports the development of topological sensibilities that are not commonly part of school curricula. Arnold & Espejo (2013) also stress that weaving practice implicates its practitioners in a kind of three-dimensional reasoning that is difficult to capture in flat images.

This research points to the powerful sensorimotor engagement involved in learning mathematics through weaverly making. But these studies only implicitly address the rich veins of overlap between mathematical practice and craft practice engendered by diagrammatic representation. This paper draws on Charles Sanders Peirce’s (2010) understanding of diagrams as inextricably linked to deductive reasoning and experimentation to explore the case of a novice tapestry weaver developing an image-based project. The paper poses the following research questions: (1) How are diagrams used in the learning processes of a novice weaver? (2) What observations can we make about the learning of mathematics inside of these diagrammatic events?

Theoretical framework: Diagrams as sites of action and experiment

Peirce (2010) pointedly understands the work of the mathematician to be about observing “nothing but the diagrams he himself [sic] constructs” (p. 4). This assertion – that the mathematician’s work is

in some sense always diagrammatic – relies on Peirce’s expansive yet precise understanding of what constitutes a diagram. For Peirce, diagrams are *iconic* – that is, they are a sign which, “by direct observation of it, other truths concerning its object can be discovered than those which suffice to determine the construction” (Peirce, 1934-58, 2.279). Essentially, for a sign to qualify as iconic, it must be ‘similar enough’ to its referent that new and surprising truths about this referent can be discovered simply through observing the icon (in this case, a diagram).

To give a helpful example, think of a triangle. Although one can construct a triangle merely by prescribing the crossing of three lines, careful observation of these three lines reveals a strange and seemingly unrelated truth: The angles contained by these lines always sum to the same measure. This example points to the active and experimental nature “observation” implied in Peirce’s definition. Observation of a triangle might involve drawing new lines, transposing angles, or rotating the diagram to look at it from a new perspective. Peirce complexifies our common notion of a diagram as a visual representation by pointing to the active and kinaesthetic qualities of diagrammatic thinking. Diagrams are designed to be worked on, transformed, manipulated. They are sites that require imaginative engagement. For Peirce, any icon – from a landscape to a linear equation – becomes diagrammatic the minute that we momentarily imagine ourselves on its mountainous slopes or move beyond its symbols to explore the stability of certain invariances held between mathematical expressions. In this sense, Peirce also understood algebraic equations to be diagrammatic. This is because, unlike other types of icons, diagrams resemble their referent through the *relations of parts*.

Peirce’s thinking has pushed researchers to recognise the ways in which diagrammatic reasoning is not only visual, but also bodily and material. De Freitas (2012) describes the ‘storying’ of a diagram as an enlivening event, one which animates the diagram’s ostensibly fixed form. Drawing on tangible experiences and ambivalent perspectives and movements, Châtelet (1993/2000) refuses to separate diagrammatic drawings from bodily gestures. Diagrams may even be invisible, made simply in thin air through gesture (Nemirovsky & Smith, 2013). In all these accounts, the diagram does not position its user or force our interpretations from a fixed perspective. Instead, the diagram calls to us and upon our experiences of the world in indeterminate ways. In this sense, diagrams do not merely represent, they act on us and in us. It is in doing so that they become generative sites for learning.



Figure 1: a & b) two diagrams ‘becoming’ topological, c) a tapestry weaving diagram

In thinking about how diagrams participate in learning events, it is important to recognise that the quality or acuity of diagrammatic thinking depends on the user’s experience with the diagrammatic form at hand. The more one works with a diagrammatic form, the more it becomes alive to direct perception. For example, Figure 1a shows what we might, at first, understand as simply a circle. Although its outline is rather thick, this figure conforms to a familiar geometric form. It is evenly

drawn, uniformly shaped. But the neighbouring diagram (Figure 1b) interrupts this interpretation. This image does not draw on our experiences of reading Euclid's *Elements*. Instead, the same thick line now curls around itself and seems to start and stop in ways that suggest continuities and connections across these breaks. These are continuities that we recognise from experiences tying our shoes or wrapping a present. In looking at this image, somehow the white space surrounding this form is transfigured. No longer a flat surface, the page takes on a dimensionality that seems to hold a fat cord which courses over and under itself. In examining this bare and nondescript cord, we experience the image not just by looking down upon a familiar scenario, but we can also find ourselves *inside* the cord – traversing this line as though moving inside it, first over and then under different segments. It is through this experience of observing Figure 1b that Figure 1a begins to take on new qualities and dimensions.

The topological example above is especially important given the range of literature suggesting that some of the most interesting mathematics related to weaving is topological (O'Brien & de Freitas, 2019; Saxe & Gearhart, 1990; Thompson, 2022). Figure 1c shows how tapestry weaving diagrams also invoke topological connections not unlike the starts and stops of Figure 1b. (This figure will also acquaint readers with the weaving terminology used in this paper, which involves vertical *warp* strings that are “filled” with “picks” of horizontally flowing *weft* yarns.) In the case study below, a novice weaver engages in diagrammatic reasoning that involves both ‘textbook-like’ geometry problems (relating to scale and proportional reasoning), as well as less familiar topological inquiries concerning connectivity, continuity, and dimension.

Methods and data: Weaving a “creeper”

The research presented here was conducted in collaboration with a national out-of-school arts program in the UK, where the author led a “masterclass” in tapestry weaving for approximately twenty young artists between the ages of 13 and 16. Outside of the author and young artists, seven adult participants learned to weave in the workshop, completing their own tapestry projects, while supporting the young artists’ work. Over the course of four weekend sessions (of 2-4 hours), each workshop participant built a tapestry loom and developed their own weaving project in the context of discussions exploring: a) philosophical questions about how mathematics relates to the real world, b) geometric and topological approaches to shape and dimension, and c) the historical significance of weaving in pre- and post-industrial England. Video data was collected with a chest-mounted GoPro camera, worn by a different self-nominated young artist each session. Because participants worked in close proximity, each week a novel set of projects were captured in action.

Analysis of the data takes a microethnographic approach (Streck & Mehus, 2005), exploring how small movements of the hand, moments of eye contact, facial expressions cohere into a learning episode. Microethnography focuses on the careful study of supposedly ‘banal’ details that can give us new means to address ‘big’ social issues (LeBaron, 2012). In this paper, we are interested in how diagrams participate in the learning process, but perhaps the larger social issue at stake concerns how to revise our perceptions of mathematics in ways that admit everyday makers as mathematical inventors. Instead of tackling such a topic through debate or explanation, microethnography concentrates on short, recorded specimens of interaction, where participants’ judgements are not

consulted. It is in looking closely at the patterns and rhythms of unconscious, habitual, and interpersonal knowledge registers that we search for new ways to understand how mathematical inquiry grows from between traces on paper, inside gestures, and between warped strings.

This microethnographic case study traces the emergence of one participant's – anonymised as Winston – weaving project. It focuses on two episodes which took place during our second session. In the previous session, participants were given several diagrams detailing various tapestry weaving techniques (e.g., Figure 5). Free to interact with these icons on their own terms, some young artists took up the diagrams as task sheets, deploying each technique in succession on their looms. Other students, like Winston, drew inspiration from lived experience and called on the diagrams only as need arose. Winston was selected for this case study because, unlike other participants, he drew a sketch in planning his work, which allows us to observe how both self-generated and 'found' signs become diagrams. Note that Isabel, the adult participant who is an important agent in the development of Winston's work, is the arts program leader. She is already familiar with Winston's artistic interests, but she too is a novice weaver, slowly developing her own ideas about how weaving works.

Episode 1: Mapping pixels to picks

Before beginning to weave, Winston draws a gridded sketch (Figure 2a). He marks out the bounding edges of a square and then subdivides the square into an orderly grid of eight-by-eight smaller squares. Winston colours in these mini squares according to the mirror symmetry of human anatomy: Four blank squares cluster to make each of its two opposing eyes. Twelve more mini squares are left white below, taking the shape of a frown or oversized moustache. This gridded image is Winston's rendering of the pixelated face of a "Creeper," an iconic character in the hugely popular gaming world of Minecraft. Figure 2b shows the Creeper as it appears inside of its computer game 'home'.

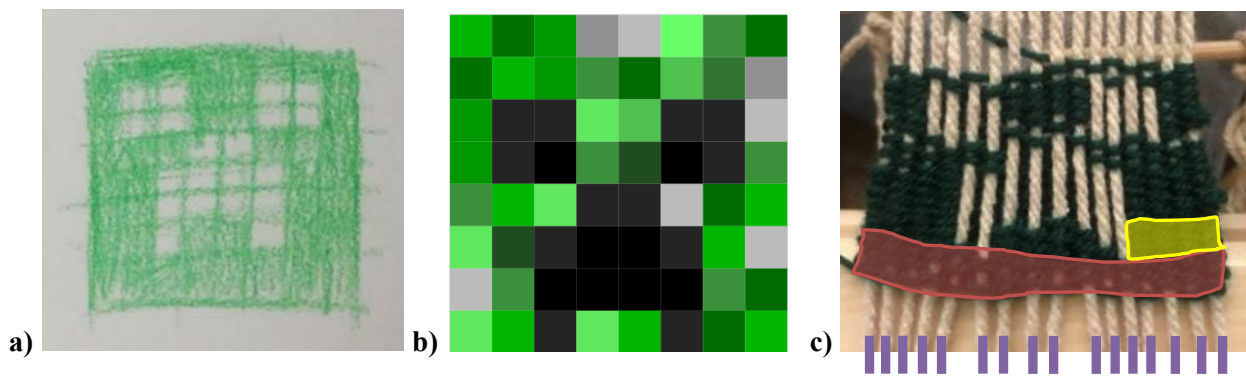


Figure 2: a) Winston's sketch, b) Minecraft's Creeper face, c) Winston's weaving, end of Session 2

After drawing this Creeper sketch, Winston begins to carefully organise his materials for weaving. He selects two colours of yarn – a dark green and dark blue – to use as weft. Next, he fastidiously adjusts his loom's warp strings, creating an evenly spaced and parallel set of lines on which to weave. Winston touch counts and separates out exactly sixteen warp strings on the left side of his loom. These strings will serve as the ground on which he will develop his Creeper weaving (see the purple lines in Figure 2c demarcating each warp string). This move seems to establish a relation that links the eight pixels spanning the width of Winston's Creeper sketch to these sixteen warp strings: Each mini-square/pixel in Winston's Creeper sketch is represented by two warp strings on his loom.

Picking a plain weave pattern (over-under-over-under) across these sixteen well-ordered warp strings, Winston presses a dark green weft yarn into place. After about twenty minutes, a thin green bar adorns his warp (red shaded region in Figure 2c). This bar correlates to the bottom-most row of pixels in his drawing. Next, Winston touch counts one, two, three, four warp strings on the right side of this bar and begins to weave over-under-over-under on just these four strings (yellow shaded region in Figure 2c). When an adult participant (Isabel) approaches, asking Winston about his project, he explains his process by pointing back and forth between sketch and weaving. Instead of verbalising the proportional relationship he has developed using number words, Winston’s explanation is indexical:

So, I’ve done that bit—there [Winston pinches the lower right portion of the sketch between thumb and index finger] ... on here [He now pinches the right-most region of his weaving (yellow in Figure 2c), as his eyes return to his sketch.]. (Session 2 recording, minute 24:22-24:26)

As Winston speaks about his efforts to weave his Creeper sketch, Isabel listens not just with her ears, but also her body. Tensing the fingers of her right hand so that they are straightened and pressed rigidly together, Isabel’s hand begins to make quick sharp cuts in front of her torso (Figure 3) as she says: “...sort of, back and forth” [24:30]. Although neither Winston nor Isabel looks at these hand movements, they reflect Winston’s understanding of how to actively “observe” his sketch as Peircean diagram – by moving “back and forth” like the mechanical eye of a digital scanner or printer head.

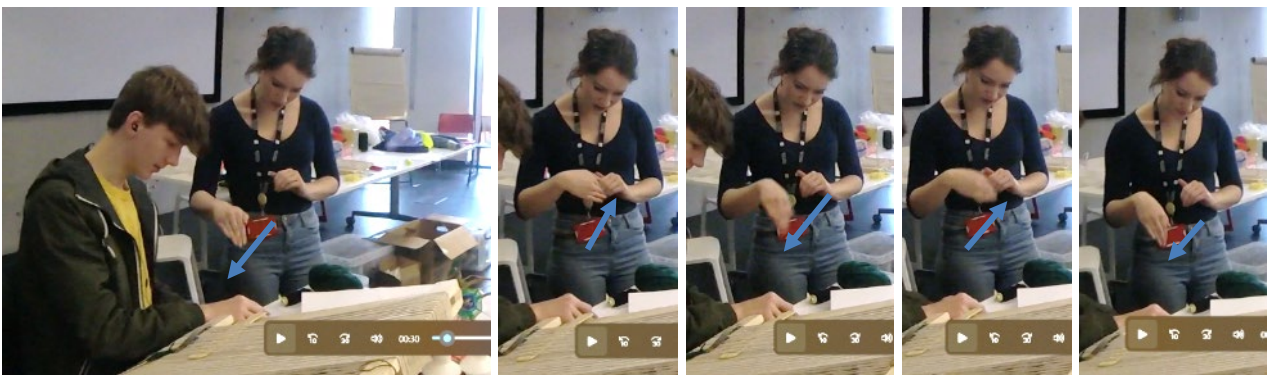


Figure 3: Isabel makes a sharp back and forth movement with her right hand. Both subjects look at the sketch, not the movement of Isabel’s hand [24:30-24:33].

Episode 2: A topological twist

While Winston wove the bottom-most row of pixels in his Creeper sketch with ease, understanding how to approach the next row of pixels – where two colours appear – proves more difficult. During their conversation, Winston tells Isabel that he is uncertain about how to proceed. After contemplating this problem, Isabel replies: “There is...umm...There is an interlocking stitch” [24:44]. As she turns to look for a copy of the worksheet showing this “interlocking stitch”, Isabel’s body begins to create another gestural movement (Figure 4). Her hands lift together in front of her stomach as she walks away from Winston’s workspace. Her forearms slide over one another – the left hand above, the right – and Winston’s eyes track her hands, as she explains that this stitch is “for when you’re having two different colours side by side” [24:48]. Turning her face and then torso back to Winston, Isabel’s hook-shaped hands catch hold of one another, and she pulls them taught.

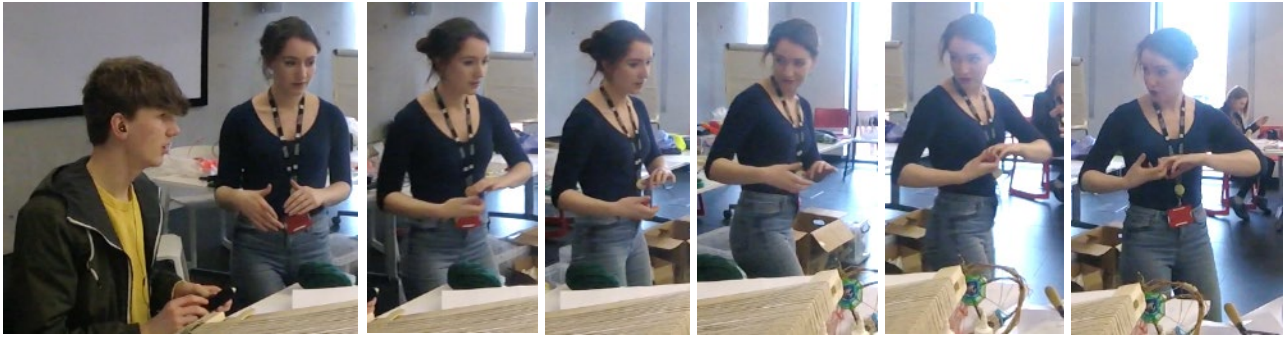


Figure 4: As Isabel begins to walk away from Winston, her hands enact the “interlocking stitch” which was diagrammed on a worksheet that Isabel shows to Winston [24:44-24:49].

After making these movements, Isabel walks to the other side of the room, returning a few minutes later with two pages of diagrams that show “the interlocking stitch” (two examples from the sheet are given in Figure 5a). Although the recording of Winston and Isabel’s conversation about the worksheet diagrams is now covered over by other conversations, we can hear them agreeing that these diagrams are confusing. Nonetheless, their conversation seems to give Winston a fresh start. During the following workshop session, he does not continue to work on his Creeper weaving. Instead, he begins a new weaving on the unused warp strings to the right of his previous efforts (Figure 5b). This weaving experiment closely follows the “diagonal” diagram that Winston discussed with Isabel.

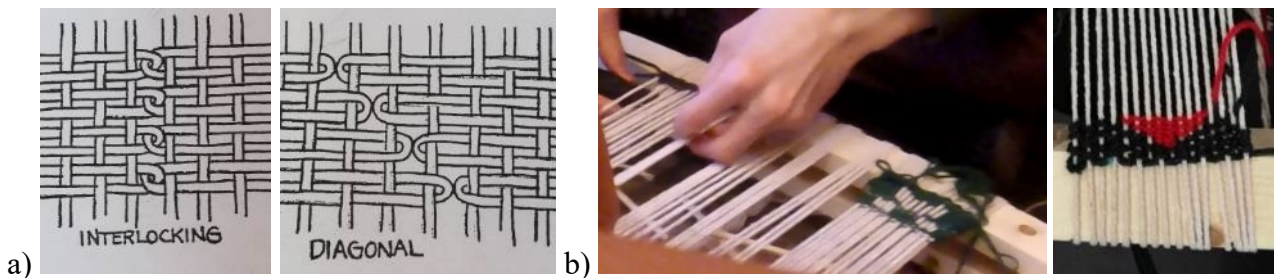


Figure 5: a) Two weaving diagrams from the workshop b) Winston’s work on a second weaving

Discussion

While Winston’s Creeper sketch was in one sense a devotional image from a beloved video game, it harboured more than symbolic weight. Tapping into the ways in which both contemporary computer images and ancient tapestry weaves are composed of discrete picture elements (or pixels), Winston quickly establishes an interpretive rule for drawing novel information from his sketch. This reading allows Winston’s sketch to become a diagram in the Peircean sense: He has found a way to manipulate this icon so that implicit information from the sketch – information not relevant to its construction – is made explicit inside his weaving project. Isabel’s hand gestures in Episode 1 emphasize these differences between in the way in which this sketch was drawn (big square cut into smaller squares and filled in) and the way in which it is activated by Winston’s weaving efforts (according to the horizontal path of the weft, moving back and forth). As the sketch becomes diagrammatic through a material encounter with the loom, Winston identifies these relations by indexically aligning sketch and weaving (“that bit—there, on here”), rather than using precise descriptive language.

Episode 2 begins with a distinctive gesture that traverses Isabel's body during her conversation with Winston. This suggestive movement and its correlated diagram seem to be eventually taken up in Winston's weaving work, but Winston and Isabel agree that these new diagrams require a stretch for their imagination. While Isabel's first gesture embraces Winston's already established interpretation of his sketch, her second gesture is more speculative, aiming to describe how a novel diagrammatic form might expand or expose what is currently "unseeable" in Winston's sketch, i.e., how to have "two different colours side by side" [24:48]. Here, counting and measurement are less important than order, continuity, and connection. Breaking from a geometric reading of Winston's sketch (regarding scale and proportional measurements), this episode opens toward a more topological approach to weaving problems, entailing new questions about connectivity and the border relations of weft yarns.

Conclusions

Commonly in literature about mathematical diagramming, a distinction is made between pictorial representations and diagrammatic ones. But Winston's work points to the ways in which diagrammatic thinking is an active and material mode of engagement with a sign, rather than a static quality inherent to a sign/image. Winston's sketch *becomes* diagrammatic through the modes in which it is taken up: A proportional mapping is brought to life by Winston's desire to render this sketch on the loom. Winston and Isabel's exchange of gestures evince them grappling to make sense of this sketch as a diagram. Later, they work together in a more experimental space to make sense of the unfamiliar topological qualities of the weaving diagrams handed out in the workshop. Inside these entooled social activities, geometric and topological problems surface and take shape through both well-practiced and nascent mathematical sensibilities that remain wedded to the materials at hand.

Unfinished by nature, diagrams always draw upon different contexts and pieces of acquired experience to generate new knowledge. This paper builds on previous research about the role of mathematics in weaving practice to explore how craft making generates rich problem spaces where mathematical ideas come alive. The data points to the way in which diagrammatic representations in the weaver's workshop support engagement with both familiar and unfamiliar mathematical ideas. In this case study, gestures sketch out these diagrammatic relations and eventually prompt further exploration of new material relations. These gestures break open the stillness of Winston's drawing, turning it into an active and experimental object, a true diagram in the Peircian sense. Importantly, then, exploring diagrammatic representations in arts-based settings can contribute to our understanding of how mathematics is learned and practiced: How might more diagrammatic action and reflection be promoted in the weaving classroom? How can this work better support the "multiple embedded patternings" (Thompson, 2022, p. 14) involved in learning mathematics?

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