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The role of automated assessment in representing students' strategies for comparing fractions

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This study describes a formative assessment platform that enables students to generate multiple responses to an example-eliciting task for comparing fractions on Cartesian coordinate system. Seventy-five students, aged 12-13 years, participated in this study. The platform automatically analyzes the characteristics of each submitted student example of ratio to a given fraction and identifies the common characteristics of the generated examples. These common characteristics are matched automatically according to predefined strategies and provided to researchers and teachers as a visual representation that should represent the students' strategic thinking in comparing fractions. The empirical results show that the automated analysis made it possible to identify various strategies employed by students. Most identified strategies were based on using a benchmark of one whole or one half or on using the same denominator; others were based on using familiar algorithms.

Keywords: Automated assessment, example-eliciting task, strategy, fraction, strategic landscape.

Introduction and theoretical background

The assessment process shapes students' engagement in learning activities and the learning strategies they employ. One challenge is to assess students' learning process longitudinally, over an extended period of time, as well as within and across a sequence of the learning process (Dawson et al., 2023). Studies show that technological assessment platforms bring new opportunities to generate reports based on automated analysis that enables tracking students' learning progress, especially their strategies, concepts, and structures (Stacey & Wiliam, 2012). Olsher and Lavie (2021) argued that researchers and teachers can learn about students' conceptual perceptions based on automated analysis of their personal example space. Personal example space is defined as the collection of examples to which the student has access at any moment (Liz et al., 2006, p. 133). Assessment platforms based on students' personal example space can reflect collective or personal mathematical conceptualizations by visual representation of a personal conceptual landscape, which is "a landscape in the sense that it gives a visual planar density complex representing student perceptions" (Leung & Lee, 2013, p. 366) that may appear as a line or region (Olsher & Lavie, 2021) and may enable us to learn about the students' way of thinking or their mathematical strategies. The present study focused on the mathematical strategies of the students that may be identified by automated analysis of comparing fractions. Mathematical strategies refer to students' methods to solve problems with mathematical content, whether their answers are correct or not (Hegedus & Otálora, 2022). Ellis et al. (2019) argued that to learn about students' strategies based on their examples, we should examine the mathematical features and properties of sets of examples. To this end, we designed an example-eliciting task (EET) in the context of comparing fractions that includes an interactive diagram and asks students to create examples by performing dragging under given constraints (Yerushalmy, & Olsher, 2020). Fraction is represented by a dot that represents an ordered pair (denominator, numerator) in the Cartesian coordinate system. Arnon et al. (2001) used this system as part of their curriculum unit as an alternative representation to more conventional representations of fractions. Their study demonstrates how students learned the concept of fraction and how such representation,

when designed as part of interactive environment, can help educators to follow student's conceptualization of fraction. We asked 75 12- and 13-year-old students to construct and submit multiple fractions that are larger than a given one. The opportunity to construct and submit multiple examples is based on different dimensions and possible variations of these examples, and may stimulate students' different ways of thinking (Leung & Lee, 2013; Olsher & Lavie, 2021).

Researchers have reported that students used different strategies to successfully compare fractions. These strategies were included in the mathematical curriculum and taught in the classroom. They can be categorized into two groups: feature-based and rule-based strategies. Feature-based strategies are reasoning strategies that take the characteristics of the two given fractions into account. The common strategies in this group are using the same numerator, the same denominator, the benchmark of one whole or one half, and the distance of each fraction from one whole. Rule-based strategies use familiar rules or algorithms, such as finding equivalent fractions by expansion or reduction algorithms to get a common denominator or numerator (Reinhold et al., 2018). Other strategies for comparing fractions were mentioned in the mathematical education literature: strategies that students develop on their own and that result in common mistakes (Schneider & Siegler, 2010), for example, considering a fraction as a larger one when its numerator, its denominator, or both are larger than the other fraction. In our study, the feature-based and rule-based strategies were predefined as attributes to identify the students' strategies.

We used STEP as a formative assessment platform that supports the design of EETs with an interactive diagram (Olsher & Lavie, 2021), in this case, for comparing fractions. Based on previous research (Leung & Lee, 2013; Olsher & Lavie, 2021), this study describes and discusses the opportunity that the platform offers for teachers and researchers to implement the formative assessment process based on automated analysis and visual representation. The term "characteristic" refers to each of the predefined mathematical properties of the fractions that the student chose to satisfy the requirements of the tasks (e.g., the fraction is less than one), and the relation between the properties of this fraction and the properties of the given fraction (e.g., the same denominator in both fractions). As a next step, after the automated analysis of the characteristics of each submitted example, STEP automatically identifies the common characteristics present in the students' submitted examples. Visually, STEP enables capturing student responses to EET and presenting the generated personal example space (the submitted examples) of each student on one screen. Next, by automated analysis, STEP identifies the common characteristics of student examples according to predefined strategies of comparing fractions, consistent with the mathematical curriculum and the studies mentioned in the literature. In the last stage, STEP presents the student's generated example space together with a visual description that represents student strategies which we call "personal strategic landscape". We aimed to determine whether the automated analysis of the common characteristics present in the students' personal example space for a single task, and automated matching with predefined strategies, can help us identify different strategies for comparing fractions. The novelty of this study lies in integrating the use of a digital formative assessment platform with task design to examine the ability to assess the student strategies. Our research question was: What strategies of comparing fractions can be identified automatically (by visual representation) based on the students' submitted example space in response to an EET?

Methods

Research setting. To examine student strategies for comparing fractions, we designed an unconventional representation task based on submitting examples. Seventy-five students, aged 12-13 years, participated in the study. Students worked on their computers on a task that involved dynamic dragging of a point on the screen. Students were asked to drag the point to a position that would satisfy a certain condition. Examples were submitted by students and stored on the platform. STEP enables automated analysis of the characteristics of each example, identifies common characteristics present in the submitted examples based on predefined strategies, and provides visual representations that represent the quantitative data of the example space. The students learned to compare fractions in a regular classroom, as provided by the national curriculum. The students were given the task and 45 minutes to complete it on STEP.

The interactive diagram (Figure 1) used in this study represents fractions in the Cartesian coordinate system as a point whose vertical coordinate is the numerator and the horizontal coordinate is the denominator (Arnon et al., 2001).

The origin and points on the vertical axis do not represent any fraction. All equivalent fractions are represented on a straight line passing through the origin (e.g., $\frac{2}{5} = \frac{4}{10} = \frac{6}{15}$). Points that exist on the line with a larger slope represent larger fractions (e.g., $\frac{6}{6} > \frac{2}{5}$). The red point (which corresponds to an X in Figure 1) represents the given fraction (a fixed point), and the green point (which corresponds to an empty circle in Figure 1) is the fraction that the student can drag freely to satisfy the requirement of the task.

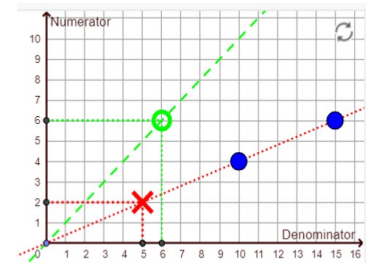


Figure 1: The interactive diagram

The students submitted their final examples after finishing the task.

The task. Students were asked to construct a fraction larger than the given fraction $\frac{2}{5}$ represented by the red point by dragging the green point. The task required students to construct multiple examples that fit the requirement of the task (10 such fractions). The task was formulated as follows: “Choose 10 fractions that are larger than the fraction $\frac{2}{5}$.” Figure 2 presents eight submitted examples by student X. The student was asked to construct his examples one by one.

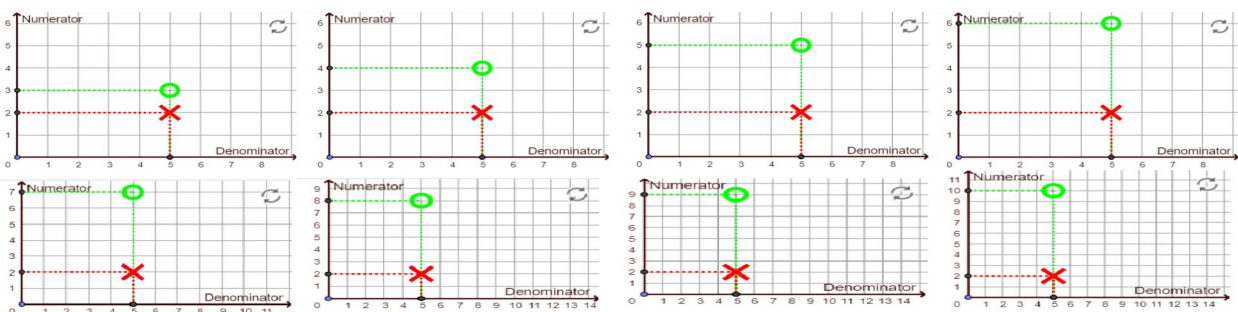


Figure 2: Eight examples submitted by student X

The personal strategic landscape presented by the assessment platform. In this study, STEP visually represented the self-generated example space of each student on one screen. In Figure 3, each blue point (which corresponds to a full circle in Figure 1 and Figure 3) represents a submitted example

of student X. STEP analyzed the mathematical characteristics of each example. Based on this analysis, it located the common characteristics present in the student's example space. The visual representations of the green line, in Figure 4, depict the personal strategic landscape and are based on a combination (automated matching) of automated analysis of the common characteristics or relations of the student's example space with predefined strategies consistent with the mathematical curriculum and literature. We examined whether each representation of the student's examples helped identify the exemplification strategy chosen.

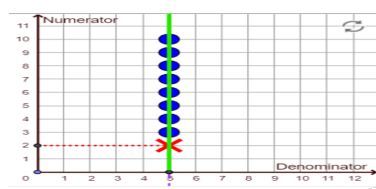
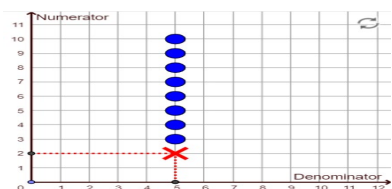


Figure 3: The student's submitted examples **Figure 4: The strategic landscape that STEP provides**

Data analysis and resources. The data used to answer the research question were based on the automated information analysis by STEP. The combined quantitative data generated by the platform included (a) automatic classification of mathematical characteristics of each submitted example; (b) identification of the common characteristics of the student's submitted examples; (c) automatic matching of the common characteristics with predefined strategies; and (d) identification of the student's strategic thinking involved in comparing fractions in the generated personal example space based on the hypothesized predefined strategy and a visual representation of a personal strategic landscape. The common characteristics were matched automatically with the predefined strategy if at least seven submitted examples had the same common characteristic (feature or rule) reflecting this strategy. Some strategies were subsets of other superset strategies; thus, when STEP identified two strategies where one was a subset of the other, it identified the subset strategy as the strategy that the students used.

Results

We designed this task to examine the students' strategies for comparing fractions visually based on automated data about the characteristics of the students' examples provided by STEP. Table 1 lists the identified strategies among those hypothesized, the percentage of the students who chose each strategy, and the automated personal strategic landscape that depicts a visual representation of the predefined strategy according to the automated analysis of the common characteristics of the student's personal example space.

Table 1: The hypothesized strategies, the percentage of the students (n=75) using each strategy, and the personal strategic landscape

Hypothesized strategy for comparing fractions	The personal strategic landscape
<p>S-correctness) This strategy is true when the student's example fulfills the requirements of the tasks, and it is a superset strategy of all the other strategies, therefore we did not take it into consideration as a student strategy. In the visual representation, each blue fraction is above the visual red representation</p>	

<p>line that represents all the fractions that are equivalent to the red fraction, which means that each blue fraction is larger than the red one.</p>	
<p>S1) 10 students (13.3%) chose their examples to compare fractions using a benchmark of one whole. It takes the shape of a line that separates between two regions. The green/red region represents all fractions that are larger/smaller than one, respectively. The line between them represents fractions equivalent to one whole. The red fraction is less than one, and the submitted examples are equal to or larger than one. Thus, each blue fraction is larger than the red one.</p>	
<p>S2) 27 students (36%) chose their examples to compare fractions by using a benchmark of one-half. It takes the shape of a line that separates between two regions. The green/red region represents all fractions that are larger/smaller than one-half, respectively. The line between them represents fractions equivalent to one-half. The red fraction is less than one-half, and the submitted examples are equal to or larger than one-half. Thus, each blue fraction is larger than the red one. This strategy is a subset of S1.</p>	
<p>S3) 19 students (25.3%) chose their examples to compare fractions by using the same denominator. It takes the shape of a line that is parallel to the vertical axes in $X=$ (the denominator of the red fraction). All submitted examples (blue) have the same denominator as the given red fraction. This strategy is a subset of S6.</p>	
<p>S4) 1 student (1.3%) chose his examples to compare fractions by using the same numerator. It takes the shape of a line parallel to the horizontal axes in $Y=$ (the numerator of the red fraction). All submitted examples (blue) have the same numerator as the given red fraction.</p>	
<p>S5) 3 students (4%) chose their examples to compare fractions based on: "The numerator and the denominator of the larger fraction are larger than the numerator and the denominator of the smaller fraction, respectively." It takes the shape of a region that represents all fractions that have a larger numerator and denominator than the given red fraction. This is a misconception, and each blue fraction can be larger or smaller than the red one.</p>	
<p>S6) 5 students (6.6%) chose their examples to compare fractions based on: "The denominator of the larger fraction is a multiple of an integer of the denominator of the smaller fraction." It takes the shape of lines that represent all fractions that are parallel to the vertical axes, $X= a \times$ (the denominator of the red fraction), where a is an integer that is not equal to zero. Thus, each blue fraction can be larger than the red one. This strategy is a subset of S5.</p>	

S7) 2 students (2.6%) chose their examples to compare fractions based on: "One unit fraction from a whole. The fraction that is closer to 1 whole is the larger fraction." It takes the shape of a line that represents all fractions that are parallel to the line that represents all the fractions that are equal to one (the green line). Thus, each blue fraction can be larger than the red one.

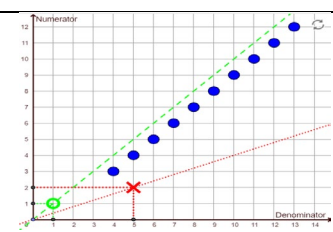


Table 2 illustrates the example space of 6 students (8%) who chose their examples using a combination of two or more of the abovementioned identified strategies.

Table 2: Students who chose their examples using a combination of two or more of the identified strategies

Visual representation (strategies)		
<p>S1+ S3</p>	<p>S1+S3+S6</p>	<p>S1+S3</p>
<p>S1+S2+S3</p>	<p>S1+S3</p>	<p>S1+S2+S3</p>

Table 3 illustrates the example space of 2 students (2.6%) about whose strategies for choosing examples we had to conjecture.

Table 3: Students about whose strategies we could only conjecture

<p>One student chose his examples by increasing the denominator of the given fraction to obtain a larger fraction. For this student, it did not matter what was the value of the numerator of the new fraction.</p>	
<p>One student chose his examples by decreasing the denominator of the given fraction, to obtain a larger fraction. For this student, it did not matter what was the value of the numerator of the new fraction.</p>	

In sum, the assessment platform and task design made it possible to identify different strategies for comparing fractions. The strategies we identified were consistent with the mathematical curriculum. Some student strategies were identified as common mistakes mentioned in the literature. In addition, we found that six students used a combination of two or more strategies, and we conjectured about the strategies used by two students. Table 4 shows the distribution of the identified strategies described in Tables 1-3, according to students.

Table 4: The identified hypothesized strategies

Identified strategies for comparing fractions	Number of students (%)
S1: Using a benchmark of one whole	10 (13.3)
S2: Using a benchmark of one half	27 (36)
S3: Using the same denominator	19 (25.3)
S4: Using the same numerator	1 (1.3)
S5: Using: “The numerator and the denominator of the larger fraction are larger than the numerator and the denominator of the smaller fraction, respectively”	3 (4)
S6: Using: “The denominator of the larger fraction is a multiple of an integer of the denominator of the smaller fraction”	5 (6.6)
S7: Using: “The fraction that is closer to one whole is the larger fraction”	2 (2.6)
Using a combination of two or more of the identified strategies (S1-S7)	6 (8)
We can only conjecture about the student’s strategy	2 (2.6)
Total	75 (100)

Discussion

We integrated the assessment platform STEP with the task design to learn visually about student strategies as they were working on an EET of comparing fractions. The empirical results show that the automated analysis helped us hypothesize about the strategies of the students. The combination of the task design and the assessment platform made it possible to represent visually the students’ strategies of choosing examples to illustrate this mathematical concept based on their personal example spaces. Some strategies for comparing fractions were consistent with the mathematical curriculum, others were mentioned in the literature as misconceptions or common mistakes. Most identified strategies were feature-based, that is, they took into account the characteristics of the fractions (for example, using a benchmark of one whole or one half, and using the same denominator), whereas others were rule-based, making use of familiar rules or algorithms, such as expansion.

The findings are consistent with the literature, which has reported that technological assessment platforms make it possible to learn about students' perceptions, strategies, and understanding (Stacey & Wiliam, 2012). This is especially true when the platform is combined with EETs designed for this purpose (Olsher & Lavie, 2021). To determine strategies the students use, we need to identify the characteristics that appear repeatedly in their submissions in sets of different and varied tasks. The limitation of this study was that we identified strategies based on a single task, therefore the study should be repeated with a sequence of tasks.

The automated analysis in this study provided information about the characteristics of submitted examples, which helped us hypothesize about the learners’ strategies. We argue that such automated information can be used more efficiently if the technological platform enables automated analysis of a sequence of tasks and activities, in order to support the planning of differential teaching or adapted learning. Future research should investigate how such automated data and visual representation can

be useful for assessment purposes and how it can be used as automated visual feedback to be provided to students to enhance their reasoning and exploration.

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