



HAL
open science

Unknown and variable: The semiotic potential of a digital balance-model

Chiara Bonadiman

► **To cite this version:**

Chiara Bonadiman. Unknown and variable: The semiotic potential of a digital balance-model. Thirteenth Congress of the European Society for Research in Mathematics Education (CERME13), Alfréd Rényi Institute of Mathematics; Eötvös Loránd University of Budapest, Jul 2023, Budapest, Hungary. hal-04407923

HAL Id: hal-04407923

<https://hal.science/hal-04407923v1>

Submitted on 21 Jan 2024

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

Unknown and variable: The semiotic potential of a digital balance-model

Chiara Bonadiman

Department of Mathematics and Computer Science, University of Florence, Italy;

chiara.bonadiman@unifi.it

The aim of this paper is to analyse a specific digital environment designed to introduce equations. More specifically, through the instruments provided by the Theory of Semiotic Mediation, I study the semiotic potential of an artifact constituted of two balances, one fixed and the other movable, to mediate the mathematical meanings of unknown and variable. Having conducted pre-experimentation, I present some excerpts to highlight artifact-signs emerging during the activities that, with appropriate intervention of an expert, could evolve into mathematical signs. An excerpt from an artifact-integrated activity with a low-achieving student is analysed in more detail to show the complexity of the relation between artifact signs and mathematical signs.

Keywords: Balance-model, unknown, variable, semiotic potential.

Introduction

Algebra has been widely recognized as a challenging topic in teaching and learning mathematics (Kieran, 1992). Specifically, Arcavi (1994) highlights the lack of symbol sense in students, even after several years of algebra instruction. Moreover, he states

Instruction does not always provide opportunities not only *not* to memorize, but also to “forget” rules and details and to be able to see through them in order to think, abstract, generalize, and plan solution strategies. (Arcavi, 1994, p. 24, italics in original)

Some scholars have studied artifacts’ use in teaching and learning algebra, showing how students can “make sense” of algebraic abstract concepts that usually create difficulties (Mariotti & Cerulli, 2001; Kieran & Drijvers, 2006). For example, as reported in Otten et al. (2019), a common metaphor to introduce linear equations is the balance model, which favours a meaningful construction of the equal sign and allows dealing with unknowns. Here I will discuss an artifact constituted of virtual balances.

In this context, the aim of this paper is to study and analyse a specific digital environment in the introduction of some mathematical concepts that cover an essential role in the domain of algebra. Specifically, I want to analyse an artifact constituted by a two-pan balance with known and unknown weights to introduce the idea of unknown and variable.

Theoretical framework

With the aim of studying the potentialities of a digital environment, I frame my analysis within the Theory of Semiotic Mediation (Bartolini Bussi & Mariotti, 2008), hereon TSM. The TSM finds its roots in the socio-cultural frame for learning developed by Vygotsky (1978). The Vygotskian approach considers signs as mediating tools with a twofold cognitive function: psychological tools for the learning subject and communicative tools for a common shared construction of knowledge.

As written in Bartolini Bussi & Mariotti (2008) “the idea of artifact is very general and encompasses several kinds of objects produced by human beings through the ages”. Some examples of artifacts proposed are sounds and gestures, utensils, books, oral and written forms of natural language, and scientific instruments. With the idea of artifact and utilization schemes, TSM combines the perspective about signs produced in the accomplishment of a given task to the content that has to be mediated. In this way, not only the relation between artifact, task and the contingent signs is highlighted, but also the link between these signs and the mathematical meanings to be mediated. This double semiotic link, defined as *polysemy* of an artifact, finds a correspondence in two parallel systems of signs. The relation between artifact and task becomes visible with the production of signs, called *artifact signs*. On the other hand, the link between artifact and mathematical culture is expressed through signs that are culturally determined, identified as *mathematical signs*. In this light, within the use of the artifact, personal meanings with reference to the aim of the task emerge; at the same time, mathematical meanings are related to the artifact’s use. This double semiotic relationship is called *semiotic potential of an artifact*. As emphasized by Bartolini Bussi and Mariotti (2008), the semiotic potential of an artifact is not naturally activated and exploited: the expert must take advantage of it to foster the transition from personal to mathematical meanings. TSM illustrates the main characteristics of a teaching sequence for a classroom to develop this evolution in the context of the didactical cycle.

Goal of this study

This study is part of a greater research project called *DynaMat, Dynamic Math for inclusive Education*. One of the main project’s purposes is investigating the impact of dynamic and interactive digital environments for students having a history of low achievement or difficulties in Mathematics. This identification was made by the students’ teacher, who has known them for at least a full school year; and agreed by the students themselves, who report struggling in mathematics. In this paper, I present the explorative study about artifacts representing virtual balances. The study consists of a first part within which I designed the artifact and conducted the analysis of its semiotic potential, while the second part is dedicated to some data collected during activities with students. This investigation, conducted within a pre-experimentation with single or pair of students, had the specific aim to study the production of artifact signs, during the accomplishment of tasks with the artifact, that, as experts, we can relate to mathematical meanings. Here, I will not deal with the possible evolution from these artifact signs to mathematical signs; that will be examined in depth in future investigations.

The balance-model: Our starting point

The starting point of our design is a digital artifact in the free online software Desmos. This artifact is constituted by a dynamic balance initially blocked with a lock, with pans composed of objects of different shapes and colours, some representing known quantities and others unknown. Within the software, the user has the possibility to assign a value to a weight by inserting the number in an “input field” and pressing the key “Let’s try!” to activate the balance. The balance will move depending on whether the sum of the weights on the left side is less than, greater than or equal to the one on the right side, depending on the user’s choice. In Figure 1 are reported two possible feedbacks for the relation between the expressions $2x+3$ and 11 . For example, assigning to the triangle’s weight a value

of 4, the scale balances (Figure 1a): it represents the equalities between $2 \cdot 4 + 3$ and 11 . On assigning a value of 6, the balance dangles to the left (Figure 1b): it represents the relation $2 \cdot 6 + 3 > 11$. The artifact also gives another feedback. Once the user presses the key and the balance moves, the lock is substituted by a green tick when the scale is in balance; when the two pans are not equal, a red X appears. From the mathematical point of view, this feedback corresponds to the function that associates to the value chosen the output 'true' (green tick) or 'false' (red X), referring to the truth of the equality between the two expressions represented by balance's pans.

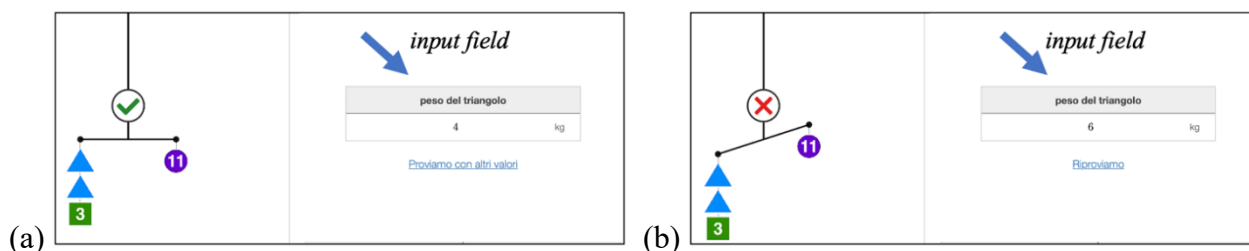


Figure 1: The feedback of the artifact when the two expressions are equal (a) and are not equal (b)

In this paper I consider 'unknown' as the specific but not known value that makes equal the two algebraic expressions involved, and 'variable' as an indeterminate quantity that can vary and to which we can assign a value. If we look at the equation as a relation between quantities, the unknown is treated as a specific number even if it is hidden. For instance, the equation $2x+3=11$ is interpreted as equality between two terms composed of numbers: the term x is also a number, even if unknown. In the relation $2x+3=11$ an expert can see x both as unknown and as variable. I think that this double meaning is essential in the process of thinking related to equation. Considering these ideas of unknown and variable, in using this balance x is a variable and the meaning of unknown as a specific number appears only when the user inserts that value in the input field and the scale balances.

For this reason, we consider a second artifact made of a fixed balance that cannot move. This scale is in balance and presents the green tick: it represents two pans having the same weight even if the triangle's weight is unknown. The artifact, for an expert, can represent the equality between two algebraic expressions depending on the same unknown. For example, in Figure 2 we can see a balanced scale which pans represent $2x+3$ and 11 . We know that these two algebraic expressions are equal, but we do not know the unknown's value that makes true this relation. In Table 1, I summarize the main features of this artifact with the corresponding mathematical meanings.

Table 1: Features and correspondent mathematical meanings of the fixed balance

Features of the artifact	Mathematical meaning
Weights with the same colour and shape with number	Constants
Weights with the same colour and shape without number	Equal unknowns
Collection of weights on a balance's pan	Algebraic expression on the left or right of the equal sign
Green tick	Equality between two algebraic expressions

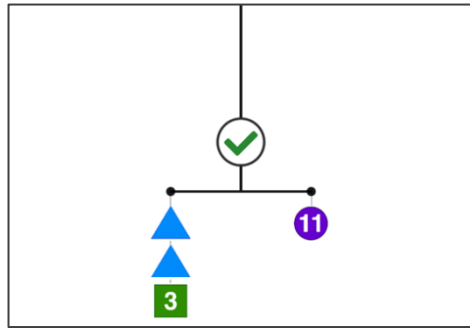


Figure 2: The fixed balance

The design of a new artifact

The artifact used in the didactical sequence is reported in Figure 3. It involves both the artifacts previously described. The need to combine them arose because our didactical aim was to mediate the mathematical meanings of both ‘unknown’ and ‘variable’. The fixed balance can mediate the meaning of ‘equation’ and related ‘unknown’. The necessity to try values to find the solution is satisfied by the mobile balance that becomes a test-balance and then mediates the meaning of ‘variable’.

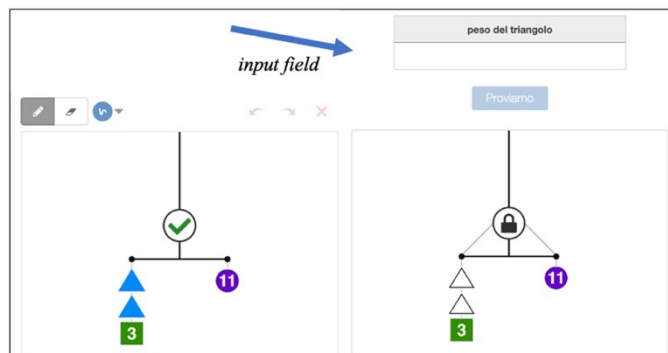


Figure 3: The complete artifact. On the left the fixed balance and on the right the test-balance

The semiotic potential of the two distinct artifacts has been previously described, while the features of the new artifact (Figure 3) and related mathematical meanings related are summed up in Table 2 (below). To highlight the differences between unknown and variable, we decided to modify the test-balance. In our artifact, weights without number are white, while, in the fixed balance, they are coloured. We wanted to mediate the idea that in the fixed balance they have a specific value while in the test balance they can assume any positive value chosen by the user through the input field. In other words, the algebraic sign x is represented by two different signs, a coloured and a white weight, that correspond to two different meanings, unknown and variable. Moreover, on the fixed balance students can draw using a digital pen. This affordance could foster the production of written signs.

Table 2: Features and correspondent mathematical meanings of the complete artifact

Features of the artifact	Mathematical meaning
Fixed balance	Relation of equality between two quantities $f(x)$ and $g(x)$ where x is unknown (specific but not known number)

Test balance	Relation (greater, less, equal) between two functions $f(x)$ and $g(x)$ with respect to the independent variable x
Fixed balance – Test balance	Different nature of x and of $f(x)$ and $g(x)$
Coloured and not numbered weight in fixed balance	Unknown
White (not numbered) weight in test-balance	Variable

A pre-experimentation

The pre-experimentation was conducted during October and November 2022 and took place in an out-of-school centre with 12 voluntary students with a history of low achievement. They all came from 10th grade classes of Italian high schools (age 14 or 15). All students have taken a preliminary interview with a researcher before the teaching experiment. Individual or pairs of students attended the activities under the guidance of a researcher during five sessions of two hours each. They had at their disposal two touch-screen tablets, one to interact with artifacts and one for writing. Artifacts used were not only the one described previously, but also others, such as dynagraph, of which description and analysis are beyond the scope of this paper. Data consisted of video-recordings of the meetings, tablets' screen-recordings, and students' written productions.

The task proposed to students, considering for example the artifact in Figure 2, was: *On the left you have a scale in balance. Can you discover the weight of the triangle? You can use the scale on the right and the button "Let's try!" to verify what happens when you vary the weight of each triangle.* It was focused on the search for the value that balances the scale, but, during activities, researcher pushed students to explore the balance's behaviour by varying the value of the unknown weight.

In light of the first pre-experimentation, I report in Table 3 some words and sentences pronounced by students during the activities with the artifact that we can interpret, according to TSM, as artifact signs. The table also presents the relations of these artifact signs with mathematical meanings we wanted to introduce. A, D, H, and L refer to four anonymised students.

Table 3: Artifact signs and related mathematical meanings emerged during the pre-experimentation

Artifact signs	Mathematical meanings
D: "A triangle is equivalent to 9." H: "A weight is worth 4."	$X=9, x=4$ (the value of the unknown)
L: "Weight that is right , that make the scale in balance."	The solution of an equation
D: " You do not know ." D: "You discover what are these two triangles"	Unknown

<p>L: “It [the white triangle] hasn’t got a weight yet.”</p> <p>D: “There is a square, that can be a not defined number.”</p> <p>H: “Neutral weights.”</p> <p>A: “I made also the spaces.”</p> <p>H: “I put a number smaller than 5.”</p>	Variable
<p>L: (describing the test-balance) “There is the same thing that there is on the other side just the triangle is not blue, so, in theory it hasn’t got a weight yet.”</p>	Relation between unknown and variable

In Table 3 we can see the emergence of artifact signs that, according to TSM, could evolve, thanks to the intervention of an expert, into mathematical signs. For example, with *“the weight that is right”* the student refers to the value that makes the scale balance, while the expert can see behind it the general idea (not referring to a specific equality) of the solution of an equation. Moreover, D. refers to coloured, not numbered, weights of the fixed balance as something we do not know and that we have to discover. On the other hand, students talk about the white shapes of the test balance as “neutral weights”, “spaces”, something that “hasn’t got a weight yet” or that “is a not defined number”. The verb “to put” suggests the action of inserting a number as a weight and a choice that can be made, in the same way we can choose the value of a variable. The last line of Table 3 suggests that L. Sees the “same thing” behind the two balances in the artifact, but she highlights the difference in the colours. The different signs spontaneously produced by the students (as the two verbs “to discover” and “to put”) could become a starting point, in a didactical cycle, towards the meanings of unknown and variable and their differences.

Now, I present an excerpt from the activities in which we will see the emergence of artifact signs and a problematic use of algebraic symbolism.

The case of Dario

During the first session conducted by the author of this paper, Dario, a low achiever in mathematics, deals with the first task (Figure 3). He reads the task and immediately states:

- 1 Dario: So, I already have an idea...
- 2 Researcher: Ok. What is your idea?
- 3 Dario: Well, I can tell that I try to calculate $11-3$, the two numbers we can see. And it turns 8. And they are two triangles... I divided, 4 and 4. 4 and 4, 8. Plus 3, 11 and 11 on the other side, so it is balanced.

Dario gives sense to the problem proposed and reasonably finds a solution. So, the researcher asks why he decided to calculate $11-3$. Dario tries to explain his strategy.

- 4 Dario: Indeed, because considering that they are the only two numbers, it is like doing, I do not know, just to say, x plus x plus 3 equal 11.
- 5 Researcher: Do you want to write something?

[...]

- 6 Dario: Well, I was thinking, I do not know... The triangles, so, I do not know... square x plus 3 is equal [he writes, Figure 4], that is, x plus x , I mean... oh god, wait because I do not remember if square x is x times x or x plus x ...

A rectangular box containing the handwritten mathematical expression $X^2 + 3 =$. The 'X' is written in a cursive style, and the '2' is a superscript. The plus sign and the number '3' are also handwritten.

Figure 4: The first written explanation of Dario involving algebraic symbolism

Dario produces the equation that models the balance, putting into words the mathematical signs that describe the fixed balance (line 4). When the researcher asks if he want to write something, he tries to algebraically manipulate the equations verbally introduced. He uses the sign x^2 (Figure 4) and, hesitating, he tries to remember if x^2 corresponds to x times x or x plus x (line 6).

- 7 Researcher: OK, so, first, we could try to understand what you want to write here.
8 Dario: That is... It is easier to do so.. let's make triangle plus triangle plus 3.
9 Researcher: Ok and here you are writing what, with respect to your balance?
10 Dario: That triangle plus triangle plus 3 is in balance with 11, so it is equal to 11 [he writes, Figure 5a].

Two rectangular boxes labeled 'a' and 'b'. Box 'a' contains the handwritten equation $\Delta + \Delta + 3 = 11$. Box 'b' contains the handwritten equation $11 - 3 = \Delta \Delta$.

Figure 5: Dario's return to artifact-signs (a) and the consequent explanation of his resolution (b)

Dario turns back to artifact signs and writes the equalities as in Figure 5, with which he carries on his explanation showing self-confidence (*"It is easier to do so"*, line 8).

- 11 Dario: So, I think to calculate 11, that are the two numbers we can see, minus 3, equal.. and you discover what are these two triangles [he writes, Figure 5b].
[...]
12 Dario: 8. Considering that they are two, I have divided. To make them add up. For me it is 4.

Dario, once he has avoided uncertainty related to algebraic formalism, goes back to artifact signs and implements practices, identifying the unknown value, that for him make sense and that as experts we relate to the process of resolution of an equation. While an expert can recognize the expression $x+x+3 = 11$ and the signs in Figure 5a (triangle + triangle + 3 = 11) as referring to the same mathematical meanings, for Dario, there is a rift between the artifact signs and the mathematical signs, and, in particular, between the practices related to them. He seems convinced that he can operate on the algebraic symbolism only by remembering how to do it: without the rule, he cannot proceed. The use of algebraic symbolism does not make sense for him; he got stuck in the attempt to remember. On the other hand, with artifact signs he does not have this perception: he proceeds with confidence, generating the steps of the procedure by himself rather than trying to recall rules to his mind.

Conclusion

In this paper I described the process of design and analysis of an artifact grounded on the balance model to mediate the meanings of unknown and variable, stressing the differences in the two balances

to highlight differences between the two mathematical meanings. The data reported in Table 3, in the frame of the Theory of Semiotic Mediation, show some potentialities of the designed artifact in fostering the production of artifact signs that can be significant starting points for possible classroom's discussions, with the goal to build significant mathematical meanings. On the other hand, the case of Dario is enlightening. He is able to explain his thinking in a meaningful way using artifact signs, while with mathematical signs he believes (in terms of Di Martino & Zan, 2011) that he needs to turn to memory. Dario's case shows us a possible fracture, especially for low achievers, between the signs of school, and signs that emerged during the teaching sequence with the artifact. In this light, more studies are needed to understand how to fill this gap within the Theory of Semiotic Mediation.

Acknowledgment

Study part of the funded project DynaMat (PRIN2020BKWEXR); experimental work conducted at CARME (www.carme.center), UNISER Pistoia Srl, Italy.

References

- Arcavi, A. (1994). Symbol sense: Informal sense-making in formal mathematics. *For the Learning of Mathematics*, 14(3), 24–35.
- Bartolini Bussi, M.G. & Mariotti, M.A. (2008). Semiotic mediation in the mathematics classroom: Artifacts and signs after a Vygotskian perspective. In Lyn D. English & D. Kirshner (Eds.), *Handbook of International Research in Mathematics Education* (pp. 746–783). Routledge. <https://doi.org/10.4324/9780203930236>
- Di Martino, P. & Zan, R. (2011). Attitude towards mathematics: A bridge between beliefs and emotions. *ZDM Mathematics Education*, 43(4), 471–482. <https://doi.org/10.1007/s11858-011-0309-6>
- Kieran, C. (1992). The learning and teaching of school algebra. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning: A project of the National Council of Teachers of Mathematics* (pp. 390–419). Macmillan Publishing Co, Inc.
- Kieran, C. & Drijvers, P. (2006). The co-emergence of machine techniques, paper-and-pencil techniques, and theoretical reflection: A study of Cas use in secondary school algebra. *International Journal of Computers for Mathematical Learning*, 11(2), 205–263. <https://doi.org/10.1007/s10758-006-0006-7>
- Mariotti, M.A., Cerulli, M. (2001). Semiotic mediation for algebra teaching and learning. In M. Van den Heuvel-Panhuizen (Ed.), *Proceedings of the 25th conference of the International Group for the Psychology of Mathematics Education* (Vol. 3, pp. 225–232). PME.
- Otten, M., Van den Heuvel-Panhuizen, M., & Veldhuis, M. (2019). The balance model for teaching linear equations: a systematic literature review. *International Journal of STEM Education*, 6(1), 1–21. <https://doi.org/10.1186/s40594-019-0183-2>
- Vygotsky, L.S. (1978). *Mind in society. The development of higher psychological processes*. Harvard University Press.