

Geogebrization: A process for dynamizing classical mathematical texts

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Introduction, rationale, and theoretical framework

With this poster, I would discuss a learning experience proposal that involves classical textbooks of the early 1900 and (online shared and shareable) *Geogebra Books* (Bea Kristinsdóttir's overview: <https://www.geogebra.org/m/ztaQMAMx>). More precisely, it presents, describes, and exemplifies a configuration of an activity focused on a process where, starting from a portion of a printed (static) mathematical text, it leads to an appropriate combination of resources from the GeoGebra Service Platform (GSP, www.geogebra.org) that unpacks the mathematics embedded in the text. This unpacking process takes advantage of all the GSP features (e.g., dynamization and animation of mathematical content, the shareability of resources), and it yields to a finished product that can be exploited in turn for communicative, popularizing, or educational purposes. I call this process *geogebrization* (GGBZ) of a mathematical text. I intend GGBZ, in which interesting semiotic features are involved, as a learning experience, i.e., an activity that engages students. Here, I consider GGBZ that realizes a GeoGebra Book. The GGBZ process under analysis, and the structure of the activity through which it unfolds, will be framed within Radford's Theory of Objectification (TO; Radford, 2021). Thus, the learning of mathematics is interpreted in terms of *domestication of the eye* where students change their perception from "spontaneous" forms of attending to objects to a mathematical one. The GGBZ process is interpreted in terms of *sensuous cognition*, an enlarged conception of mind and consciousness that includes imaginative, perceptual, and sensorimotor features in which natural language and its metaphorical structures are embodied and dynamized taking advantage of all the GSP affordances. Moreover, the TO conceives the history of mathematics not as something external to the learning process. It is a necessity to understand our *historical* and *cultural* human nature; it *must* be part of the learning process (Radford & Santi, 2022). Finally, TO considers the teaching-learning activity as characterized by (i) an *object*, part of a teaching project, (ii) one or more *goals*, to be reached while developing toward the object, and (iii) *tasks*, which configure the activity to be designed. Each task comprises some problems that lead to *questions* to answer or *actions* to be done.

The configuration of a geogebrization activity: an example

The object of the presented activity is to visualize dynamically the geometrical content of a mathematical text on the basic tenets of projective geometry (improper elements, projections, and sections). The goal of the activity is to transform excerpts of the original printed static text of Guido Castelnuovo (1905), a textbook on projective geometry, into a GeoGebra Book. The complete set of original text involved (approximately 7 pages selected among the first 30) has been segmented into "thematic units" that represent the problems that compose the tasks. Each task leads to *questions*, e.g. the exploration of a given combination of proper/improper elements, or to *actions*, e.g. the realization of a described construction or the creation of new resources having a communicative purpose (i.e. where original text and related dynamic visualizations are rearranged to make texts themselves

dynamics and more usable for an external reader). In some cases, solutions are provided, obtained either by exploiting excerpts composed of text or images. The configuration of the activity (i.e. the tasks) is organized as a GeoGebra Book (English version: <https://www.geogebra.org/m/ksxsjc3v>). The poster will present, and discuss in the light of TO, some examples of a pair of undergraduate students engaged in it. As we will see, the mathematical content of the text (general, an archetype) becomes sensuous (w.r.t. sensuous cognition): GSP embodies cultural knowledge, and it is through interaction with its affordances that students' process of domesticating the eye occurs.

Castelnuovo's (1905) textbook as a starting point for geogebraization: why?

The motivation is both cultural and didactical. In the late 1800 – early 1900, in Europe, geometry experienced a revolution and an enormous development characterized by a particular perspective change: the focus shifted from the study of single entities to the study of invariants in families of entities. This switch of attention implies a “dynamic” geometrical eye which characterizes “advanced geometry” even today. The role of the Italian School of Algebraic Geometry was crucial for the evolution of geometry as we know it (Brigaglia & Ciliberto, 1998). Castelnuovo was an exponent of such a school, and the 1905 textbook is one of his first didactical works for the first-year students attending his course. The book begins with “point, line, and plane” and the geometric space never exceeds three dimensions. Nevertheless, the content is permeated from the very first words by the dynamic geometrical eye that substantiates and justifies the subsequent developments of geometry. Textbooks like Castelnuovo's are peculiar. Indeed, geometry is affected in this period by a semiotic revolution, see the work of Sylvester and Cayley around 1850, extended with the advent of category theory in about 1950. Thus, like other textbooks of the same period, there is still a strong presence of natural language used to describe highly visualizable and dynamic geometric contents, not always accompanied by an image nor always symbolically formalized. From this point of view, GeoGebra, considering only the *Graphics View*, offers an environment that seems coherent with the synthetic geometrical content of Castelnuovo's textbook. Moreover, GeoGebra Books offers a possible environment to propose a new edition of it (in Europe it has been in the public domain since Jan. 1, 2023), providing strength and foundation for having in mind the product's communicative purpose.

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