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# Piloting an innovation for developing students' written mathematical communication competency

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*We use a single key case to understand whether a research-based innovation to improve students' formal written mathematical communication has implementability and achieves desired outcomes for the target population. We investigate how and in what way students' communication competency can develop when teachers use an innovative teaching model targeting formal written communication. We found that students' formal written communications of solutions to problems have progressed in relation to the qualities of efficiency, clarity, arguments and justification, and correct mathematical notation. We also found that the teacher and his students adapted to the teaching model relatively quickly, which can be attributed to the implementability of the innovation. When scaling up, the teacher-researcher contact level needs to be down-scaled, putting new requirements on the contact-, material-, and organizational factors of influence.*

*Keywords: Implementation research, teaching model, written communication, communication competency*

## Introduction

Language and communication are at the core of all learning. Communication is also the core of the implementation project *Teaching Written Mathematical Communication* (TWMC) that started in January 2021. While the focus of the project TWMC is teaching for progression in written mathematical communication, the goal is to increase learning across all mathematical competencies. The implemented innovation consists of two parts: a framework for evaluating and assessing students' formal written communication and a teaching model. The framework includes three dimensions of qualities (efficiency, clarity, notation) and four content elements (problem description, calculations, arguments and justifications, conclusion) for evaluating and assessing progression in students' written formal mathematical communication (for more information, see Teledahl et al., 2023). The teaching model exists in a few variants but typically consists of six phases, where the final phase is a whole class discussion of student-produced written communications of different solutions to a mathematical problem. In this paper, we aim to examine whether the innovation is possible to implement without too much effort for teachers and students (Jankvist et al., 2021), so-called implementability, and if the innovation achieves desired outcomes for the target population (Century & Cassata, 2016). We use a pilot teacher as a single key case to illuminate this phenomenon. The key case is chosen for its capacity to exemplify the analytical object of the inquiry (Thomas, 2011), which in this study is the innovation enactment in classroom practice. Our research question is: *How and in what way can students' communication competency develop when teachers use an innovative teaching model targeting formal written communication?*

To orient the reader, we begin with a theoretical background for the project as a whole. Then, we move on to delve deeper into the teaching model and its implementation, followed by an existence proof of progress in students' formal written communication. We close the paper with reflections on the pilot phase and how we can use them when scaling up the project.

## **Theoretical background**

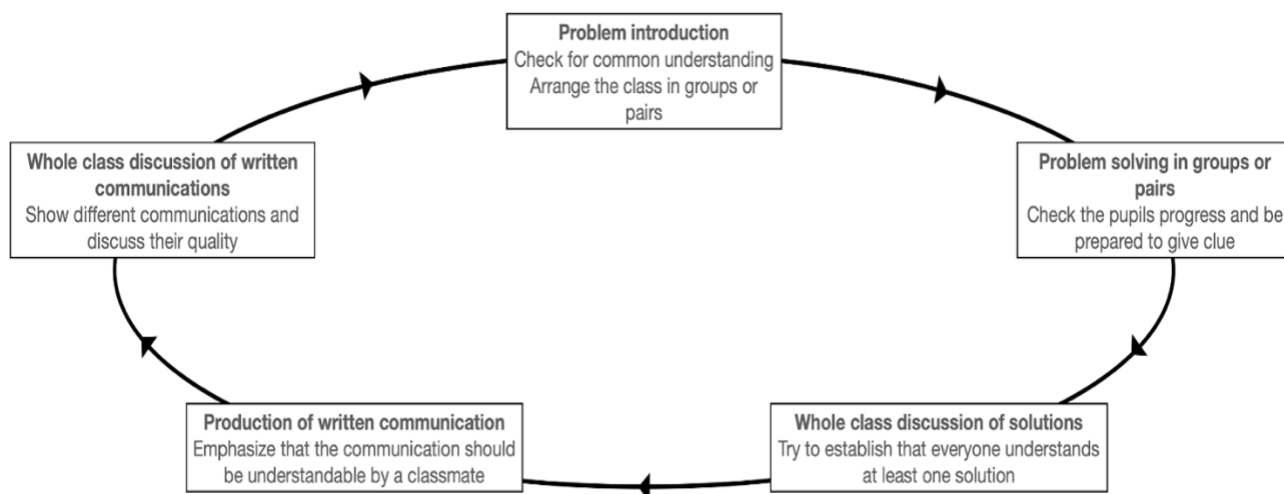
The project is based on two theoretical premises. The first is that communication is a distinct ability that can be developed, separate from other skills (Wittgenstein, 1953/1986). The second premise is that the ability to communicate consists of different elements, including infrastructural, graphic, linguistic, semantic, pragmatic, cultural, and social resources (Blommaert, 2013). The project focuses on written communication, which, like other genres, has evolved over time and relies on specific norms for combining communicative resources to create meaning (Veel, 2005; Schleppegrell, 2007).

For the development of our framework, we did a systematic review of existing research (Teledahl et al., 2022). The purpose of the review was to find research-based frameworks that suggest criteria for evaluating the quality of mathematical writing produced by students. The findings show that multiple frameworks describe different aspects of students' mathematical writing, but only a few focus on different quality levels. Building on prior work, we developed a framework for evaluating and assessing students' formal mathematical written communication (Teledahl, 2023). A framework for progression in communication competency needs to consider that communication in mathematics has unique historically constituted linguistic, graphic, and cultural characteristics that make mathematical written communication a relatively stable genre. At the same time, mathematical written discourse is also influenced by local context, infrastructure, and social contexts. The relative and social element of communicative ability is theoretically grounded in Wittgenstein's idea of language games (Wittgenstein, 1953/1986). We acknowledge that in a language game, participants work together over time to develop rules for how communication takes place and what communicative choices are valued, so this is highly dependent on context and social interaction. Wittgenstein (1953/1986) argued that there is a collective and more or less universal component to this, based on the fact that, as humans, we share experiences, and there are, therefore, not an infinite number of possible interpretations. Competent communication can be based on how communication is defined and the time frame for assessment, but the most common criteria are based on efficiency and appropriateness. The innovation takes into account the universal component specific to mathematics, based on the formal mathematical written and symbolic language developed in the past 200 years (Solomon & O'Neill, 1998), as well as the social and contextual component (Barwell, 2005), such as locally agreed-upon rules for what constitutes a mathematical proof (Stylianides, 2007). The focus on the mathematical communication competency aims to give students opportunities to invoke mathematical notions and concepts, terms, results, and theories and use mathematical representations (Niss & Højgaard, 2019). We developed a teaching model to support teachers' work with formal written communication. Further background and descriptions can be found in Teledahl et al. (2023).

## **The teaching model**

The teaching model is inspired by a well-researched model for teaching (Sterner et al., 2020). Our model describes how structured teaching can be carried out over two lessons that focus on 1) the

students' presentations of the solution of a mathematical problem with the goal that all students shall understand at least one solution to the problem, 2) the students' production of formal written reports of one chosen solution, and 3) a whole-class discussion about the quality of these written productions. Sometimes, there are variations of which phases go in which lesson, but the phases come in the order prescribed below.



**Figure 1. The teaching model**

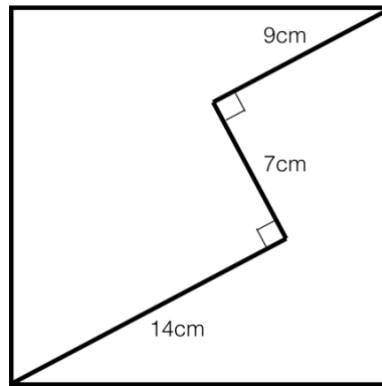
We exemplify our pilot teacher's innovation enactment in practice by describing his work with the phases in the teaching model. At the time of reporting, the teacher (first author, David) had tried the model twice during the spring of 2021 and four times in the fall of 2021. The teaching described below from March 2022 is built around a geometry problem.

### **Innovation enactment in practice**

When the problem (Figure 2) was introduced to the whole class, David's main concern was to ensure that all students shared a common understanding of the problem. Common understanding was important because if the class has different opinions about the problem and what needs to be done, it can be difficult to conduct a whole-class discussion. After the introduction, problem-solving began. The students worked independently on the problem for approximately 10 minutes before switching to pair work. The students worked on the solution and handed them in when they thought they were ready. Since the students at this point knew that the lesson was of a particular type with communication in focus, they did their best to communicate the problem solution to be 'comprehensible to an absent classmate' already in the first written draft<sup>1</sup>. The first lesson of two is now complete.

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<sup>1</sup> While this enactment differs slightly from the basic description of the model shown in Figure 1, it is an expected adaptation of the model when students get accustomed to the model.



**Figure 2. What is the length of the side of the square?**

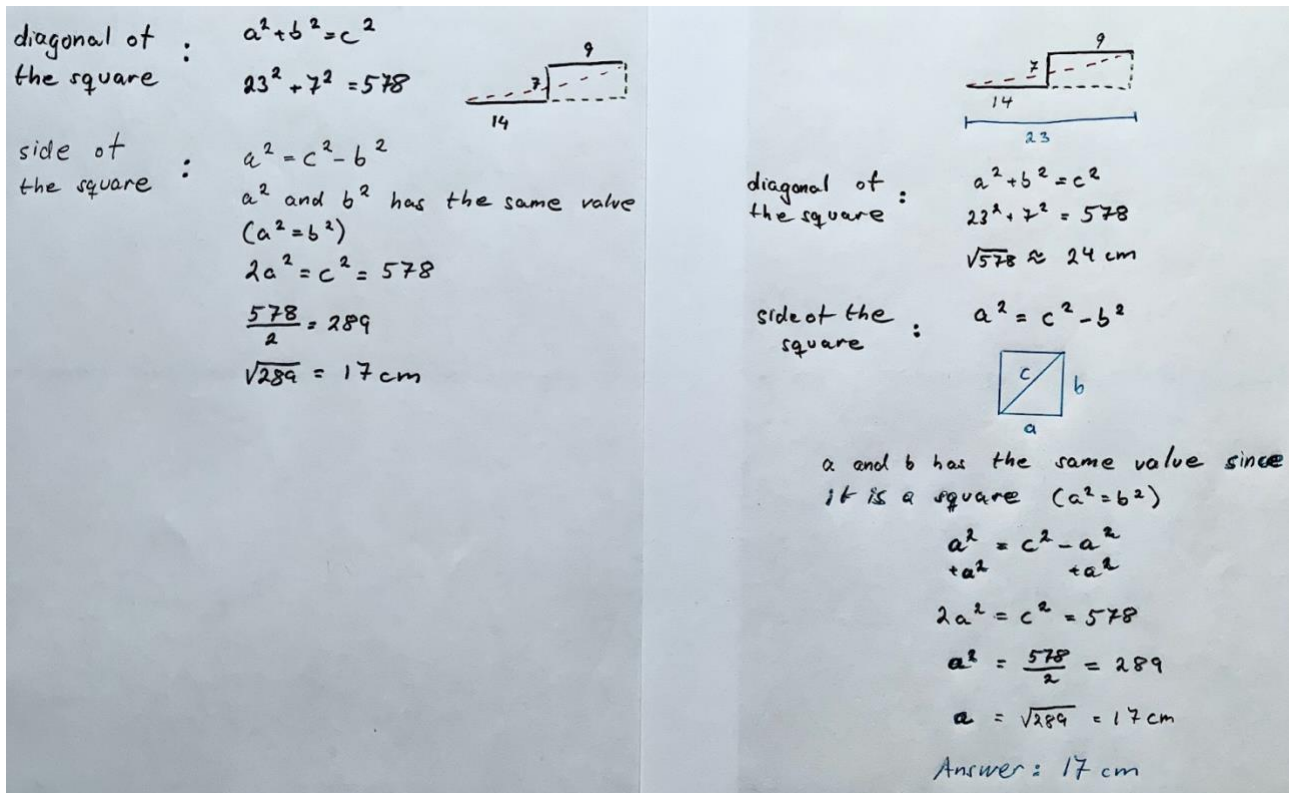
Within a week, the problem-solving lesson was followed up. Meanwhile, students' work has been discussed with the innovation designers (second and third authors of the article). These discussions identified the shortcomings and merits of students' communication. Solutions that showed good examples, common mistakes, and interesting reasonings received special attention.

In the second lesson, three different written communications of problem solutions are distributed digitally to the students. Based on the discussion with the innovation designers, the solutions were selected to illustrate some interesting points regarding the quality of communication. The students worked in groups or pairs to identify strengths and weaknesses in the distributed written communications of the solutions, followed by a whole-class discussion about the students' observations. The goal was for the students to understand at least one solution to the problem and pay attention to the communicative qualities and elements that could be improved.

Following our model, the students worked on improving the written communication of the solution to the problem. The instruction to the students was that the solution should be so clear that an absent classmate would fully comprehend how the problem was solved. As pointed out above, since the students were already familiar with the model, they, during the solution phase, did their best to include communicative elements and clear reasoning. Still, now they are instructed to make further improvements. Note that this step allowed you to choose a different solution than the one you made yourself. If students see a smarter, swifter solution, they can write it down, with the further revisions they want to make to the written presentation. This is important for two reasons. First, students gain the opportunity to expand their conceptual fields by using reasoning beyond their scope before instruction, and second, by improving someone else's solution, the students who did not solve the problem fully can still participate in the work with written communication. The improved communications of the solution were handed in for a second whole-class discussion.

### **Reflections on implementability and identified student achievements**

Figure 3 below shows an example of a pair of students' solutions and written communication to the problem displayed in Figure 2.



**Figure 3: Example of the solution of the problem, to the left, and the final communication of the solution to the problem, to the right**

Regarding communicative elements in the framework, the solution (to the left) lacks a *problem description*, a problem statement, and full information about the routes in the sketch. Further, the second row needs to include an *argument* for what 23 symbolizes. In row 4, an *argument* for why  $a^2 = b^2$  needs to be included. Also, the conclusion could be clarified. On the other hand, the appliance of the Pythagorean theorem did not need an extra explanation since its applicability was considered to be shared knowledge in the class of why, concerning quality elements, it is *efficient* to omit that *justification*.

To the right in Figure 3, we can see that the students have clarified the *argument* that 23 cm is the base of the triangle used to calculate the diagonal. Furthermore, they have argued that a square exists where the Pythagorean theorem is applicable. Mathematical *notation*, i.e., incorrect use of equal signs, has been corrected, and an explicit conclusion exists. The *problem description* is still missing. There were thus still opportunities for further improvement, but the communication (to the right) shows clear improvements compared to the first draft (to the left).

As for implementability, David found that enacting the teaching model revealed that reading other students' solutions was more demanding for the students than he expected. It turned out that many of the students had great difficulties with reading and following other students' solutions, as well as suggesting improvements to make the communication of the solution easier to understand. This is the effect we intend the teaching model to have, and the mechanism is that the difficulties of

understanding others' communications should act as inspiration to clarify one's communications in the future. However, despite the initial hurdles, it turned out that students' difficulties could be dealt with quickly in the whole-class discussions by highlighting particular qualities in problems that all students share a common understanding of. We interpret the rapidly transient resistance on students' behalf as a strength concerning implementability

An overall reflection from the pilot phase is that David has noticed an increasing overall standard of students' communication competency during the pilot phase of implementation<sup>2</sup>. The improved written communication is not only visible in mathematics but also in physics. One of the earliest identified achievements from this series of lessons was getting the students to comment on the calculations to get a numeric result. The students quickly learned that including calculations was a necessary condition to produce a communication of a solution possible to follow for other students. Initially, most students neglected to acknowledge the importance of putting the calculations in context with *arguments*<sup>3</sup> necessary for *clarity* and coherent reasoning. As mentioned above, many early attempts were difficult to understand, even for the students themselves, when revisiting their solution a week later (an indicator of lack of *clarity*). According to David, addressing this common shortcoming in selected problems gave good structure and sequencing, the use of mathematical notions and concepts, terms, results, and theories, and the use of mathematical representations in students' written communications.

### **Implications for scaling the project**

To understand whether an innovation has implementability for teachers (Jankvist et al., 2021) and achieves desired outcomes for the target population (Century & Cassata, 2016), we asked the question: *How and in what way can students' communication competency develop when teachers use an innovative teaching model targeting formal written communication?* We found that during the project's pilot phase, students' formal written communications of solutions to problems have progressed in relation to *efficiency*, *clarity*, *arguments* and *justification*, and correct mathematical *notation*. We interpret that the students adapted to the teaching model relatively quickly so that the innovation can have the intended effects; at least in this small-scale pilot-style context, the innovation has a high degree of implementability.

We will now review the implementation experiences described above from a scaling perspective. As mentioned, the described case is from a pilot implementation phase. The innovation has two parts: (1) the relatively formalized model for teaching formal written mathematical communication and (2) the framework for characterizing students' formal written mathematical communications. In the case of David, the teaching model was utilized from the start. In a collaboration between the teacher and the developing researchers, it was realized that the teaching models probably need to be adapted once

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<sup>2</sup> David also teaches the class in physics.

<sup>3</sup> The terms in italics are elements from a two-dimensional framework for assessing formal written mathematical communications we are developing. We do not have space to present the framework here, but still, use the terms since they are relatively self-explanatory.

the students get used to it. Still, it was also realized that the basic mechanisms behind the model probably were quite sound. One such mechanism is that reading mathematics is quite challenging, so being forced to read other students' writing will motivate students to write clearly. The tight teacher-researcher collaboration helped reinforce the validity of some underlying project ideas but also led to modifications and expansions of the teaching model. These results align with what Aguilar et al. (2023) characterize as affordances associated with small-scale implementations. In parallel with the small-scale implementation described here, a handful of similar implementations with the individual teacher have been going on, with similar results. The framework part of the innovation only had a peripheral role in the case described here and in the others. The framework was never explicitly shown in material form. It was only present as a set of underlying ideas leading our collaborative discussions on qualities in students' communications. This low requirement on the material factor of influence (i.e., text, video, etc., describing components of the innovation) is also typical for small-scale implementation projects, according to Aguilar et al. (2023)

But what about scaling up? Aguilar et al. (2023) describe three roles medium-scale implementations can have and what characterizes such projects. The type that most closely describes the next step for TWMC is a medium scale with deliberate multipliers. In terms of Aguilar et al., this put new requirements on the contact, material, and organizational factors. Because of the scaling, the teacher-researcher contact level needs to be down-scale. Concerning the material factor, the components of the innovation (framework and teaching model) need to be, at least partially, communicated in material form, like text and video. Regarding the organizational factor, the rather loose collaboration with individual teachers must be replaced with an organization that involves specifically educated advisors (multipliers) and teacher groups working together.

Expected research results from this ongoing scaleup are getting an even better idea of which aspects of the teaching models are actual determinants for the functioning of the model and further knowledge on how the framework can help teachers understand progress in formal written communication so that they without researcher support can select and discuss students' work in a way that helps the students develop their formal written mathematical communication competence.

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