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Evaluating the enactment of a scripted teaching model

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From an implementation perspective, there is a need to research the effects of large-scale projects. We focus on a project that engages over 1,000 teachers who implement a teaching model via highly scripted teacher guides. In this single case study, we analyzed video recordings of a third-grade teacher's innovation enactment of a teaching cycle. We found that the teaching deviated from the implementation program determinants. We discuss potential reasons for the teacher's lack of fidelity to scheme theory.

Keywords: Case study, determinants, innovation enactment, large-scale project, scheme theory.

Introduction

Thinking reasoning and reckoning (TRR) is a teaching model and a large-scale implementation project running in fourteen Swedish municipalities with around 1000 teachers. TRR relies on a quite detailed action model, that is, “a systematic plan for arranging staff, resources, settings, and support organizations to reach a target group and deliver intervention services” (Chen, 2012, p. 18) that builds on a detailed stakeholder scaffolding (Helenius, 2022) where all levels in the municipality are engaged in supporting teachers to implement the TRR teaching model. A fundamental part of the action model is detailed teacher guides describing how each lesson is intended to be taught.

The goal of this project is that students should learn more mathematics, which is relatively easy to evaluate. The question for the present paper, however, concerns teaching. If the teaching does not match the TRR model, then disappointing student results need not reflect the teaching model but rather a problem with the action plan. Teacher's adherence to teaching models is usually discussed in terms of fidelity, but as Century and Cassata (2016) argue, understanding the nature of implementations of pedagogical innovations requires researching “why adaptations are made, where implementation challenges come from, and how the nature of adaptation matters for outcomes” (p. 200). Examining adaptations, as suggested by Century and Cassata, is best done in qualitative case studies, which means that even in large-scale projects such as TRR, “large-scale studies often need to be supplemented with case studies that illustrate in-depth how different challenges in the implementation have been handled” (Ahl, Aguilar, et al., 2022).

In the present paper, we present a single case analysis of videotaped teaching complemented by an analysis of student work. We examine the following research question: *How can a teacher's adaptation of a teaching cycle in TRR be characterized?* In doing so, we interpret the teacher's adaptations through the Documentational Approach of Didactics, DAD (Trouche et al., 2020), and examine why certain adaptations to TRR might be made and in what light they make sense.

Theory

The Documentational Approach of Didactics, DAD, can be used for studying teachers' goal-oriented work through their interactions with resources and artifacts used in their teaching (Trouche et al., 2020). A goal of DAD is to comprehensively understand teachers' work by developing what it might mean to "use" resources. The concept of resource encompasses any artifact with the potential to mediate teaching activity, such as a whiteboard, slide show, resource book, and multilink cubes. When an artifact becomes incorporated into the handling and thinking scheme, i.e., the "[i]nvariant organization of behavior for a certain class of situations" (Vergnaud, 1998, p. 229) of an individual, there are two processes involved. The *instrumentation process* is when the affordances of the artifact shape the individual's practice. The *instrumentalization process* is when an individual uses the artifact with her knowledge and dispositions to fulfill her needs and goals. When the artifact is a teaching resource, the teacher's resource design work where the instrumentation and instrumentalization processes interact is called documental genesis. "In terms of processes, during the interaction with a particular resource or sets of resources, teachers develop their particular schemes of usage with these resources" (Trouche et al., 2020, p. 2). The product of the documental genesis is called a document. A document is thus a result when a resource or resources are appropriated and reshaped by a teacher in a way that reflects their professional experience with using resources.

The invariant is the organization of behavior in teachers' utilization schemes consisting of goals and expectations, rules of action, operational invariants, i.e., concepts-in-action (to categorize and select information), and theorems-in-action (to infer from the available and relevant information), possibilities of inference (Vergnaud, 1998). Teachers' schemes of usage, i.e., their invariant behavior for certain classes of situations, are reflected in teachers' routines in classroom practice. While schemes are mental processes that we cannot see, these processes are reflected in individuals' behaviors, which is why we can research which schemes of usage an individual has. When we work with behavior change, we need to set up hypotheses for how existing schemes should change when adapted to the new situation and how schemes supporting the desired teaching could be. We describe the desired change of routines in terms of hypothetical schemes.

A description of TRR and the analyzed teaching cycle

The Swedish innovation, the TRR teaching model for arithmetic for students aged 6-10 (grades 0-3), consists of units called cycles, and for each grade, there are 20 cycles grouped in four themes. Each cycle is described in a very detailed way in a teacher guide. Further, each cycle covers three lessons, which involve six phases with distinct teacher-student roles: a collective chanting phase, a whole-class introduction, pair work, whole-class follow-up, individual documentation, and a final whole-class follow-up. TRR covers roughly half of all mathematics taught in grades 1-3. The cycle under scrutiny here is from grade three and is called 'Regrouping and Algorithms'. In the TRR, additive arithmetic is processed by handling addition and subtraction simultaneously. TRR uses two concrete and mental models introduced in first and second grades to deal with additive arithmetic, a *number line model*, which originates in the embodied experiences of positions, distances, and movements (not dealt with in the cycle analyzed here) and a *part-part whole bar model* which originates in the embodied experiences of item collection and item construction and deconstruction.

The cycle under analysis here deals with standard algorithms for addition and subtraction (see students' work in Figure 1 for examples). These kinds of standard algorithms have already initially been introduced in Year 2 under two different cycles. We have no information about how the students have worked with such algorithms since those two cycles, over one year earlier. TRR introduces the algorithms by letting students present a completed algorithm and then discussing how the underlying addition/subtraction task can be represented with multilink bars representing ones and tenths and then regrouped to illustrate what happens in the algorithmic process. We hypothesize that when the algorithms are introduced, the students are already very well versed in thinking in terms of the bar models, both constructed with multilink cubes and in iconic draws, but also well versed in what we call grouping and regrouping arithmetical expressions, for example grouping 18 into $18=15+3$ or regrouping $15+3$ as $15+3=10+8$.

In the pair work, the students are to choose an addition or subtraction in the form of an arithmetic expression and then represent it with a bar model, a symbolic calculation using something other than an algorithm, an algorithm, and then explain the algorithm in their own words. All in predefined places on a worksheet. The individual documentation is a three-page worksheet with six tasks, A-F (see Figure 1 for the layout). The tasks are additive situations of slightly different complexity formulated in words, inspired by Vergnaud's (1982) classification. For each task, students are then to represent the situations as a bar model, an arithmetic expression, and calculate the answer using an algorithm or some other method.

Method

Data comes from three fully transcribed consecutive lessons with 9-year-old children (grade 3), video-recorded by the teacher herself, in autumn 2022. The teacher has a formal teacher education and has been involved in the TRR program since the pilot phase in the autumn of 2018. The present paper deals with phase 6 in the TRR cycle described above.

We do not compare the teacher's actions with the actual instructions in the teacher guide, as teachers may deviate from the instructions but are still in line with the program's underlying intentions. As a basis for our analysis, we instead explicate the relevant aspects of the TRR cycle in terms of so-called determinants, which are elements from the TRR's program theory, that is, the "leverage or mechanism upon which [the program] can develop a treatment or intervention to meet a need." (Chen, 2012, p. 18).

We present seven determinants of relevance in the sequence for the teaching of phase 6. We differ between classroom organization determinants, denoted by D_x , and content organization determinants, denoted by dx (Ahl, Helenius, et al., 2022).

D1. Building on students' productions. The teacher should review the students' work from phase 5 before phase 6 so the teacher can organize a discussion capturing all relevant mathematical ideas in the class and where students are engaged by explaining their actual work.

D2. Using student's errors. With reference to D1, the discussion should elaborate on the student's errors.

D3. Whiteboard organization. Divide the board into six sections for comparison between tasks.

d1-2. Discussion focus. Focus on arithmetic expressions (d1) and algorithms (d2).

d3-4. Comparing across tasks. Differences and similarities between the six additive situation and corresponding arithmetic expressions should be discussed (d3) and in particular, the situations corresponding to open additive arithmetic expressions like $a+_b$ and why this expression need to be dealt with as a subtraction when using the algorithm (d4).

To analyze D1, we classified every instance of student-teacher interactions as ignored, acknowledged, or undertaken, following the method described in Ahl, Helenius et al. (2022). An interaction could be a single student statement or a short, coherent communicational exchange. Interactions where the teacher acknowledged the student's contribution, but the interaction did not influence the following discussion were classified as acknowledged. Interactions were classified as an undertaking where the teacher takes the student's contributions and uses them in the subsequent discussion. An ignored interaction is when the teacher gives no feedback on a student's interaction.

Results

When referring to determinants in our presentation below, we will use the notation (Dx+) and (Dx-) to denote that some teaching action either reinforces (+) or counteracts (-) the determinant. Before presenting our analysis of phase 6, some previous events are highlighted. First, in the initial whole-class introduction (phase 2), the teacher leaves out the discussion on the subtraction algorithm and instead discusses an additional addition example. Second, the teacher modifies the worksheet for the individual documentation (phase 5) to only include tasks A and D (d2-) instead of A-F and to include a line for "Answer:". She also verbally emphasized students should include *both* a calculation using the algorithm and some other means to calculate, where the original instruction was to use an algorithm *or* some other method. Some but not all students followed this request.

As a basis for our analysis, we will first discuss the student's work, which the teacher is supposed to build phase 6 on D1.

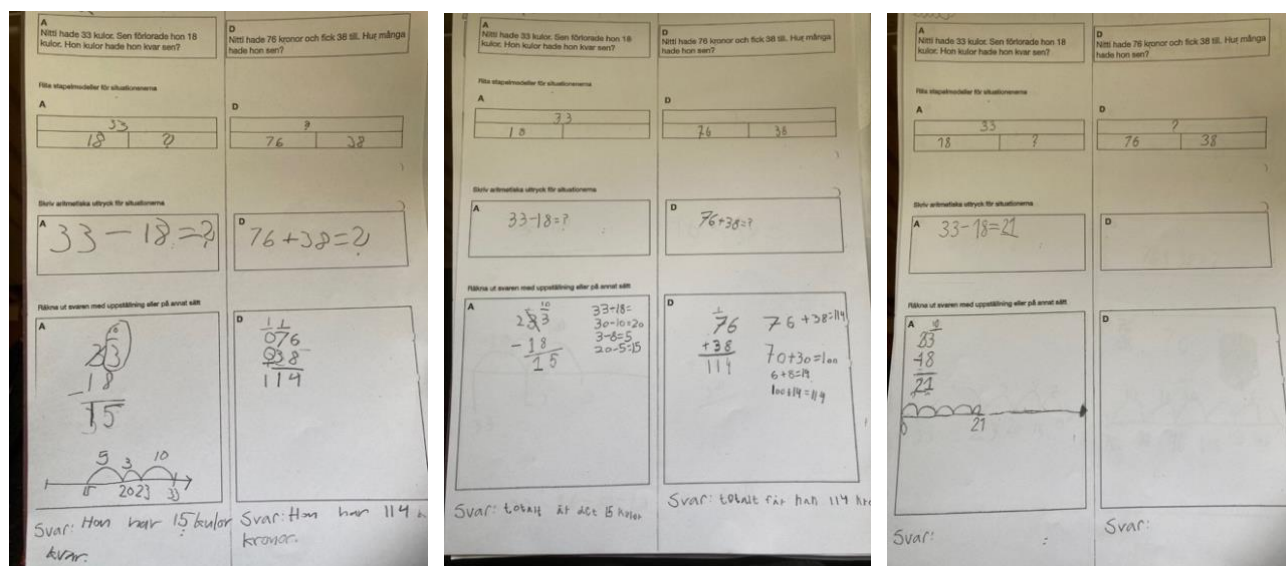


Figure 1. Examples of students' work

The two tasks the teacher selected are seen in Figure 1. The tasks translated into English at the top read as follows: ‘Nitti has 33 marbles. Then she lost 18 marbles. How many did she have then?’ and ‘Nitti had 76 crowns and got 38 more. How many did she have then?’. The other text on the worksheet reads: ‘Draw a bar model for the situations’, ‘Write arithmetic expressions for the situations’ and ‘Calculate the answer with an algorithm or in some other way’. At the bottom, the teacher has inserted an indication: ‘Answer:’

In total, 11 and 10 of the 19 students, respectively, did a correct subtraction and addition algorithm, which means there were several errors present. All except two students produced a correct bar model and a correct arithmetic expression. In Figure 1, three examples are presented. Students apply the subtraction algorithm in all three despite the teacher omitting it in phase 2. The example in the middle displays an incorrect algorithm. To the right, task D is not completed. All three examples show a correctly produced bar model and an arithmetic expression, often using “?” for the missing value. To the left, we see a complementary number line-based calculation based on a decomposition of $18=10+5+3$, where 3 has the role of matching up the decomposition of $33=30+3$. So, $33-18$ is represented as a movement from 33 backward by 10 (to 23), then by 3 (to 20), and then by 5 (to 15). All this is in accordance with how groupings and regroupings of numbers have been dealt with in TRR since grade 1 and with how the number line has been introduced as a representation of positions and movements. When the third lesson started, the teacher excused herself for “copying the wrong paper,” which included a subtraction task despite not discussing the subtraction algorithm in the earlier phase. The teacher may have intended to choose two addition tasks, fitting her modified worksheet. The teacher asks for some additional pair work (not included in the TRR instructions): “to really look at the [subtraction task] together and give each other tips”. After the TRR specified chanting, students work in pairs for around seven minutes. The teacher breaks the pair work 12:45 into the lesson to start with the content in phase 6. She asks a student to read out task D and writes the task on top of the whiteboard. Then she asks a student how the bar model should be drawn and leads a discussion on that because it is an addition; the two parts are known, and the whole is missing. The teacher then writes the arithmetic expression on the board (D1-). There is no discussion on the arithmetic expression (d2-).

The teacher moves to the algorithm and asks Evin to explain how to write the algorithm and then how to proceed with the calculation. Evin responds: “Six and eight... eeh... fifteen...no”. Now, the teacher leads Evin through a rather long discussion before reaching 14. The teacher let students respond to her writing on the whiteboard instead of having students talk about their own work (D1-). The discussion proceeds until answer 114 is reached. The general tendency is that the teacher asks students to respond to her questions instead of asking students to report on their work (D1-).

When we analyzed the teacher-student interaction in phase 6, we found 84 interactions in total. Of these, 3 were of the type ignored, 76 were of the type acknowledged, and only 5 were of the type undertaken. As mentioned, undertaken is an interaction type when students’ input is built on by the teacher in the following discussion. The few examples of undertakings are rather simple. At one point, the teacher asks for methods other than the algorithm to deal with the addition $76+38$. Evelyn responds, “I didn’t do it, but you could use regrouping.” In a question-response type dialogue with Evelyn, the teacher then goes through how a grouping of the number in tenths and ones could be used

without building on the student's previous work (D1-). A second example is when Nick suggests using a number line, leading to a question-response type dialogue with the student. Both these interactions result in a solution strategy ending up on the whiteboard. The regrouping strategy to the right in the left whiteboard in Figure 2 is a case when the teacher actually seems to build on students' actual work since the student Miro had actually produced this regrouping strategy involving moving a 2 from 76 to 38 in his individual documentation (D1+).

Soon after the number line strategy is written on the whiteboard, the teacher erases the whiteboard and moves on to discuss Task A. The discussion largely follows the same pattern but does not include an example of a regrouping. Of note is that both discussions focus on all representations instead of the arithmetic expressions and algorithms (d1-, d2-) and, in particular, focus on various solution strategies instead of comparing across problems (d4-).

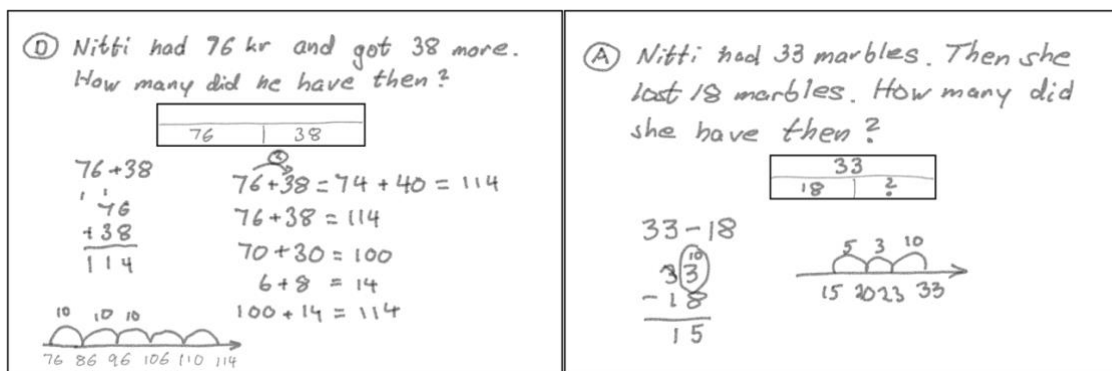


Figure 2. Whiteboards after the teacher has finished the discussion of tasks D (left) and A (right)

Discussion

We have shown that a very experienced teacher with TRR also departs from the teacher guide's instruction. In many ways, the teacher builds on TRR-specific models for dealing with additive arithmetic. The students' work shows that the students are well acquainted with TRR-specific reasoning; in this sense, the teacher builds on her students' thinking. Yet, how the teacher adapts the teaching radically breaks the TRR cycle determinants. To make sense of the teacher's adaptations, we will interpret the teacher's actions using scheme theory and consider two likely interacting schemes that the teacher holds because they represent widespread ideas and practices in Sweden.

The first scheme concerns the use of representations. Representation is an important concept in the teacher's scheme, as well as in TRR. A theorem-in-action in the teacher's scheme is that connecting representation is a significant feature in creating conceptual understanding. Widespread Swedish curriculum resources support this idea (e.g., McIntosh, 2008). Under this assumption, a rational goal for a summarizing discussion of a teaching cycle is to summarize the representations that have been in use for each task treated. A routine, or rule of action, that accomplishes this is to go through task for task and ensure all relevant representations end up on the whiteboard. A scheme encompassing the above would explain many of the teacher's actions. With a scheme like this, it is, for example, unreasonable to deal with six tasks in the available time, which can explain the choice to focus on two tasks only. The intentions for the analyzed and discussed TRR cycle phase are mainly to discuss

different additive situations and the algorithms in detail, and the six additive situations included in TRR are set up to support such a discussion.

The second scheme concerns the role of tasks. Even if all teachers do not have an explicit idea of the role of tasks, a typical view is that a task is something students should solve. The solution should eventually be generalizable to a set of similar tasks. The goals and anticipations of a summarizing session would then be to have students see different ways to solve a task. Solution strategy would be a relevant concept in such a task scheme. A theorem in action could be that multiple solution strategies reinforce students' skills for solving addition and subtraction tasks. In such a scheme, dealing with several tasks solved with the same strategies could be considered irrational. It may also be rational to ensure that correct solutions end up on the whiteboard rather than to extract and discuss erroneous thinking from the students' work.

In TRR, all tasks, or this case, a set of tasks, are designed to support the whole class discussion. Tasks are firstly designed to drive all students to think and focus on the mathematical phenomenon under discussion, and the goal and anticipation is that the totality of students' work, including errors, should provide enough variety to form a basis for such a discussion. In TRR, the role of tasks is to provide bases for discussion of some involved mathematical phenomena.

From our examples, it is clear that the teacher does not enact with acceptable fidelity to the program determinants. Can it be the case that the teacher's adaptations are improvements relative to the TRR intentions? After all, the teacher knows her students and is very experienced. We think following the TRR determinants would have been better, though. When looking at students' work, it seems almost all of them already know how to use regroupings, number lines, and also how to produce arithmetic expressions describing the two simple additive situations in tasks A and D. Therefore, we think that it would have been more worthwhile focusing more on the nuances between slightly more complex additive situations and arithmetic expressions (d4), using the suggested whiteboard arrangement (D3) and focusing on the algorithms (d2), which many students were unsure about and in particular building student interaction on algorithm errors students made (D2). We also think building more on students' actual work (D1) makes it easier to generate a more active interaction, which is more attractive to the students.

Lack of fidelity is often explained by the fact that the innovation does not align with the teachers' beliefs about what mathematics learning is and how teaching is done (Ahl et al., 2023). We do not believe that this explanation holds for this teacher as we know she is very dedicated, with beliefs in tune with the TRR project. We consider a better explanatory model can be given if we look more closely at the teacher's usage schemes as above. Schemes tie together expectations, routines, and rationalities.

Learning something new means adapting schemes into new schemes (Vergnaud, 1998). In demanding change models like TRR, whole routines for classroom practice might need to be replaced. So even if one believes in a new teaching model, as schemes are the mental backbone of routines, they will remain and continuously ask to be realized into behaviors, observable as old routines contaminating a new teaching model. A gap opens between the teacher's intentions and her behavior, most of which are too brief and automatic to be controlled (Sfard, 2023).

Regarding IR on the effectiveness of TRR, the consequence of the program determinants not being implemented in teaching practice is that we cannot evaluate whether the teaching model is effective or has flaws. For a credible evaluation of TRR, we see the necessity of doing case studies on large-scale projects because it is impossible to capture the nuances of enactment in quantitative studies.

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