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# From design principles to implementation heuristics: A networking of theories perspective on implementation research

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*With an outset in design-based research, this paper coins the notion of 'humble implementation heuristics' as the next progression of design principles prior to implementation (at scale). Based on using 'networking of theories' as a meta-practice to design research, the paper discusses the potential of this approach to the implementation processes of innovation, exemplified by a specific case of design-based research presented at the previous CERME.*

*Keywords: Implementation research, networking to theories, design-based research, representation.*

## Networking of theories and design research

Networking of theories is a research practice initially described at CERME4 (Kidron et al., 2018). It may be defined as a meta-research practice (Bikner-Ahsbahs & Vohns, 2019), often applied in combination with other methodologies, e.g., design research (DR) (see Alberto et al., 2019). Eight networking strategies are provided and described in Prediger et al. (2008), stretching from *understanding own/others* to *integrating locally* and *synthesising*. Two strategies, those of *coordinating* and *combining*, focus on how the use of theoretical perspectives may help to understand an empirical phenomenon. When coordinating, the theoretical perspectives must be compatible, so that the basic theoretical (and philosophical) assumptions are not be too far apart. The demand for compatibility in the theoretical basis does not appear to the same extent when combining. According to Niss and Jankvist (2022), "A theory is a *theory of something*, i.e., it deals with certain sorts of *objects* and *phenomena* and includes terms for these" (p. 17). In DR, studies aim at developing both theory and practice. For example, a DR study may seek to develop and describe design principles as part of *predictive theory elements*. Design principles thus relate to the aim of DR as design principles are both grounded in practice and theoretical perspectives, involving the development of both (Prediger, 2019). Principles that are not yet tested in practice and refined and consequently only supported theoretically, are considered heuristic (van den Akker, 1999) and *humble* (Prediger, 2019).

In Bach et al. (2022), we applied networking of theories as a meta-practice to a DR study. The aim was to develop *humble design heuristics* (HDH) into design principles (see section below). In continuity, in this paper, we aim to explore networking of theories in regard to implementation research (IR). Inspired by DR, we want to coin the idea of *humble implementation heuristics* (HIH) with an outset in our previous work. Such HIH would then be a precursor for a set of resulting implementation principles. The first underlying *hypothesis* here is that such HIH may serve as an informative 'link' between the developed design principles and an initial implementation process (small scale or large scale). Another *hypothesis* we have is that networking of theories as a meta-practice has a role to play here as well. Hence, the research question of the paper: *Based on the*

*specific empirical case of Bach et al. (2022), what might a set of ‘humble implementation heuristics’ look like, and in what ways may the formulation of these benefit from a networking of theories perspective?* Implicit in this question is of course a demand for an attempt to define HIH. For this purpose, we draw on IR theoretical constructs (Koichu et al., 2021)—constructs that may potentially themselves be subject to networking. We begin with a description of the empirical case.

### **Empirical case: From humble design heuristics to design principles**

Based on our literature review concerning students’ representation competency when using digital technologies, we recommended five HDH (Pedersen et al., 2021). Here, we utilised van den Akker’s (1999) definition of design principles, which includes a heuristic nature and may be explained as:

If you want to design intervention X [for the purpose/function Y in context Z], then you are best advised to give that intervention the characteristics A, B, and C [...], and to do that via procedures K, L, and M [...], because of arguments P, Q, and R (van den Akker, 1999, p. 9)

Three out of the five HDH were applicable to the specific task in Bach et al. (2022):

1. Including the linguistic register by formulating the task in natural language and asking for answers in natural language.
2. Breaking objects and windows of the tool into smaller units by introducing them gradually.
3. Using sliders, dragging and tracing for explorations of different representation and representation forms’ reciprocal relations. (Bach et al., 2022, p. 2)

We then developed the three HDH into design principles based on empirical testing involving two students using a dynamic geometry template. Within our study, we relied on the Danish competency framework, KOM, which is of a cognitive nature and concerns what it means to master mathematics (Niss & Højgaard, 2019). KOM defines *mathematical representation competency* as “the ability to interpret as well as translate and move between a wide range of representations [...] as well as of the ability to reflectively choose and make use of one or several such representations [...]” (p. 17)

KOM’s description of the representation competency was not enough to understand the situation of students’ use of the dynamic geometry environment and their interactions with the digital representations. Finer grained theoretical perspectives were needed. We thus applied a networking of theories practise to coordinate two other finer grained theoretical perspectives to KOM (Bach et al., 2022): (1) a semiotic register approach, which was used to analyse the representations involved (Duval, 2017); and (2) the instrumental approach to mathematics education, which was applied for analysing students’ use of the digital tool (e.g., Trouche, 2005).

The case presented in Bach et al. (2022) involves two students who use a pre-set GeoGebra template. While investigating the template, the students communicate about the relationship between a point  $P$  and a rectangle  $ABCD$ . See Figure 1. Based on a parallel analysis of the two students’ work, the revised design principles were formulated in the following way:

If you want to design teaching with a digital tool for exercising and developing the representation competency in schools, then you are best advised to gradually relate all four registers (linguistic, symbolic, figurative and graphic) [from Duval (2007)] in a task sequence when using digital tools.

Collaboration between students is essential for all three characteristics and procedures. This may be realised by:

1. including the linguistic register (both written and oral) throughout the task (i.e., when formulating the task, exploring the environment and representations, and writing answers), and being aware of how natural language appearing in the environment relates to the intentions of the task, as the linguistic register is often neglected in a digital tool, although it is an important register expressing the relations between representations, formulating interpretations of different representations and for developing an artefact into an instrument.
2. breaking objects and tool windows into smaller units by introducing different representational registers and components of the artefact gradually (e.g., hiding the algebra view), because gradually introducing representations makes it possible to identify the specific characteristics of a representation and to help the students' processes of instrumental genesis by limiting their explorations to specific parts of the tool.
3. using sliders, dragging and tracing for explorations about different representations and the treatments and conversions caused by the dynamic features since these features make it possible to quickly move and translate between representations, and for developing conjectures about the representations' reciprocal relations. (Bach et al., 2022, pp. 6-7)

<p>Investigate the construction on your computers by dragging point <math>A</math> in the figure. Describe the relationship between point <math>P</math> and the shown figure. For point <math>P</math>, describe what characterises the <math>x</math>-coordinate and the <math>y</math>-coordinate. Explain why the relationship exists.</p>	
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**Figure 1: Illustration of the task and the students' pre-set GeoGebra template (Bach et al., 2022, p. 4). The tasks and GeoGebra template were based on tasks presented in Johnson and McClintock (2018)**

The networking practice was applied to inform the choice of well-fitting theoretical constructs (see Prediger et al., 2008) around KOM to be able to coordinate these. The two other theoretical perspectives shared the cognitive background of KOM, yet they concerned different objects, i.e., mathematical representations and the use of digital tools, respectively. Within our study, we found that the strategy of coordinating and DR were well suited for one another, since both aim to develop theory and understand practice. In addition, the networking practice appeared to strengthen the theory development in DR (Bach et al., 2022).

## Implementation in mathematics education

Rogers (2003) defines an *innovation* as “an idea, practice, or object that is perceived as new by an individual or other unit of adoption” (p. 12). Koichu et al. (2021) perceive an *implementation in mathematics education* as “an ecological disruption to a particular mathematics education system, through the gradual endorsement of innovation in conjunction with an action plan aimed at resolving what is perceived as a problem by (at least some of) the stakeholders involved” (p. 986). They describe the implementation process of the innovation in terms of the innovation proponents and the innovation adapters. Initially, the proponents have the ultimate agency of both the innovation and the plan of action. Over the course of the implementation, the adapters experience some or all of four sub-processes: “(1) constructing agency over the innovation, (2) gradually changing within-community communication or across-community communication, (3) gradually changing practice so that it accommodates the innovation, (4) adapting the innovation to their needs and aspirations” (p. 986). These four sub-processes reflect back on the innovation proponents, also in terms of the evolution of the innovation, the plan of action and not least the theories underlying the development of these. Koichu et al. (2021) remark that the implementation process is iterative and that it comes to an end once it stops being perceived by stakeholders as an ecological disruption.

As argued on several occasions (e.g., Jankvist et al., 2021), an articulated ‘Theory of Change’ (ToC) is a crucial aspect in any implementation process for measurement of the ecological disruption. A ToC is a collection of theoretical constructs—from general IR or from ‘local’ mathematics education—that may be used to guide design, the phases of implementation and the evaluation of an innovation. A ToC may also encompass a description of a set of assumptions. Jankvist et al. (2021) state that theory-driven evaluations have two key components: a conceptual one explicating a program theory or model; and an empirical one that is to “investigate how the programs (or innovations) cause intended or observed changes and outcomes” (p. 1048).

### Discussion 1: Developing humble implementation heuristics

Referring back to van den Akker (1999), we may rephrase his advice in terms of IR to define the HIIH for our implementation study based on Bach et al. (2022), relying on Koichu et al. (2021):

If you want to implement design principles (X) seeking to exercise students’ representation competency, when using digital tools in mathematics classrooms for the purpose (Y) of scaling the DP to a larger population of mathematics teachers (Z) in lower secondary schools in Denmark (e.g., in a municipality), then you are best advised to *anticipate possible implementation issues regarding mathematical representations competency and the use of digital tools across schools and classrooms*. This may be realised by:

- (A) clarifying the purpose of the innovation, the context of using digital tools and the mathematical content in play to the stakeholders (e.g., schools and teachers) *by* (K) designing possible tasks and adapter guidelines to be implemented by the adapters *because* (P) having tasks and teacher guidelines may help adapters to access and take agency of the innovation. The tasks built on the design principles, and teacher guidelines help the adapters with different knowledge and experiences about mathematical representation and use of digital tools.

- (B) supporting the development of communities of practice focused on this theme *by* (L) gradually introducing discussions groups of stakeholders (e.g., adapters at schools and across schools), as well as access to researchers/experts, mathematics counsellors or “super teacher users” with respect to digital tools and mathematical representations (both physical and online communities are established) *because* (Q) sharing experiences between adapters and collaboration may ease the implementation of the innovation and allow adapters to gain from other adapters’ experiences.
- (C) helping adapters to assess their own practice using didactical theoretical perspectives on mathematical representations and the use of digital tools (as part of the ToC) *by* (M) gradually presenting ToC-based objectives and rationales of tasks to be implemented and their aligned intended learning outcomes *because* (R) the adapters must evaluate according to the didactical theoretical perspectives (i.e., the ToC).
- (D) contextualising the tasks and the design principles, presenting possible learning trajectories available, *by* (N) adapting these to the one’s own classrooms *because* (S) adapters may experience difficulties when implementing the design principles depending on the involved classrooms, and on previous teaching experiences related to both the content and the use of digital tools.

The ToC for this implementation study would build on the semiotic register approach (Duval, 2017) and the instrumental approach (Trouche, 2005) as part of the design principles from Bach et al. (2022). Hence, from an IR point of view these theoretical perspectives make up our ToC. When assessing and evaluating the implementation, the ToC becomes an important constituent.

Our HIIH are to be tested in a large-scale implementation study. In DR, these heuristics may traditionally include both design principles and learning trajectories (see Prediger, 2019). Yet, so far we have not touched upon the learning trajectories, which are closely related to the mathematical content of the study (in our case, functions). Learning trajectories may thus be more difficult to present in a generalised manner for all lower secondary classrooms, as students in 7-th grade are likely to have other learning trajectories than e.g., students in 9-th grade.

If to describe HIIH to any IR study, we may again draw on van den Akker (1999):

If you (the proponent) want to implement innovation X [for the purpose/function Y in context Z, for adapters], then you are best advised to take into account the ToC in relation to the characteristics A, B, C, and D [being aware of different kinds of stakeholders], and to do that via procedures K, L, M, and N, because of arguments P, Q, R, and S.

In such cases, (A) concerns a *layered communication* (i.e., communication regarding the purpose of the innovation, the content of the innovation, and ways that stakeholders can use the innovation according to the intentions of the proponents) *by* (K) “constructing adapters agency over the innovation” (Koichu et al., 2021, p. 986) *because* (P) easy access and agency of the innovation to mathematics teachers, i.e., adapters, with different scope of engagement at different times is essential. (B) emphasises the necessity of communities of practice (i.e., different levels of communities with access to experts or “super adapters”) *by* (L) “gradually changing within-community communication or across-community communication” (p. 986) across local adapter groups and, likely, having access

to online communities *because* (Q) communities foster imperative communication, which supports adapters in the implementation process according to their needs. (C) entails that adapters have the possibility of assessing their own practice according to the ToC *by* (M) “gradually changing practice so that it accommodates the innovation” (p. 986) *because* (R) the adapters require a mechanism fostering awareness/focus on how to implement the innovation successfully into their own practices. (D) focuses on the flexibility of the innovation *by* (N) “adapting the innovation” to one’s own “needs and aspirations” (p. 986) *because* (S) implementation of the innovation depends on the contexts in which the innovations are implemented.

To sum up, DR and IR differ in various ways, e.g., DR is likely to be more local than IR. van den Akker (1999) remarks that developmental research (e.g., DR) does not aim for implementation, but rather at developing prototypical interventions. IR, however, aims for implementation of these interventions. Still, the two research approaches are relatable, since IR may naturally follow DR.

## **Discussion 2: The potential contribution of a networking of theories practice**

When using the networking strategies of coordinating and integrating locally, the results are of course local. In IR in general, however, results may be more global, have more context, etc. Hence, it is in this spectrum that the potential of relating networking of theories (strategies) and IR may arise.

In our DR study, we used the semiotic register approach and the instrumental approach to develop and adjust our design principles (Bach et al., 2022). From an IR point of view, the semiotic register approach and the instrumental approach act as our ToC. Hence, these theoretical perspectives act as key components when designing and assessing the innovation. Networking of theories brings practices that create consistency between the innovation to be implemented (i.e., the design principles) and how we wish to assess that. Meanwhile, IR then allows for less local testing of the theoretical work. Although both networking of theories and IR address much more fundamental issues, they both also share a focus on communication. Networking of theories includes an aim of intercommunication to the field of mathematics education research, i.e., the basic assumptions of the involved theoretical perspectives must be described and discussed (Prediger et al., 2008). In IR, a successful implementation requires that basic assumptions of innovations are described clearly, and that the adapters are aware of what we wish to assess and how. Hence, providing particular practices from networking of theories focusing on presenting basic assumptions may also support clearly described innovations and well-established and coherent ToCs. Both IR and DR have theory elements of different kinds. In Prediger’s (2019) perspectives on DR, e.g., categorial, normative, predictive, descriptive and explanatory theory elements, these theory elements are acting together throughout a project. As described in Bach et al. (2022), the networking of theories then provides practices to choose reflectively the theoretical perspectives as well as to relate different theoretical perspectives. The use of networking of theories in IR may help to achieve consistency in the involved conceptual component. IR may also strengthen networking of theories, since networking theories often are considered to be local in organisation. Nevertheless, it appears relevant to stress again that the theoretical perspectives connected/networked in IR are not global, unified theories. Overall, IR may be fruitful for testing the connectivity between theoretical perspectives, also in large-scale studies.

## Final remarks on the idea of humble implementation heuristics

Let us return to our two hypotheses, i.e., (1) HD between the developed design principles and an initial implementation process (small scale or large scale), and (2) that networking of theories as a meta-practice has a role to play in relation to such HIH and perhaps IR in general. Besides formulating specific HIH based on the study of Bach et al. (2022), we have illustrated that HIH may be formulated using a similar structure as van den Akker's (1999) definition of design principles. Of course, such HIH may cover both learning trajectories and design principles (Prediger, 2019). We suggest that formulating HIH may assist proponents in being more specific about the aim of their innovation as well as its implementation. Although HIH are indeed humble in their outset, they aim to be refined as the project progresses. The meta-practices of networking of theories are often applied to other research practices. Applying networking to IR has promising potential with respect to *consistency* and *alignment*. In our DR study, we networked the semiotic register approach with the instrumental approach to develop and adjust design principles. This theoretical work was our ToC in the IR study. If studies focus on networking local theoretical perspectives as part of the ToC, its implementation may be more likely to be consistent with respect to the basic assumptions. Being specific about the ToC from the beginning of an IR study may result in a larger degree of alignment between the different phases and sub-processes of the implementation—and HIH may be a construct to assist the achievement of this. We prophesy that it is indeed worth looking closer at the unexplored potential in an interplay between the IR and the networking of theories practice.

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