

Pre-service teachers' knowledge of students' misconceptions and difficulties about functions

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Keywords: Functions, misconceptions, pre-service teachers (PSTs).

Introduction

In this paper we will focus on PSTs' knowledge of misconceptions and difficulties (M/D) during a university course. The important aim for us is that every PST is aware of known M/D before they start designing and teaching their lessons, so that they can prevent these M/D in their students. PSTs' knowledge of M/D that students may encounter when learning functions in secondary school is part of the subdomain Knowledge of Features of Learning Mathematics in the Mathematics Teacher's Specialised Knowledge (MTSK) model (Carrillo, 2018).

This study is part of the research within the FunThink project, which focuses on the development of functional thinking. In this study, we focused on PSTs' knowledge of students' M/D about functions. The research question is: Did the course aimed at developing PSTs' functional thinking, designed within the MTSK model, help PSTs to scaffold their knowledge of students' M/D about functions?

Method

The course, which aimed to develop the functional thinking of PSTs, was attended by 13 PSTs (4 men and 9 women) aged 22-23. All PSTs had a bachelor's degree in mathematics. The course was compulsory, conducted in the winter semester of the academic year 2022-2023 and consisted of 26 lessons divided into 13 sessions. It focused on the 6 subdomains of MTSK, in relation to the development of PSTs' functional thinking. To collect the data, we used the research tool developed in the FunThink project, which includes 8 mathematics tasks from the textbooks and 9 questions (some questions follow these tasks). To find out the progress of the PSTs, the same research tool was used for both the pre-test (given by the PSTs during the first session - at the beginning of the course) and the post-test (given by the PSTs during the last session - at the end of the course). In the study we will analyse the written answers and notes of the PSTs to the following question from the research tool: What learning difficulties and misconceptions do you expect when teaching functions?

Based on the theoretical framework, together with the research question and the PSTs' response to the highlighted question from the research tool, we created a coding scheme that included 16 different categories of M/D. We adopted 6 codes from Leinhardt et al. (1990): What is and is not a function (F/N); Linearity (LIN); Representations of functions (REP) divided into 3 subcategories - One representation of function (1REP), Linking representations (LREP) and Modelling (MOD); Concept of variable (VAR); Notation (NT); Continuous vs. Discrete graph (CvsD). The other 4 codes refer to students' M/D related to 4 aspects of function (1A, 2A, 3A, 4A) according to Pittalis (2020); and the remaining categories arose from the need to classify all PSTs' responses. The frequency of codes in the pre-test (*) and post-test (*) is shown in the Table 1. We have produced a manual for other project solvers, in which each code is given a description and a concrete example of a PST's response.

Table 1: Classification of pre-test and post-test misconceptions and difficulties reported by PSTs

PST	Categories of misconceptions and difficulties																Σ
	F/N	LIN	1REP	LREP	MOD	VAR	NT	CvsD	1A	2A	3A	4A	GEN	Oth+	Oth-	BLK	
Abby	**			*		**	*	*	**	**	**	**	*	*			6/9
Betty			*	**			*				*	**	**	*			4/3
Caleb	*	*	*	*	*	*			**	**	**	**	*	*			6/8
David	**	*	*		*	*	**		*	*	**	**	**	*	*		4/10
Eli	*	*	*		*	*			*	*		**					2/7
Fiona			*	**		*	*				**	*		**			3/5
Grace	*		**				*				**	*	*	*			3/4
Heidi	**		*	*	*						**	**	*	*			5/4
Ian	**	*	*	**	**	**			*	*	**		*	*			6/8
Joe	*		**	*		*	*		*	*	*		*	**			3/6
Kate			**	*	**		**				**	*	*				4/5
Lisa			*	*		*	**				*	*	**	*			3/4
Macy	**	**	*		*		*		*	*	**	*	*				4/8
Σ	6/8	1/5	7/8	6/4	4/5	2/8	4/8	0/1	3/6	2/7	11/10	7/10	8/6	9/3	0/1	0/0	

Results and conclusions

As can be seen in Table 1, by the end of the course almost all categories had increased in frequency and almost all PSTs reported a greater variety of different M/D in the post-test. The best results in both the pre-test and the post-test were observed for categories 3A and 4A. On the other hand, the category CvsD had the weakest representation. Only Abby mentioned this kind of M/D once in the post-test. The LIN, LREP and MOD categories were also poorly represented. Our results suggest that consideration should be given to adding tasks and activities for PSTs that would also more closely reflect these typical M/Ds reported in several studies dealing with teaching functions. Based on the above analysis, we plan to modify the above course for PSTs to achieve better results. It is important for teaching functions that PSTs have the best knowledge of students' M/Ds and reflect this knowledge in the preparation of the lesson and also during the actual delivery of the lesson.

Acknowledgment

This work was supported by the “FunThink Erasmus+ Project” (2020-1-DE01-KA203-005677).

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