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Theorizing teacher's knowledge on connections from the MTSK perspective

Milena Policastro¹ and Miguel Ribeiro²

¹Escola Comunitária de Campinas, Campinas, Brazil; mitapolicastro@gmail.com

²State University of Campinas, UNICAMP, Brazil

This paper presents a theory of teacher knowledge on mathematical connections from the MTSK perspective. We identified structural elements and unifying ideas from previous descriptors of teachers' knowledge in division and measurement, establishing relationships between different elements of teachers' knowledge within each topic and between them. This analysis resulted in specialized knowledge packages that provide a detailed understanding of the structural elements and unifying ideas underlying teachers' knowledge about connections for each mathematical topic.

Keywords: Teacher's specialized Knowledge, MTSK, connections.

Introduction

Teachers' knowledge is a crucial factor in both their practice and students' understanding and outcomes (Hill et al., 2008). Different conceptualizations recognize the importance of 'making connections' as part of teacher's knowledge content (Ball et al., 2008; Carrillo et al., 2018). For example, the Mathematical Knowledge for Teaching (MKT) perspective highlights the teacher's understanding of the relationship between mathematical topics, while the Mathematics Teacher's Specialized Knowledge (MTSK) conceptualization, in turn, views connections as relationships between concepts and considers them as part of the teacher's knowledge in different subdomains. However, the two conceptualizations agree on the significance of connections in mathematics teaching, which leads to deeper understanding of mathematical structures, known as structural thinking (Mason et al., 2009).

To build on the idea that teachers promoting connections in the classroom can lead to diverse learning opportunities (De Gamboa et al., 2020), it is important to consider the teacher's own knowledge of connections. This requires not only a deep understanding of the concepts, properties, foundations, and connections within a single topic and across topics, but also an investigation of the types and nature of these connections, as well as the moments and contexts in which the teacher becomes aware of them and uses them to guide students' learning.

Measurement and division are essential mathematical topics that form the foundation for understanding arithmetic, proportional reasoning, and algebra. Understanding the key elements of measurement and division is crucial for a thorough understanding of these subjects.

In this context, we seek answering the following questions: (1) *What are the key elements of teacher's specialized knowledge related to the topics of division and measurement?* (2) *What kind of relationships are observed between the content of such specialized knowledge?*

Theoretical background

Connections from the Mathematics Teacher's Specialized Knowledge (MTSK) perspective

The MTSK conceptualization recognizes the specialized nature of a teacher's knowledge, assuming two domains: mathematical and pedagogical content knowledge (Carrillo et al., 2018). The domains are further divided into six subdomains, three of which relate to the Mathematical Knowledge (MK). The MK subdomains include Knowledge of Topics (KoT) and Knowledge of the Structures of Mathematics (KSM), both of which deal with connections within and between mathematical concepts, respectively. Intra-conceptual connections (element of KoT) focus on the relationships between mathematical foundations and properties within a concept, while inter-conceptual connections (KSM) refer to connections between different mathematical concepts and their generative role in the construction of mathematical items (Carrillo et al., 2018). These connections can increase complexity or simplify mathematical contexts.

The Knowledge of Topics (KoT) includes knowledge of multiple definitions, senses, and representations of mathematical concepts, as well as organization of mathematical facts and rules. Knowledge of the Structures of Mathematics (KSM) includes a deep understanding of mathematical topics and their relationships with other structures. Originally, the specifics of teacher's knowledge included in the KoT was studied from four categories. However, recent results (Policastro & Ribeiro, 2023) showed the necessity to separate one of the categories – Definitions, properties and foundations (Carrillo et al., 2018) – into three. In this sense, this study assumes six categories in KoT: *Definitions, Foundations, Properties, Procedures, Registers of representations and Phenomenology and applications*. In KSM, from four categories, is possible to explore the content of teacher's knowledge of connections between different mathematical topics: *Connections based on simplification; Connections based on increased complexity; Auxiliary connections; and Transverse connections* (Carrillo et al., 2018).

Division and measurement topics: the mathematical contexts for looking into connections

Measurement is a crucial part of Mathematics that involves quantifying and analyzing real-world concepts and attributes. It should be integrated into early childhood education to lay the foundation for future mathematical concepts such as arithmetic, proportional reasoning, and algebra (Szilagyi et al., 2013). A *sense of measurement* (Szilagyi et al., 2013) should be a central focus in teaching, as it relates to developing the essence of the concept of measuring (Mariotti & Fischbein, 1997) and important mathematical concepts, associated with Big Ideas in Mathematics.

The act of measuring involves assigning a numerical value to a measurable attribute of an object, phenomenon, or process (Berka, 1983). This value is obtained through comparison of magnitudes of the same nature and quantification of the number of iterations (Berka, 1983; Clements & Stephan, 2004). Unit, magnitude, measurement value, and iteration are key constructs in the measuring process and an effective teaching of these concepts (Barrett et al., 2011) can help students understand the differences between discrete and continuous quantities (Gao et al., 2000) and the two meanings of division (Fischbein et al., 1985).

Division poses challenges in both teaching and learning (Roche & Clarke, 2013), often stemming from misconceptions about key elements like dividend, divisor, quotient, and remainder (Correa et al., 1998). Students also face difficulties due to the complexity of division algorithms and reasoning behind each step (Correa et al., 1998). Policastro & Ribeiro (2021) note that teachers' limited grasp of division's interpretations might lead to a neglect of the reasoning behind calculations, focusing solely on the algorithm's mechanical execution. This approach overlooks the necessity of establishing strong foundational concepts critical for comprehensive division comprehension.

Division and measurement play a similar role in constructing mathematical structure, as seen when analyzing their fundamental concepts. This is further emphasized by their connection to Big Ideas, which integrate multiple mathematical concepts into a unified whole (Charles, 2005) such as comparison, quantification, equivalence and estimation. The content of teacher's specialized knowledge in the scope of measurement and division was a recent focus of research (Policastro & Ribeiro, 2023, 2021) resulting in the following descriptions.

Table 1: Examples of descriptions of the teacher’s specialized knowledge content included in KoT

Categories included in Knowledge of Topics (KoT)	<i>Definitions</i>	KoTd1 (measurement): Knowing the definition of measurement: comparison between magnitudes of the same quantity, followed by the quantification of one of them (the quantity to be measured) in terms of the other (unit of measurement). KoTd2(measurement): Knowing the definition of unit of measurement: a magnitude of a quantity with which another magnitude of the same quantity can be measured.
	<i>Properties</i>	KoTpp1 (measurement): Knowing that the magnitude of any quantity (or unit of this quantity) is necessarily expressed by a continuous amount.
	<i>Foundations</i>	KoTff1 (measurement): Knowing that the constructs "compare", "iterate", "accumulate", and "quantify" are fundamental to the activity of measuring any quantity with standardized or non-standardized units of that quantity. KoTff2 (measurement): Knowing that the unit of measure and the whole to be measured must be of the same nature, that is, they are expressed by magnitudes of the same quantity.
	<i>Phenomenology and applications</i>	KoTph1(measurement): Knowing that performing a measurement is related to comparing magnitudes of the same quantity, in terms of a quantification - one of the magnitudes in relation to the other. KoTph2 (division): Knowing the division as phenomenon through its interpretation as a measurement: dividing is comparing the magnitudes of the dividend and divisor, in order to quantify the number of times the magnitude of the divisor needs to be grouped to compose the magnitude of the dividend.
	<i>Procedures</i>	KoTmp1 (measurement): Knowing that having a unique unit of measurement is a necessary condition to perform a measurement, even if that unit is the result of a composition of units (unitizing). KoTmp2 (measurement): Knowing that comparison is a necessary but not sufficient condition for measuring.

The knowledge packages

Liping Ma (1999) introduced the concept of "knowledge packages" to describe the connections teachers make between various topics while teaching, finding that some teachers were aware of these connections and could articulate their components, while others were not. Additionally, the structure

of the knowledge packages varied accordingly with the teacher's level of conceptual or procedural understanding.

"Knowledge packages" refer, therefore, to groups of interrelated mathematical knowledge on topics consciously invoked by teachers based on their understanding of related ideas and procedures. To better explore a teacher's knowledge and its connections, it is important to identify mathematical concepts, principles, and properties related to each topic and how they interrelate. This work reconciles Ma's (1999) original idea with a theoretical perspective that focuses on the specifics of teacher's knowledge (Carrillo et al., 2018), leading to the view of "specialized knowledge packages" as relationships between components of a teacher's specialized knowledge.

Context and methodology

This study is part of a broader research that aimed to explore, describe, and detail the teacher's knowledge specificity in the topic of division and measurement. Primary and secondary teachers (21, in total) participated of six training sessions (8 hours/session) of a teacher training program in Brazil, aiming at developing teachers' knowledge on such topics. In this context, teachers' knowledge was explored and detailed (Policastro & Ribeiro, 2023, 2021) from their productions (oral and written statements) for a set of teacher training tasks (1 related to division, and 4 related to measurement) designed according to MTSK conceptualization. The data considered for the present study encompasses the set of descriptions of the teacher's specialized knowledge content obtained from the aforementioned research.

For the analysis, two sets of descriptors were consolidated and displayed in a single table (e.g., Table 1) to provide an overview of their content and facilitate a comparison of their features. The focus was first on descriptors that belonged to the same category. Through several reading sessions, the structural elements and unifying ideas in the descriptor's content were identified, allowing for a comparison of similarities and differences. This process resulted in a new level of synthesis outlining the structural elements and the unifying ideas related to the topics of division and measurement.

Table 2: Examples of structural elements and unifying ideas observed in the teacher's specialized knowledge content, related to the topics of division and measurement

	<i>Division</i>	<i>Measurement</i>	<i>Unifying ideas</i>
<i>Structural elements</i>	Dividend	Whole to be measured	DISTRIBUTION; COMPARISON; QUANTIFICATION;
	Divisor	Unit of measurement	
	Quotient	Numeric value of measurement	
	-----	Quantity	COMPARISON; QUANTIFICATION;
	Discrete quantity	-----	QUANTIFICATION
	Continuos quantity	Magnitude	QUANTIFICATION; CONTINUITY
	Algorithm	Iteration	GENERALIZATION

It is noteworthy that the structural elements in question encompass constructs, concepts, principles, attributes, methods, and depictions that shape and organize the content of mathematical topics.

Meanwhile, unifying ideas play a key role in mathematical comprehension, serving as the glue that ties together concepts and processes.

From this synthesis it was possible to map the relationships between the content descriptors. The first level of examination considered the relationships within each mathematical topic and afterwards between topics. The relationships were identified using two criteria: common structural elements and corresponding unifying ideas. This led to a set of knowledge descriptions for both intra- and inter-conceptual connections. The connection descriptions were numbered and labeled as "Intra n (topic)" or "Inter n (topic1-topic2)". Here the relationships are presented in diagrams, each one associated with a specialized knowledge package.

Results and discussion

In the context of measurement, we identified that the constructs "whole be measured", "unit of measurement", "numeric value of measurement", "quantity", "magnitude", and "iteration" are structuring elements of the content of knowledge related to: i) what is measuring - the phenomenon; and ii) definitions of measurement and unit of measurement (Berka, 1983); iii) the foundations of the measuring activity (Clements & Stephan, 2004); iii) properties associated with the measurement topics (Barrett et al., 2011); iv) the procedures and characteristics of the results of a measurement (Berka, 1983; Clements & Stephan, 2004). The ideas of "comparison", "quantification", "numeric relationship", "groupings", "equivalence", and "proportionality" are also highlighted as structural and unifying elements of the content of knowledge associated with the topics of measurement.

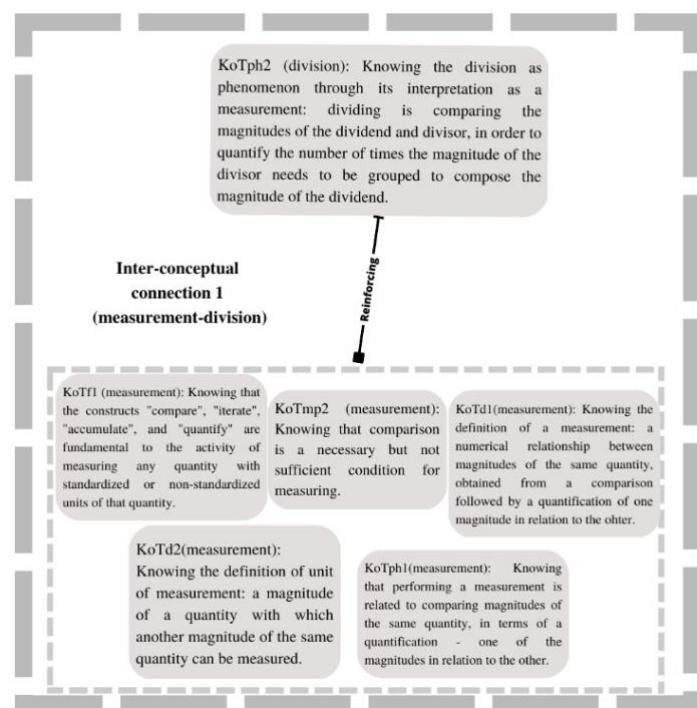


Figure 1: Specialized knowledge package related to the intra-conceptual connections on measurement

For example, understanding the act of measuring as a phenomenon is foundational for understanding that measurement is defined as a numerical relationship obtained through the comparison and quantification of magnitudes of the same quantity. Defining measurement as a numerical relationship between magnitudes captures the essence of the concept of measurement (Mariotti & Fischbein, 1997) through the ideas of numerical relationship, comparison, and quantification. However, this essence of the concept of measurement becomes meaningful if supported by the "sense of measurement" (Szilagyi et al., 2013) – (Intra-conceptual connection 1).

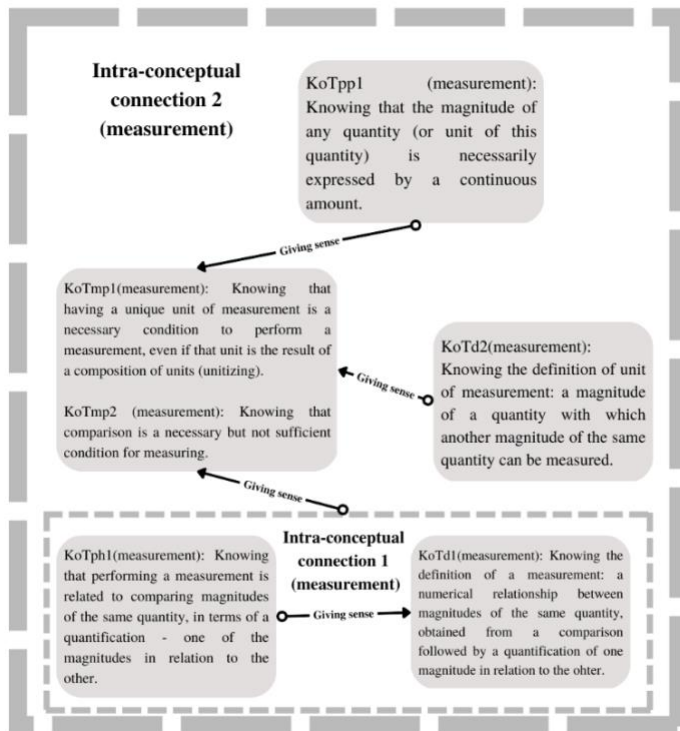


Figure 2: Specialized knowledge package related to the inter-conceptual connection between measurement and division

Besides, given that the Procedures category encompasses not only the teacher's knowledge of "what" and "how" to do something, but also the "why" and "when" of using certain procedures, it's important to note that knowledge of the concept of measuring, including how to define a measurement (Intra1), the definition of a unit of measurement (KoTd2), and the nature of magnitudes as continuous quantities (KoTpp1), is crucial for attributing meaning to knowledge about the necessary and sufficient conditions for conducting a measurement (KoTdp1 and KoTdp2) – (Intra-conceptual connection 2).

Understanding division as a measurement operation involves recognizing the dividend and divisor as representing "the whole to be measured" and "unit of measurement," respectively, expressed through continuous quantities (Gao et al., 2000). This interpretation, associated with KoTph2 (division), requires knowledge related to measurement concepts (KoTph1), definitions of measurement and units (KoTd1 and KoTd2 - measurement), and conditions for measurement (KoTdp2). Teachers must establish this connection to avoid confusion between division and measurement, as they are distinct despite sharing underlying concepts like "comparison," "quantification," "iteration," "accumulation," and "numeric relationship."

As we aimed to explore the content of teachers' knowledge within these descriptors and the connections observed, we were always interested in seeking the relationships between the structural elements and unifying ideas present in this knowledge. From this process, new descriptions related to the intra- and inter-conceptual connections emerged.

Table 3: Synthesis of descriptions of teacher's specialized knowledge of connections, related to division and measurement

Intra-conceptual connections	<p>Intra1 (measurement): Knowing that measuring is a phenomenon that involves comparing magnitudes of the same quantity in terms of quantifying one of them based on the other, and that this phenomenon contributes to assigning meaning to the concept of measurement, which refers to the numerical relationship obtained from this comparison followed by quantifying magnitudes of the same quantity.</p> <p>Intra2 (measurement): Knowing that to carry out a measurement, it is essential to have a unit of measurement, which can be composed of other units (unitizing), since the unit of measurement, like any magnitude, is expressed by a continuous quantity. The unit of measurement should be compared to the whole being measured in terms of quantification to obtain a numerical relationship between the magnitudes of both.</p>
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Inter- conceptual connection	Inter1 (measurement-division): Knowing that dividing is not measuring, but this phenomenon is supported by the knowledge of the "measuring" phenomenon, when it is associated with knowledge of the definitions of measurement, unit of measurement, and that measuring is not just "comparing", but involves other constructs such as "iterating", "quantifying", "accumulating" to obtain a "numerical relationship" between magnitudes.
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The descriptions related to intra-conceptual connections on measurement show how the teacher's knowledge can be structured and organized to shape and give meaning to the measuring phenomenon and the roles of the measuring unit, its multiples, and submultiples. In the case of the inter-conceptual connection, it is visible the role that structural elements and unifying ideas play not only in supporting the knowledge of division as a measure, but also in establishing a well-defined boundary between what is dividing and what is measuring.

Final comments

Adopting the MTSK conceptualization and a special focus on the mathematical connections lead to the deepening the understanding of the knowledge packages and the role of each one of the knowledge elements. The mathematical connections are perceived as the result of the relationships between structural elements and unifying ideas within a teacher's specialized knowledge. A further step to follow concerns theorize these connections, demanding identifying the structural elements and unifying ideas of the topics leading to a deeper comprehensive description of teacher's knowledge. Also, a further step refers to understand the dynamic perspective of teachers' mathematical knowledge highlighting the relationships between different specialized knowledge packages making connections visible.

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