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Investigating the relation between primary school teachers' beliefs, self-efficacy, and knowledge: The case of fraction multiplication and division

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There is a mutual relationship between teachers' beliefs and how they teach because beliefs can both cause and result in teaching practices. The purpose of this study was to explore teachers' beliefs about learning, teaching and self-efficacy about multiplication and division of fractions. The subjects of the study were 83 pre – and in – service teachers, who answered a digital questionnaire. To do so, we adopted well-accepted theoretical frameworks that define aspects of teachers' beliefs. The results showed that teachers shared an investigative and student-centered orientation and believed that multiplication and division of fractions should adopt a conceptual and procedural-fluency approach. Further, regression analysis showed that specific aspects of beliefs are predictive factors of self-efficacy and teachers' knowledge, while self-efficacy and knowledge were negatively correlated.

Keywords: Beliefs, self-efficacy, content knowledge, fractions.

Introduction

Research findings demonstrate that teachers' views influence their instruction, just as their sentiments about mathematics, as students influence how they instruct mathematics (Philipp, R.A., 2007). It is noteworthy that teachers' beliefs have a reciprocal relationship with teaching practices, since research showed that they can be both the causes and the results of teaching practices. Teachers' thoughts and experiences about the nature, teaching, and learning of mathematics determine their decisions regarding how to tackle a challenging math topic, such as fraction operations. Teachers admit that fractions are one of the trickiest concepts to teach. This is either due to their inability to conduct fraction operations skillfully alone or to a lack of conceptual comprehension and procedural performance (Thompson & Saldanha, 2003). The purpose of this study is to explore primary school pre-service and in-service mathematics teachers' beliefs, self-efficacy, and knowledge, with a focus on multiplication and division of fractions.

Theoretical Background

Teachers' beliefs have a guiding role in the teaching of mathematics, as their views have an impact on the choices they make and the actions they take during teaching. A well-accepted belief model regarding the nature, teaching, and learning of mathematics was proposed by Ernest (1989). Ernest's model presents three aspects of teachers' belief system. The first one is beliefs for the content, which consist of the nature of the domain, the significance of the domain, the structure of the domain in relation to other domains, effort and time. The second one refers to beliefs for learning, which refers to how knowledge is constructed, and the third one is beliefs for teaching, which refers to activities.

In the present study, to concretize beliefs about the nature of the concept we utilize Kilpatrick's (2001) model. Based on it, mathematical proficiency consists of five intertwining strands: (a) conceptual

understanding of mathematical concepts, operations, and relationships; (b) procedural fluency-ability to perform procedures accurately; (c) problem solving—formulating, representing, and solving problems; (d) reasoning—logical thinking, reflection, justification, and explanation; and (e) productive disposition—seeing mathematics as useful and logical.

Teachers' Beliefs about Teaching and Learning Mathematics

Teachers' beliefs about the teaching and learning of mathematics refer to their conception on how mathematics should be taught, how mathematics should be learned, what actions and mental processes should be involved in learning mathematics, as well as what activities are appropriate for learning mathematics (Ernest, 1989). Teachers' beliefs about teaching and learning mathematics are extremely important as research findings assert that teachers who have negative beliefs about teaching mathematics cannot strengthen their learning mathematical environment (Charalambous, 2015).

Teachers' beliefs regarding how to teach and learn mathematics are categorized into two groups. The teacher-centered view (or traditional teaching), suggesting that students passively receive knowledge from teachers, and mathematics teaching is considered as a process of knowledge transfer. The student-centered view (or constructivist teaching) that adopts teaching methods based on students' active participation in knowledge construction (König et al., 2014).

Teachers' Self-Efficacy

Self-efficacy involves peoples' expectations of their own skills than about their actual abilities. Similar views are presented by other researchers, who argue that self-efficacy refers to an individual's beliefs about their ability to perform mathematics-related tasks and to teach in a way that promotes mathematics learning (Charalambous, et al., 2008). Teacher's self-efficacy might affect the learning objectives he/she sets, the teaching tactics he/she employs, and the way he/she responds to unexpected events (Sowder, 2007). As a result, teachers who have higher self-efficacy put more effort and persistence into teaching their students, place a greater emphasis on academics in the classroom, and provides multiple types of feedback (Chang, 2015).

Teachers' Knowledge

Teachers' knowledge is a promising component in the effectiveness of their instruction and students' learning (Charalambous, 2015). According to Kilpatrick et al. (2001), teacher knowledge goes beyond just subject-matter expertise. In essence, instructors' knowledge can be divided into three categories: general pedagogical knowledge, subject knowledge in mathematics, and content knowledge, which are considered crucial for effective mathematics instruction (König et al., 2014). Content knowledge in mathematics includes both conceptual understanding of the structure and organization of mathematics as an integrated whole and the conceptual knowledge of the fundamentals of the thematic areas (Blömeke & Delaney, 2012).

The Present Study

Purpose and Research Questions

The purpose of this study was to investigate teachers' beliefs regarding the content, the learning, and teaching of fraction multiplication and division, and the relation of their beliefs with their self-

efficacy and content knowledge. The research questions are: (a) What are teachers' beliefs about learning and teaching multiplication and division of fractions and (b) Do teachers' beliefs about learning and teaching multiplication and division of fractions predict their self-efficacy and content knowledge of multiplication and division of fractions?

Participants – Procedure

The sample consists of 83 teachers (16 men and 67 women; 65 pre-service and 18 in-service). Additionally, 65 individuals stated that they have no experience, 9 individuals stated that they have 1-5 years of experience, 2 individuals stated that they have 6-10 years of experience, 3 individuals that they have 11-15 years of experience, and 4 individuals stated that they have more than 15 years of experience in teaching Mathematics.

The data was collected by means of an online questionnaire administered to a sample of pre-service primary school university teachers at the University of Cyprus, as well as to in-service teachers. The questionnaire was sent by e-mail, by the research team, explaining the purpose of the research and that their participation is voluntary.

Measures – Data Analysis

The questionnaire used consists of four parts. The first part was developed based on Ernest's model (1989). It consisted of 42 multiple-choice questions to measure teachers' beliefs about mathematics content (31 questions), mathematics learning (7 questions) and mathematics teaching (4 questions). The items used are based on Ernest's model for measuring teachers' opinions on the significance of the domain, effort and time as well as the connection with other domains. Teachers' beliefs about the content of mathematics were measured by items referring to the development of conceptual and procedural understanding, the ability to formulate and solve problems, the promotion of explanation of students' thinking, and how important they think the topic of multiplication is and dividing fractions. The aspect of productive disposition was measured with the items about the domain of significance. Table 1 presents examples from the different parts of the questionnaire. The high score for conceptual understanding equals knowledge rich in connections and for procedural fluency equates to knowledge characterized by a variety of methods and flexibility in their use. Also, if teachers' beliefs score about problem solving is high that means that teachers consider problem solving as a multidimensional process. In addition, if teachers' beliefs about reasoning had high score, it means that they consider that it involves reasoning processes, assessing the reasonableness of the answer, and searching for relationships. For significance of the domain, if teachers got a high score equals to the importance teachers give to the thematic unit. At the same time, the teachers also answered questions regarding how they connect the students' pre-existing knowledge with the underlying thematic unit, as well as the time and effort put in. The high score about connections equates to connecting fractions to other concepts and procedures such as problem solving and decimals and if teachers got a high score to their beliefs about time and effort equals to using more teaching time to teach multiplication and division of fractions. We also used items referring to the way they approach teaching of multiplication and division of fractions, and the materials they use. High score in teachers' beliefs about teaching involves utilization of materials, representations, problems, connections with everyday life, and for learning relates to student-led learning.

Table 1: Questionnaire Item Examples

Part 1 - Beliefs about mathematics content, teaching and learning (Likert Scale from 1 (strongly disagree) to 7 (strongly agree))	
Conceptual Understanding	Students should know that the math sentences $\frac{1}{2} \times \frac{1}{3}$ and $\frac{1}{3} \times \frac{1}{2}$ correspond to different problems.
Procedural Fluency	To multiply fractions, it is enough to know a quick calculation rule (numerator over numerator and denominator over denominator)
Problem Solving	To solve problems involving multiplication and division of fractions it is important to be able to construct one's own problems based on mathematical propositions.
Connections	Teaching multiplication and division of fractions is easier than addition and subtraction because it is not connected to other concepts such as conversion to identical fractions.
Reasoning	In addition to knowing the rules for performing calculations with fractions, it is important for students to explain the process they follow.
Significance	Multiplying and dividing fractions involves knowledge of rules, procedures and skills that are important to everyday life.
Time and Effort	Solving problems with fractions takes much more time and practice than solving problems with whole numbers.
Teaching	In teaching multiplication and division of fractions, the teacher must use appropriate materials, applications, and representations.
Learning	Knowledge of multiplying and dividing fractions is built through investigation of appropriate problem situations
Part 2 – Self-efficacy items (Likert Scale)	
Self-efficacy beliefs	I know how to teach multiplication and division of fractions effectively. Even if I try hard, I won't teach multiplication and division of fractions as well as the rest of the concepts.
Part 3 – Content Knowledge	
Teachers' knowledge	I have $\frac{3}{9}$ of a Kg of raisins and will divide them fairly to make 3 identical cakes. What part of Kg of raisins will each cake have? a. $3 \div \frac{3}{9}$ b. $\frac{1}{6}$ c. $\frac{3}{9} \div 3$ d. $\frac{6}{9}$

The second part of the questionnaire was an adaptation of the Mathematics Teaching Efficacy Belief Instrument (MTEBI) to measure teachers' self-efficacy. The items of this instrument were modified to refer to fraction multiplication and division and consisted of 21 Likert scale questions statements. Half of the items referred to teachers' feelings and how comfortable they feel when teaching multiplication and division of fractions. The rest of the items described teaching methods. High score indicates that teachers feel comfortable teaching multiplication and division of fractions. The third part of the questionnaire consisted of 10 multiple-choice questions to measure teachers' content knowledge of multiplication and division of fractions. The questions were one step and two step fraction problems with multiplicative structure. Scoring was based on correct and incorrect answers. The last part of the questionnaire examined background information of the participants (gender, age, status, years of teaching experience). As described above, most of the items were based on the models of Ernest (1989), Kilpatrick (2001) and the Mathematics Teaching Efficacy Belief Instrument (MTEBI). We developed content-specific items to measure teachers' beliefs regarding multiplication

and division of fractions. To strengthen the validity of our measurement tool, we computed the content-validity index of the individual items (Taherdoost, 2016). We asked three experts to examine the content validity of the questionnaire items and evaluate the extent to which the developed items aligned with the adopted models. We kept only items with a content-validity index of at least 0.75. Further, the questionnaire was trialed in a pilot study with a sample of 20 pre-service teachers. The results of the pilot study provided feedback to make additional modifications.

For the analysis of the data, we used the statistical package SPSS. We calculated the mean value for each dimension of the beliefs system based on the items used. To answer the first question, we used descriptive statistics. To address the second question, we ran two parallel multiple regression analyses, having self-efficacy and content knowledge as dependent variables. Teachers' beliefs and their status (pre-service or in-service) served as independent variables. Finally, we used Pearson Correlation Analysis to determine the relationship teachers' self-efficacy and knowledge.

Results

We calculated the mean value and the standard deviation of pre- and in-service teachers for every aspect of their beliefs. The measurement scale of each statement ranged from 1 to 7. Thus, 4 was set as a reference point. The mean value of pre-service teachers in beliefs about teaching was 4.73, indicating that they believe that teaching of multiplication and division of fractions should incorporate materials and a variety of representations. The corresponding value for in-service was 4.33. In respect to teachers' beliefs about learning, the mean of pre-service teachers was 4.90 and of in-service 4.81, indicating that teachers believe that the learning of multiplication and division of fractions should be student-centered. The standard deviation for teaching and learning was around 0.50 for both pre and in service teachers. In respect to conceptual knowledge, the mean value of pre – service was 5.64 and 5.68 for in - service, which suggests that they argue that knowledge should be rich in connections. Similar results were also presented for procedural fluency, where it appeared that teachers believe that knowledge should be characterized by a variety of methods and flexibility in their use, as the value of pre - service was 4.57 and 4.01 for in - service. The mean value for problem solving was 5.88 for pre-service and 5.62 for in - service, indicating that they consider problem solving as a multidimensional process. The mean value for reasoning was 5.35 for pre-service and 5.03 for in-service. In respect to teachers' beliefs regarding the significance of the subject, the mean value of pre-service teachers was 3.54, while the mean value for the in-service teachers was 3.33, indicating that they think that multiplication and division of fractions in not so important for primary school. This is shown by the fact that their averages were marginally below the general average. Both, pre-service teachers (\bar{x} = 4.54) and in-service teachers (\bar{x} = 4.32) support the connection of fractions with other concepts. Finally, teachers' beliefs about time and effort were high, with pre-service having a mean value of 5.50 and in-service 4.72. Standard deviation of the categories of beliefs about the content were around 1 for both groups.

Table 2: Means and Standard Deviation for factors.

	Pre - Service		In - Service	
	Mean	SD	Mean	SD
Beliefs about Teaching	4.73	(0.61)	4.33	(0.47)

Beliefs about Learning	4.90	(0.79)	4.81	(0.46)
Beliefs about the Content				
- Nature of the domain				
- Conceptual Understanding	5.64	(0.90)	5.68	(0.80)
- Procedural Fluency	4.57	(0.83)	4.01	(1.02)
- Problem Solving	5.88	(0.88)	5.62	(0.80)
- Reasoning	5.35	(0.68)	5.03	(0.75)
- Significance of the domain	3.54	(0.99)	3.33	(0.80)
- Structure of the domain	4.54	(0.66)	4.32	(0.63)
- Time and Effort	5.50	(1.10)	4.72	(1.14)

Table 3: Hierarchical Regression Analysis for the Variables Interpreting Teachers' Self-Efficacy and Knowledge

Independent Variances	Self - Efficacy		Teachers' Knowledge	
	B (SE)	Beta	B (SE)	Beta
Constant	1.674 (0.548)		0.624 (0.241)	
Status	-0.182 (0.132)	-0.137	0.157 (0.058)	0.284**
Beliefs about Teaching	0.116 (0.110)	0.126	-0.083 (0.048)	-0.220
Beliefs about Learning	0.185 (0.089)	0.241*	0.073 (0.039)	0.230
Beliefs about the Content				
- Nature of the domain				
- Conceptual Understanding	0.029 (0.78)	0.046	0.056 (0.34)	0.213
- Procedural Fluency	0.018 (0.067)	0.029	-0.061 (0.029)	-0.238*
- Problem Solving	-0.136 (0.084)	-0.211	.0029 (0.037)	0.107
- Reasoning	0.257 (0.101)	0.323**	-0.047 (0.044)	-0.143
- Significance of the domain	0.156 (0.064)	0.268*	-0.008 (.028)	-0.034
- Structure of the domain	-0.030 (0.104)	-0.035	-0.039 (.045)	-0.111
- Time and Effort	0.042 (0.057)	0.088	0.021 (.025)	0.106

Notes: $R^2 = .43$ (self-efficacy), $R^2 = .60$ (teachers' knowledge), * $p < .05$, ** $p < .01$, *** $p < .001$

To answer the second research question, two multiple regressions were conducted, where in one the dependent variable was teachers' self-efficacy and in the other teachers' knowledge. In both models, teachers' beliefs about learning, teaching, content and teachers' status served as independent variables. Table 3 presents multiple regression analysis coefficients and the percentage of variance explained. The independent variables entered the model of teachers' self-efficacy explained 43% of the variance. Beliefs about learning, significance of the domain and reasoning were predictive factors of teachers' self-efficacy. The regression coefficients were $r = 0.241$ for teacher beliefs about learning, $r = 0.323$ for reasoning and $r = 0.268$ for significance of the domain. Regression analysis for teachers' knowledge showed that the independent variables entered in the model explained 60% of the variance. Teachers' beliefs about procedural fluency and status had the greatest contribution, with the

first variable equating to 0.238 standard deviation increase in content knowledge and the second entailing 0.284 standard deviation increase over the dependent variable. The importance of the status of teachers reveals that their content knowledge is affected by whether they teach in schools, since they must read and prepare properly for their teaching. Finally, we examined the relation between teachers' self-efficacy knowledge. It was found that there is a statistically significant negative correlation between these two factors ($r = -.309, p < .01$).

Conclusion

Previous research showed that teachers' beliefs about their self-efficacy, the way they teach, and the way students learn affect their mathematics performance. In the present study we investigated teachers' beliefs for multiplication, and division of fractions, based on well-established theoretical frameworks. Further, we investigated which aspects of teachers' beliefs affect their self-efficacy and knowledge. Analysis showed that teachers believe that multiplication and division of fractions learning should be student-centered, and that teaching should utilize materials and representations that facilitate conceptual understanding. However, analysis showed that in-service and pre-service teachers believe that this topic should get less importance in schools. This finding is interesting and should be examined in respect to their knowledge and self-efficacy about fractions.

Regression analysis was used to examine whether teachers' beliefs about teaching, learning, and the content of multiplication and division of fractions can be predictors of teachers' knowledge of fractions and their self-efficacy beliefs. Analysis showed that teachers' self-efficacy can be predicted by their beliefs about learning, reasoning and the importance given to teaching multiplication and division of fractions. In particular, their beliefs about reasoning made the greatest contribution to the model predicting teachers' beliefs about their self-efficacy in teaching multiplication and division of fractions. Therefore, emphasis should be focused on teachers' opinions about the importance of the topic and the reasoning processes involved in multiplication and division of fractions. Teachers feel more comfortable to teach this topic if they think that it is an important one. This finding is consistent with other research results that showed that teaching beliefs are linked to self-efficacy (Peker & Ulu, 2018). Analysis also showed that the status of a teacher (pre or in service), as well as their beliefs about procedural fluency are predictive factors of teacher's knowledge. This finding agrees with other research, which have established the interaction of teachers' beliefs and their knowledge (Charalambous, 2015). The findings indicate that lower levels of knowledge are related with higher value for procedural fluency in multiplying and dividing fractions. This finding shows low level of knowledge might be explained teachers' belief that knowledge of procedures is important to multiply and divide fractions. Finally, we found a negative correlation between teachers' self-efficacy and teachers' knowledge. This result leads us to the conclusion that the lower the teachers think their abilities are in teaching multiplication and division of fractions, the more aware they are of the difficulty of the specific topic. Thus, these teachers tend to study and prepare more for teaching multiplication and division of fractions, which explains their knowledge levels, which have been shown to be quite high.

The above results should be interpreted having in mind the limitations of the research. Further analysis is needed to validate the measurement tools used in the study, in a larger population. In

addition, the sample was small and not representative, as undergraduates comprised most of the sample relative to postgraduates and graduates. Thus, the results obtained cannot be generalized to the wider population, as they need further research to be confirmed.

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