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► To cite this version:

Maria Mellone, Arne Jakobsen, Miguel Ribeiro, Alessio Parlati. Ethical dimension in the use of interpretative tasks in mathematics teacher education: fraction division. Thirteenth Congress of the European Society for Research in Mathematics Education (CERME13), Alfréd Rényi Institute of Mathematics; Eötvös Loránd University of Budapest, Jul 2023, Budapest, Hungary. hal-04407634

HAL Id: hal-04407634

<https://hal.science/hal-04407634>

Submitted on 20 Jan 2024

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Ethical dimension in the use of interpretative tasks in mathematics teacher education: fraction division

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We introduce a novel approach aimed at enhancing secondary school prospective teachers (PTs) specialized mathematical knowledge for teaching fraction division. The approach is firmly rooted in the principle of fostering a communitarian ethics environment. Through our research we show that the implementation of a specific interpretative task among PTs not only nurtures their interpretative knowledge (IK) but also fosters growth of a communitarian ethics environment. Our methodology is characterized by a structured alternation between small groups and a larger collective group of PTs. Here we present findings that show the development of IK within one of the small groups of PTs.

Keywords: Ethical dimension, interpretative knowledge, mathematics teacher education, tasks.

Introduction

Recently the obvious as well as dazzling reflection that all educational practices implicitly or explicitly incorporate an ethical dimension produces challenges and dilemmas also in mathematics education (Radford, 2021). In this perspective, we propose a study aiming to enhance a particular form of specialized knowledge in PTs, that is Interpretative Knowledge – IK (e.g., Mellone et al., 2020), also fostering the perspective of community ethics and inclusion.

Ethical dimension in educational practice

Many researchers—in particular in the field of Critical Mathematics Education—have studied and reflected on the ethical dimension underlying mathematical education practices (e.g., Radford, 2021). In these studies, ethics is considered as a discipline that not only expresses and analyses judgment criteria relating to one's own and other's behaviors, but also more generally as the set of norms and values imposed by the institution one belongs to, for example, the school. With this as a starting point, regardless of the pedagogical model of reference, the teaching and learning process involves the relationships between people and is inevitably an ethical event. This can take place in different forms, e.g., in classroom interaction in the relationships of power and subjection, in the relationships of authority and obedience, or solidarity relations (Radford, 2021). Therefore, human beings are continuously influenced by the contexts in which they act—they are interdependent in the ways they come to know and feel about each other. Radford (2021) reflects on the ethics embedded in the theory of objectification, according to which learning is a bodily, material, and socially active process, through which students encounter historically mathematical forms of thought that are culturally determined. The theory of objectification also allows for studying the desirable ways in which the activity of joint labour can take place. In this joint labour, students and teachers work hand in hand to produce something together. The sense of working together—from the simple interaction with other people—makes joint labour a social activity in which learning takes place through a truly

collective process. In this view, the teacher is dethroned from the traditional role of mediator, assistant, or coach that sociocultural theories usually reserve for him. This patriarchal role is replaced with the vision of a teacher who works with students by struggling, suffering, and finding joy with their students. Radford (2021) calls the ethical dimension proposed by the theory of objectification for *communitarian ethics*, based on responsibility, commitment, and care. Communitarian ethics guide the pedagogical acts in the classroom so that teachers and students can explore new critical learning spaces together and promotes commitment, inclusion, discussion, and respect between people (Radford, 2021).

Teacher education: the interpretative approach

For this study, we refer to the model of the Mathematics Teacher Specialized Knowledge (MTSK, Carrillo et al., 2018) that considers all the mathematical knowledge for teaching as specialized and, places teachers' beliefs in the core of the model and we focus on Knowledge of the Structure of mathematics (KSM) and Knowledge of Practices in Mathematics (KPM). As part of an ongoing international cooperation research on mathematics teachers' education, the first three authors developed the theoretical construct of Interpretative Knowledge – IK (e.g., Mellone et al., 2020, Ribeiro et al., 2016). The term IK refers to a deep and broad mathematical knowledge that allows teachers to consider the potential of students' productions, exploiting their ambiguities, intuitions, differences, and errors (etc.) and using them in the classroom as an opportunity for learning. Indeed, although the research has shown that the opportunity of making mistakes and learning from them is an important aspect of the teaching and learning of mathematics (e.g., Borasi, 1996), still a great part of mathematics teaching practice in school is mostly focused on students' correct performance of procedural exercises. While the importance of making mistakes, learning from them, and exploring different strategies in problem-solving activities are crucial aspects in developing a more effective teaching practice, it is also important from an inclusive perspective.

To develop PTs' IK, a particular type of tasks is used (Interpretative Tasks), in which PTs are asked to interpret some students' mathematical productions to design possible feedback and suitable classroom work centered on these productions (Mellone et al., 2020). Over the years a particular methodological approach called the Small group–Collective–Small group (SCS) cycle (Jakobsen et al., 2022) was tuned to develop PT's IK. Using the SCS-cycle, the PTs first work on the interpretive task in small groups (from two to four), afterwards the task is discussed in large collective mathematical discussion orchestrated by the educator and, finally, the small groups return to work on the same interpretative task after around one/two months. In the SCS-cycle methodology the discussion by the educator, is crucial, conceived as a generator of new specialized mathematical knowledge for all PTs and the educator him/herself (e.g., Jakobsen et al., 2022). Therefore, albeit with different objectives, PTs and teacher educators should both recognize an authentically transformative experience in their practice, and research on IK has also explicitly aimed at investigating this aspect (e.g., Ribeiro et al., 2016).

The task used in this study deal with fraction division. Often the educational approaches to this mathematical topic are unbalanced and focus only on syntax and procedures: "Traditionally, fraction division has often been taught by emphasizing the algorithmic procedure "invert and multiply"

without considering students' ways of constructing division of fraction knowledge" (Lee, 2012, p. 46). Teachers rarely understand the conceptual underpinnings fraction division and then it is difficult to provide situations in which students can bring forward various contexts of meaning that would help them to build conceptual knowledge on this topic. In particular, it is difficult for teachers to link the fraction division algorithm to the meanings typically associated with division: the quotative division (how much or how many groups of the size of the divisor are contained in the dividend) and the partitive division (sharing the dividend into groups of the size of the divisor, how much or how many one people or one thing gets) (e.g., Fischbein et al., 1985). In this direction, we have involved PTs in work to develop their specialized knowledge about the mathematical argument behind the procedure "invert and multiply" (KSM), together with possible meaningful contexts in which the procedure can be interpreted (KPM). Moreover, in our perspective, this work is linked with the ethical issues introduced earlier because training teachers to be able to justify syntactically and semantically this particular algorithm (as other algorithms) means preparing them to have a more inclusive educational approach. In other words, we are referring to an inclusive pedagogy horizon, also declined for the mathematics education field, that in its broader definition (e.g., Göransson and Nilholm, 2014), is linked with school systems' and communities' commitment and capacity to accommodate all students. We are referring for example to the students who are not willing to execute algorithms without have the opportunity to reflect on them, as well as those considered to have a specific learning disorder, ensuring all students participation and effective learning processes (Asenova et al., 2023). In this sense, our approach is conceived as a context for developing a communitarian ethics environment in school.

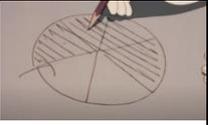
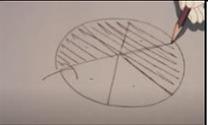
Methodology

The context of the study is the "Elementary Mathematics from a Higher Standpoint" course held in the Spring of 2022 as a part of a Master's Degree in Mathematics at an Italian University. The course consisted of 24 classes of two hours, spread over three months and deals with different topics such as the formal mathematical constructions of number sets, specific literature articles on mathematics education that investigate some school topics of mathematics in the light of the historical evolution of mathematics, different according to the different cultural contexts, and its current disciplinary structure. The students attending this course had completed a Bachelor's Degree in Mathematics or Physics. It is a non-compulsory course, but it is typically chosen by students who intend to become secondary teachers, due to which all participants are considered to be PTs. The sample of the study is composed of the 27 PTs who attended this course which was held in a hybrid way, due to local regulations related to the COVID-19 pandemic. The online teaching was recorded. During the collective part of the SCS cycle seven students attended remotely.

The interpretative task used in this study is inspired by a scene from a Japanese animated film entitled "Only Yesterday" (Original title おもひでぼろぼろ, written and directed by Takahata in 1991). At the end of one of the first classes of the course, a 4-minute excerpt from this movie was presented to the PTs. In this excerpt, a fifth grader, named Taeko, returns home with a poor grade on a math test regarding fraction division. The mother then asks the older sister to help her. The sister, looking at Taeko's mistakes and score on the math test and thinking she is alone with the mother, asks worried and loud: "Isn't it that Taeko has some mental problems?", while Taeko listens to this statement. From

our ethical perspective, this part of the excerpt is particularly important as it helps the PTs to empathize with the state of marginalization Taeko can feel, contributing for a change in PTs' beliefs. After this scene, there is an interaction between Taeko and the sister in which, considering the operation $\frac{2}{3}$ divided by $\frac{1}{4}$, the meaning of fractions division is discussed. In the sequel a table containing some of the images of the cartoon excerpt and with the dialogue between Taeko and his sister concerning the division (Table 1).

Table 1: Sequences of screenshots of the cartoon excerpt presented in the interpretative task

				
<p>Taeko: What does it mean to divide a fraction by a fraction? Two-thirds of an apple [she draws an apple in three equal parts and colours two of them]. To divide it by one-fourth, it would mean to share two-thirds of an apple among four, and look how much each one gets, isn't it? [She divides the two-coloured parts representing $\frac{2}{3}$ into four parts and she divides the uncoloured third into two parts, dividing each third into two parts]. One, two, three, four, five, six, each one takes one-sixth.</p> <p>Taeko's sister: Wrong, wrong, wrong this is to multiply.</p> <p>Taeko: How is it possible? How can multiplying make the quantity decrease?</p> <p>Taeko's sister: Dividing $\frac{2}{3}$ of an apple by $\frac{1}{4}$ means [she is unable to continue]. However, it is because you want to use the apple that you don't understand: multiplication works linearly, while for the division you have to invert, just remember this!</p>				

After watching this excerpt, the educator asked the PTs to work at home in the small group on the following questions: *How can you assist Taeko find meaning to fractions division Taeko suggested?*

Following the SCS-cycle methodology (Jakobsen et al., 2022), the educator asked the PTs to work on this question in small groups of three (nine PTs groups, and the choice of the composition of the groups was left to the PTs) and to prepare some slides to share their work with the rest of the PTs during the following class. The following class was dedicated to the collective mathematical discussion orchestrated by the educator about the several different interpretations of the episode proposed by the PTs. At the end of the course, about two months from the initial small group work, the teacher educator asked the small groups of PTs, as part of the discussion for the final evaluation, to write an email to Taeko that could help her to develop an understanding of the fraction division starting from her example, completing in this way the SCS cycle.

In the sequel, we present a case study by focusing on one of the groups of students, group 1, composed of Manuela, Vittorio, and Maura (pseudonyms). Using a phenomenological approach (Marton & Pang, 2008), we trace the development of their IK by specifying the progressive enrichment of sub-domains of the MTSK model and the emergence of their ethical sensitiveness.

Data analysis

During the beginning of the class devoted to the collective mathematical discussion, each group uses a short presentation to share their work. All groups presented the mathematical structure reasons behind the procedure "invert and multiply" (KSM sub-domain), declaring that they had never thought of them before. Moreover, all groups presented examples about the quotative division, and all the groups, except Group 1, refer to the division between $\frac{2}{3}$ and $\frac{1}{4}$. In the next transcript excerpt of the collective discussion transcript, Vittorio presents the work of Group 1 using a PowerPoint presentation slide (in Figure 1 the slide they used):

- Vittorio: Let's consider the number line and suppose we want to make $\frac{8}{3}$ divided by $\frac{1}{3}$. This is like asking how many one-third long jumps I need to make to get to eight-thirds from zero, that is, eight jumps (he presents the slide in Figure 1).
- Educator: In your opinion, this meaning context is referring to a quotative or a partitive division?
- Vittorio: Quotative division! It's like asking how many thirds are contained in eight-thirds. This example would also help us to answer Taeko's objection: here, I divide but the number increases.
- Educator: The quantity number does not increase. In a multiplicative relationship, the numbers refer to different physical quantities. They are not comparable since eight-thirds represent "things", one-third represent "things at a time" and eight are the "times".
- Vittorio: Right. However, I can only explain through a quotative division context and not by a partitive one because in that case, it is as if the apples were increasing!

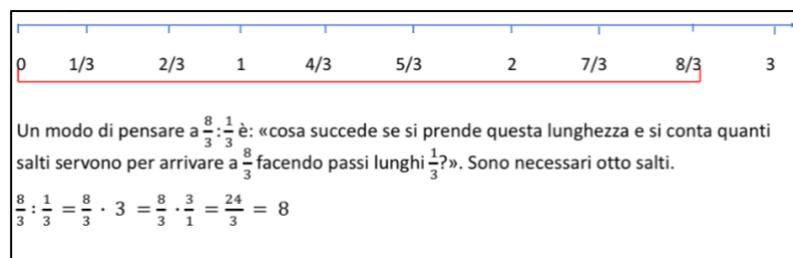


Figure 1: A way of thinking of $\frac{8}{3} : \frac{1}{3}$ is: <<What's happened if one considers this length and counts how many jumps that are needed to arrive at $\frac{8}{3}$ jumping steps of length $\frac{1}{3}$?>>. 8 jumps are needed

In this first excerpt, we can see how Vittorio's group shows an example illustrating $\frac{8}{3} : \frac{1}{3}$ relating to the quotative meaning of division (KPM sub-domain). It is interesting how Vittorio's group decides to refer to a different division in comparison to the one discussed by Taeko and her sister, showing the desire to find a meaningful context independent of the particular numbers in play. Furthermore, Vittorio clarifies how this meaningful context can also assist Taeko in dispelling doubt about whether multiplying the quantity decreases it and dividing increases it. Afterward, the educator raises the issue concerning the dimensions of the magnitudes involved in a multiplicative relationship as a tool for reflecting on the irrelevance concerning the increase or decrease of quantities. Due to space constraints, we are unable to provide detailed report of the discussion transcripts. However, it is worth noticing that all the semantic examples presented for fraction division revolved around quotative division. Only one group, distinct from Group 1, attempted to interpret it as partitive division, but their explanation was somewhat brief, leaving their interpretation somewhat unclear. Furthermore, the discussion was notable divers in terms of the various contexts in which the examples were

proposed by the different PTs groups. The discussion also included ethical reflections about the experience of “mathematical marginalization” of students who seeks to understand the rationale behind algorithms.

Two days following the collective group discussion and one hour before the next class, Maura, a member of Group 1, emailed the educator a slide prepared by their group. In this slide (Figure 2) they present an interpretation using a quotative division context for $2/3$ by $1/4$.

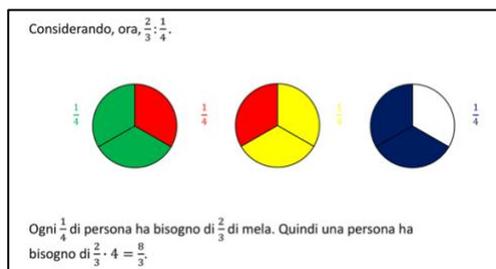


Figure 2: If we consider $2/3 : 1/4$ [...] Each $1/4$ of a person needs $2/3$ of an apple. Then a person needs $2/3 \cdot 4 = 8/3$

At the beginning of the next class, the educator provides a synthesis of the discussion also showing the new interpretation of Group 1 (Figure 2) and linking it to intuitions that came from other groups during the discussion.

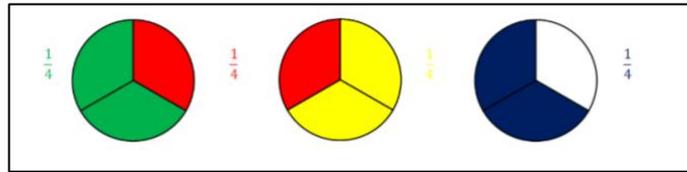
Next, we examine the output from Group 1 during the final phase of the SMS cycle, which took place approximately two months after the preceding phases. During this phase, the PTs collaborated in a small group to compose an email to Taeko, aimed at assisting her in understanding the concept fraction division based on own example. Below, we present selected excerpts from the email crafted by the group:

Dear Taeko,

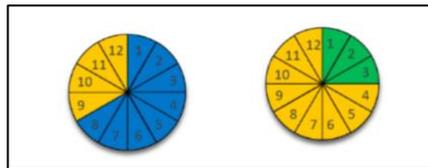
We heard you had some difficulty with fraction division. Don't worry, that is normal. Mistakes are not only inevitable, but they are also a key part of your learning process. It is precisely from your mistake that our exploration started, which allowed us to refine a knowledge we already thought we possessed. You don't resign yourself to doing something you don't understand, not because you can't do it.

Syntactically, everyone knows that to divide two fractions, the first fraction must be multiplied by the inverse of the second [...]. We clarify the algebraic meaning of the division of fractions. When you execute the division, in particular between fractions, you are searching the number that multiplies for the divisor giving you as a result the dividend. Coming back to your specific numbers $(2/3) \div (1/4)$, we are looking for the number m/n that is $(m/n) \cdot (1/4) = 2/3$, and multiplying both members for 4 we obtain $m/n = (2/3) \cdot 4$.

In the context of partitive division, and referring to your exercise, we interpret the division by using the labels “things”, “times” and “things per times”. We have $2/3$ of a thing every $1/4$ of the time. So, for re-building the entire “time”, we have to compute $(2/3) \cdot 4 = 8/3$.



While in the context of quotative division by content we ask ourselves how many quarters of an apple are there in two-thirds of an apple. We decide to subdivide this surface of the circle into twelfths, which corresponds precisely to the least common multiple between 3 and 4. So let's rewrite $2/3$ as $(2/3) \cdot (4/4) = 8/12$ and $1/4$ as $(1/4) \cdot (3/3) = 3/12$. We are wondering, therefore, how many times the $3/12$ is contained in the $8/12$. They are exactly contained: 2 times + $2/3$ of time = $8/3$ of time.



We understand your frustration of not being able to fully understand the meaning of these "rules", which at times seem imposed from above, almost as if they were written on divine tablets [...]. We appreciate your way of thinking and view of mathematics and not passively accepting what is proposed to you. We leave you with a quote from Hans Freudenthal: "Mathematics is the courage to always ask why".

In their approach, Group 1 initially offers a rationale the “invert and multiply” procedure of fractions division, grounded in the algebraic perspective that view division as a search for a number such that multiplied by the divisor, it equals the dividend (KSM sub-domain). For the semantic interpretation, they made a deliberate choice to first introduce the contextual understanding of partitive division. This understanding was developed two days after the second phase of the SCS-cycle and centred on more general concepts such as “things”, “times” and “things per times” (KPM sub-domain). Notably, when addressing quotative division, Group 1 opted not to present the same contextual understanding as in the previous two phases. Instead, they chose to reference the same numbers ($2/3$ and $1/4$) and context meaning (the apple), used by Taeko (KPM sub-domain). Finally, it’s worth highlighting the careful selection of words by Group 1 in their communication with Taeko. This choice reflects a sensitivity that appears to resonate with the communitarian ethics described by Radford (2021).

Conclusion

The educational challenge in addressing ethics is not rooted in enforcing specific behavioural norms; rather, it revolves around creating classroom environment conducive to the emergence of new forms of relationships. In this direction, the development of teachers IK becomes an indispensable prerequisite for fostering inclusive teaching practices. This study has revealed the potential to simultaneously cultivate the PTs IK and their ethical sensitiveness. Indeed, the implementation of “Taeko interpretative task” has created the conditions for Group 1 to engage in a collective learning, as exemplified in their final communication with Taeko. They recognized that it was from Taeko’s mistake, that their exploratory journey commenced “It is precisely from your mistake that our exploration started, which allowed us to refine a knowledge we already thought we possessed”.

Implementing the “Taeko interpretative task” empowered this PTs group to construct appropriate contexts that bridge two interpretations of division within the KPM sub-domain, concurrently fostering a structural understanding of the algorithm within the KSM sub-domain. We borrow the beautiful quotation from Group 1 email: “Mathematics is the courage to always ask “why”, which is the first step in working in a perspective of communitarian ethics and inclusion.

Acknowledgement: This study was partially financed by CNPq (404959/2021-0) and PID2021-122180OB-100 from Spanish Science and Innovation Ministry, and RED-MTSK supported by AUIP.

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