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# Two pre-service mathematics teachers' beliefs about mathematics and its teaching and their lesson plans for linear function topic

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*One of the Intellectual outputs of the FunThink Erasmus+ project is a course for pre-service teachers. In this context, we conducted the study focused on understanding the beliefs of pre-service teachers in relation to the lesson plans they developed before and after the course. Their beliefs were elicited through a semi-structured interview and analysed using the analytical tool developed by Carrillo and Contreras. Our results showed that their beliefs about mathematics and its teaching were strongly related to their lesson plans. Especially with those created after the course when knowledge about mathematics teaching and learning was expanded. Although the course was based on the inquiry approach to teaching mathematics, their lesson plans emphasised the spontaneous or technological tendency. Therefore, further research and adjustments in the course are needed so that the pre-service teachers' beliefs are considered and addressed.*

*Keywords: Teachers' beliefs, function, lesson plans.*

## Introduction

Our nascent research community (in the sense of our university) has been collaboratively investigating in-service and pre-service teacher knowledge for a few years. To some extent, we have admitted - by moving from Deborah Ball's (et al., 2008) MKT model to José Carrillo's (et al., 2018) MTSK model - that we consider a belief system to be a valid part of "it". According to Depaepe et al. (2013), the MKT model has been criticised for several reasons, one of which is that MKT ignores teachers' beliefs about mathematics and its teaching, which deeply influence teachers' classroom practice. The MTSK model includes teachers' beliefs, which are placed at the centre of the model to emphasise the reciprocity between teachers' beliefs and their knowledge (Carrillo et al., 2018). However, in our own research, we have left this issue untouched. In a way, this is a copy of CERME's practice, where we deal with the popularity of TWG20 by splitting it into two subgroups: one on knowledge and the other on beliefs and identity. Our recent experience - described in this paper - has shown us the limits of this simplification and the great need to study beliefs and knowledge together. For this occasion, we will only describe the research on the beliefs of pre-service teachers (PSTs). However, in a broader perspective, we aim for more complex connections, similar to what Vasco and Climent (2017) did. They linked teachers' knowledge, as described by the MTSK model, to the beliefs of two university lecturers in linear algebra. In doing so, they take some steps towards understanding "how and why teachers do what they do in the classroom".

On the one hand, we approach this paper from our perspective as educators of mathematics teachers. We have designed our study to inform the development of the teacher training course described below. On the other hand, there is a researcher's perspective that needs to enrich the understanding of pre-service teachers' knowledge growth and learning. Both views will be discussed at the end of this paper, where we will return to the link between knowledge and beliefs. But first, let us introduce

the theory that is important for understanding mathematics teachers' beliefs about mathematics and mathematics teaching.

## **Mathematics teachers' beliefs**

To study the pre-service teachers' beliefs, we drew on the classifications in Carrillo and Contreras who provided the system of categories and descriptors that as they wrote, "make a more detailed characterization of the teacher's beliefs of mathematics and its teaching easier" (1994, p. 152). This choice of scheme is conditioned by the (future) use of the MTSK model. The authors propose separate tendencies for beliefs about mathematics and for beliefs about mathematics teaching. The scope of the paper does not allow us to present all the details, so we describe each tendency in a simplified way.

To identify teachers' *beliefs about mathematics*, they proposed three tendencies:

- (1) Instrumentalist – mathematics as a set of useful algorithms, which are not a matter of discussion. Its aims are mostly external.
- (2) Platonist – mathematics as a "perfect" and beautiful world of ideas, which is logically structured, it is possible to discover it, not to create it. It does not need to be applied externally to be relevant (internal aims).
- (3) Problem solving – mathematics as a human creation that is developed by solving problems and through social interaction. The aim of mathematical knowledge is intellectual development.

The descriptors are organized into three categories which are: type of knowledge, aims of mathematical knowledge, and means of development of mathematics.

For the identification of the teachers' *beliefs about mathematics teaching*, there are four tendencies:

- (1) Traditional – teaching mathematics as a transfer of teacher's knowledge into students' minds using repetitive methods. Students accept the information provided by a teacher.
- (2) Technological – teaching mathematics with a strong emphasis on its applicability to different areas of life. Students reproduce and imitate what a teacher shows and explains to them.
- (3) Spontaneous – teaching mathematics as experimentation free from prescribed results. Students do things promoted by a teacher.
- (4) Investigative – teaching mathematics as planned research. Students – guided by their teacher – reflect and pose questions.

For these tendencies, authors established the following six categories: methodology, subject significance, learning conception, student's role, teacher's role, and assessment.

In the next section, we will explain the methodology that helped us to answer the following research question: What are the beliefs of PSTs about mathematics and mathematics teaching, and how do they relate to their lesson plans?

## **Context and methods**

This study was conducted within the FunThink Erasmus+ project (Enhancing functional thinking from primary to upper secondary school). The ongoing project focuses on the development of functional thinking. This is defined as thinking in relations, dependencies, and changes; it involves

noticing, generalising, and abstracting relations between co-varying quantities, representing these relations through language, symbols, and graphs, and using generalised representations in problem-solving situations (Pittalis et al., 2020). One of the Intellectual outputs of the project is the course for pre-service teachers. Its different modules were tested in different project countries. The reflective course took place at the Pavol Jozef Šafárik University in Košice, Slovakia, in the Fall of 2022. This course was compulsory, consisted of 26 teaching hours, was attended by 14 students, and was placed in the first semester of their master's studies. Therefore, it was the first course of the PSTs whose content focused on the *didactics* of mathematics, as the goals of the course were mainly related to pedagogical content knowledge. The PSTs experienced different learning environments based on the principles of inquiry, tool use, embodiment and situatedness, observed and reflected on classroom recordings, etc. The course was strongly based on investigative beliefs about mathematics education. They also attended the other compulsory course, which focused on general pedagogy and *didactics*. During this course, they carried out classroom observations. They were also instructed to design a lesson plan but in a slightly different way to our course. This course was not under the authors' control. We mention it to provide a context for some of the findings.

All PSTs submitted two lesson plans – one developed at the beginning of the course (pre- lesson plan) and the other developed afterwards (post- lesson plan). Both were limited to the topic of a linear function definition, which is a topic for grade 9 in Slovakia. The second one was part of the final assessment. The lesson plans were also collected as data for our research. These lesson plans were not used in the classroom. We would also like to highlight the fact that the students had no previous teaching experience. Furthermore, four PSTs were interviewed in the middle of the semester. They were selected for this procedure based on their pre- lesson plans. In the paper the authors analysed the data collected from two PSTs because they showed different tendencies – let us call them Janko and Klára. Klára studies mathematics and geography, she chose to study mathematics because of her own interest in it and because of the great future opportunities to find a job as a mathematics teacher. After her studies, she would like to work at a lower secondary school. Janko studies mathematics and physics, he also chose to study mathematics because of his deep interest and perceived success in it. He used to take part in mathematical competitions and was active in a community of young mathematicians. Unlike Klára, he would like to teach at the upper secondary level and – if possible – work with talented students. The data were collected with the consent of the participants. They were informed about the scientific and developmental purpose of the research.

The semi-structured interview transcripts were subjected to content analysis (Krippendorff, 1989). An analytical tool developed by Carrillo and Contreras (1994) was used for deductive category application. Firstly, one of the authors, with the help of atlas.ti software marked units of analysis – a sentence or a few sentences which she understood as a belief statement. Subsequently, both authors coded data separately, using the codes based on the analytical tool. To each statement, one or more codes could be assigned. Afterward, their coding was synchronized including extending, deleting, or merging the belief statements.

The lesson plans were analysed by both authors considering the response to the specific questions in the interview. For example, if the PST clarified during the interview how they would proceed with a

particular task and it was not directly written in the lesson plan, it was taken into account. The authors assigned one or more tendencies to each of the analysed lesson plans.

## Results

Tables 1 and 2 provide an overview of the tendencies that were manifested in Janko and Klára's pre-lesson plan, interview, and post-lesson plan. The lesson plan sections are not about identifying the beliefs of the PSTs, but about the visible approach to teaching mathematics (MT). This is because we cannot clearly distinguish between beliefs and knowledge. In the interview section, we present tendencies characterizing/describing PSTs' beliefs about mathematics (BaM) and their beliefs about mathematics teaching (BaMT). For each tendency, the number of code frequencies that occurred in individual interviews is indicated. The total number was 40 in both cases. As the number of code frequencies increases, the colour of the cell intensifies.

### Janko

Janko's pre-lesson plan started with the definition of a linear function, followed by three tasks and two activities. In the first task, students had to express the equation of the linear function ( $y = ax + b$ ) from the given definition. The second task was to construct a graph of the function from the equation. The third task was to construct the formula of a linear function, given two functional values. This was followed by the first activity in which the students were asked to play with the prepared GeoGebra applet. They could experiment with the coefficients in the equation of the linear function and observe the changes in the graph. The last activity concerned the construction of graphs of non-linear functions. There was no connection to real life in this lesson plan. Janko prepared a lesson for talented students.

**Table 1: Coding results for Janko**

Pre- Lesson plan			
MT:	Spontaneous		Traditional
Interview			
	Motivated students		Unmotivated students
BaM:	Platonist (2)	Problem solving (2)	
			Instrumentalist (1)
BaMT:	Spontaneous (15)	Traditional (2)	Technological (9)
			Investigative (9)
Post- Lesson plan			
MT:	Spontaneous		Traditional

As we can see in Table 1, the categories Spontaneous and Traditional were assigned to his pre-lesson plan, with Spontaneous dominating. The reasons for this were as follows:

- the definition of a linear function is dictated by the teacher as a fact (MT – traditional – teacher's role),
- in individual activities, it is not always clear what and how exactly the students should do, the teacher encourages their activity (MT – spontaneous – methodology, teacher's role),
- the students are expected to be interested, to play with the activities (MT – spontaneous – student's role),
- emphasis on adapting to students, and their needs, supporting discussion (MT – spontaneous – teacher's role).

His post- lesson plan barely differed in content from the pre- lesson plan. There was one new task for determining functional values using the equation. The change was in the order of the tasks and activities as well as in the formal aspect of the lesson plan. More explanatory notes were added to this lesson plan, including notes on how to proceed with more or less able classes.

Regarding Janko's beliefs, we see that he shows different tendencies when talking about motivated students ("those who want to", "those who are good at it") and unmotivated students ("they have other interests"). In the case of motivated students, we see a clear tendency towards spontaneous teaching of mathematics:

- 1 Janko: ... but to do it together somehow. Not in a way that the students would just sit there, and I would do everything. I would ask them. "OK. What does this parameter do? Make suggestions! We move this or we move that. Try to say how it changes. Let's show it! Now try to suggest - what do we want to get there? What do we need to change here?" I don't know, I call it playing.

(MT – Spontaneous – teacher's role = to promote, and – student's role = to play)

We see that Janko wants to convey to this group of students what is more mathematical, and thus more abstract, more structured - in accordance with Platonist beliefs about mathematics.

- 2 Janko: This is for a class that already has that interest ... they are hungry for it. So, it's a bit more mathematical. That there was a definition first and then we had three tasks and some activities.

Among students who are not motivated, we see Janko's tendency towards Technological and Investigative beliefs.

- 3 Researcher: And if they were less motivated, would you rearrange it?  
3 Janko: Probably, I would start with the activities, first the inquiry. Then somewhere in the middle of the lesson, I would define it and recalculate it. Hey, maybe one task would be missed. So, more time would be spent on this motivation. To increase their interest.

(MT– Investigative – methodology)

- 4 Researcher: And how would you, for example, show them why (*mathematics is not useless*)?  
4 Janko: Maybe through some of those applications. Also, to show him that when he is OK with it, when he solves maths problems, he can also do more difficult tasks, not just what he did in class. It will give him the necessary skills he needs to go on.

Moreover, Janko showed a tendency towards Traditional mathematics teaching regarding the role of the curriculum, which is given from the outside and must be followed.

- 5 Researcher: And after a month of these different lessons, would you go back to them? (*formulas*)  
5 Janko: I probably wouldn't go back completely, but it's necessary to include them (*formulas*) ... it's probably in the standards, that's what the students are supposed to know.

## **Klára**

Klára's pre- lesson plan started with a short frontal revision of the concept of function. This was followed by the first task where the students had to decide which of the 6 functions listed was a linear function. Then the definition of a linear function was given, and 4 further tasks were suggested. The

second task was to find the y-intercept. The third task was to draw the graph of the linear function using the points of intersection and check whether some given points lie on the line. The fourth task was to find the domain of the function from its equation. The last task was to construct the appropriate graph for the equation of the linear function. There was no connection to real life in this preparation.

**Table 2: Coding results for Klára**

Pre- Lesson plan		
MT:	Traditional	
Interview		
BaM:	Instrumentalist (3)	
BaMT:	Technological (21)	Traditional (8)
		Investigative (8)
Post- Lesson plan		
MT:	Traditional	Technological

As we can see in Table 2, her pre- lesson plan was clearly assigned to the Traditional category, for the following reasons:

- the definition of a linear function is dictated by the teacher as a fact (MT – traditional – teacher’s role),
- aligning with textbooks with the motive to keep the required standards (MT – traditional – methodology),
- school mathematics = formal mathematics (MT – traditional – subject significance).

Her post- lesson plan was different from the pre- lesson plan. She started with two tasks with a real context (car speed, heating/cooling of water). She described these tasks as motivating. Only after solving these tasks, she suggested the definition of a linear function and assigned the first and third tasks from the pre- lesson plan.

As for her beliefs, we can see that the Technological tendency dominates in the beliefs revealed during the interview. She often emphasised that the connection between mathematics and real life gives meaning to mathematical knowledge.

1 Klára: [reaction to the pre- lesson plan] I really liked how we worked on the learning environments with Veronika (*the lesson plan designed within the FunThink project*). And I hadn’t thought of it that way before. For example, to show them in real life. I just opened some grade 9 textbooks and I looked at the examples there. How they actually define it there, because I wanted to do it accordingly. ... I used them with the sole purpose of finding the tasks and then trying to put those tasks in some sort of order. It didn’t occur to me to explain what it’s for, let’s show how it actually works. I think it is a much better way to start, for example, with functions like giving some tasks right away.

(MT – Technological – methodology = practical objectives, and – teacher’s role = to explains, and – subject significance = applicability)

Here we see Klara’s self-reflection and her subsequent shift from the Traditional methodology to a more Technological one, which is more in line with her expressed beliefs about mathematics.

2 Klára: Well, in my opinion, mathematics is used by absolutely everyone in their everyday life: when we work with money or when we go to bake a cake and

we need to measure flour, milk and yoghurt, so we have to know some of these things. So, I think the point of mathematics is to teach them, like everyone else in the world, the basics so that they can just count and somehow live their lives.

(Mathematics – Instrumentalist – aims of mathematical knowledge).

The Traditional and Investigative tendencies are equally represented, while the Spontaneous tendency did not manifest itself in the data obtained. This presence of two significantly different tendencies allowed us to observe a certain inconsistency.

- 3 Klára: I like it when teachers go through a lot of tasks with us in class. When they give a basic definition or something like how it's done, or they give some examples to explain it, and they go through all kinds of tasks that they want us to do. Because you also have teachers who give you, for example, three examples and you think about it, and I don't know what.

(MT – Traditional – teacher's and student's role)

Here the need for a traditional approach to teaching is manifested - the teacher as a transmitter of information, the student as a passive receiver.

- 4 Klára: And then maybe the more difficult stuff. The stuff that they don't seem to be able to use in life, ... maybe just because they are interested, they just want to explore, they want to come there, they want to know. I think so.

(MT – Investigative – student's role)

## **Discussion and implications for future research**

Returning to our research question – *What are the beliefs of PSTs about mathematics and mathematics teaching, and how do they relate to their lesson plans?* – we can see that two PSTs, whose beliefs are the subject of this paper, held different tendencies of beliefs about mathematics and mathematics teaching. Klára's pre- lesson plan was not designed according to her beliefs. Due to a lack of knowledge about possible applications of linear functions or different ways of organising the lesson (see 1 Klára), she proposed to teach this topic in a traditional way. However, she was obviously inclined towards technological mathematics teaching (see Table 2). Therefore, when her knowledge increased during the course, she used it to adjust her lesson plan to be "the best" according to her belief tendency. Janko's pre- lesson plan was done in a way that copied his beliefs about a good lesson. He decided, as clarified during the interview, to design the lesson plan for "talented" students. As can be seen in Table 1, for this group of students he believes that the spontaneous approach (with some traditional elements) is the most motivating (see 2 Janko). Thus, the changes were made only in the structure and formal requirements (received in the course of general pedagogy and didactics).

As the course was designed to support an investigative approach to teaching mathematics, one might be disappointed by the lack of investigative approach in the post- lesson plans. However, after the beliefs revealed in the interviews, one should not be surprised. The study of beliefs is sometimes dismissed because of the lack of proven impact on a person's behaviour. In the case of Janko and Klára there is a clear connection between their beliefs and their post- lesson plans. Of course, designing a lesson plan is not the same as teaching. This close relationship could possibly be explained by the Theory of Planned Behaviour (e.g., Bosnjak et al., 2020). We can theorise that the



lesson plans describe the PSTs' *intentions* to teach in a particular way rather than the actual *behaviour* of teaching in that way. Therefore, the link between beliefs and lesson plans is more direct than between beliefs and teaching. To support this explanation, we need to adjust the design of our next study.

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