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Design of a “framework-document” for training schoolteachers to teach mathematics

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In France, the lively question of training content has been brought back to the forefront by the arrival of a new recruitment examination and a new reform of pre- and in-service schoolteacher training. Since 2013, the institution has listed general skills that teachers should master to practice their profession, but it does not mention specific knowledge required to teach mathematics. In the absence of a sufficiently precise framework, the tricky issue of required knowledge is left to teacher trainers. Faced with this lack, a collective of trainers and researchers to which we belong, the COPIRELEM group, has developed a framework document for schoolteacher trainers in mathematics education. In this paper, we study the design process of this resource.

Keywords: Teacher training, professional development, primary education, knowledge base for teaching, curriculum design.

Introduction

In France, a new reform of pre- and in-service teachers training has been introduced during the year 2021, correlated from 2022 with a change in the format of the national schoolteacher recruitment examination. These institutional changes lead primary schoolteacher trainers to rethink content of training. For a decade now, a competency framework document (MEN, 2013) exists in France that teachers should master to carry out their profession. This document constitutes a benchmark for teacher training. It presents general professional skills which are grouped into five items: mastery of disciplinary knowledge and its didactics; mastery of the French language; constructing, implementing, and leading learning situations; organising and ensuring a group's mode of operation; evaluating pupils' progress and achievements. Even if it makes it possible to identify the major issues of training, this competency framework does not state specific issues involved in the teaching of a given discipline, in particular the mathematical knowledge for teaching required in the exercise of these skills. Regarding mathematics, Celi, Masselot and Tempier (2019) note the absence of a reference shared by trainers to define their assessment content in pre-service teacher training. They also point out that no framing constraints are imposed in terms of the mathematics-training program. This absence of a specific framework suggests a great diversity of content in the training offered to schoolteachers. The curriculum of this recruitment examination (MEN, 2021) yet corresponds to those of the primary school (6-11 years old), the middle school (11-15 years old) and the 'Numbers and calculations' part of the mathematics high school curriculum (15-16 years old): it is obviously not sufficient to enable trainers to identify all aspects of mathematical knowledge for teaching. Moreover, the French organisation of pre- and in-service teacher training involves trainers of various statuses who have different experiences and knowledge of teaching in primary schools and/or training

of primary schoolteachers: primary and secondary schoolteachers, mathematics education researchers... The absence of a curriculum may be an issue for schoolteacher trainers entering the profession, especially for the many mathematics teachers from secondary schools who have little knowledge of teaching mathematics in primary schools when they become trainers. The COPIRELEM group (which we belong), tried to fill the gap: this commission¹ which has acquired certain legitimacy among schoolteacher trainers over its fifty years of existence, offers opportunities for national dissemination of resources (Guille-Biel Winder & Tempier, 2017). The commission then embarked on the creation of a resource proposing benchmarks for the schoolteachers training in mathematics teaching. This resource aims to homogenise training content to have shared benchmarks in all French training centres, and to support trainers to identify training content. The design of this type of resource seems to be poorly documented. It raises questions about both content and form. In this paper, we rely on the work of Demeuse and Strauven (2006) to study the design of this resource. These researchers define the concept of *teaching or training curriculum* on the work of Taba (1981) and Roegiers (1997), as an institutional document corresponding to an action plan which aims to provide a “comprehensive, planned, structured and coherent vision of pedagogical guidelines for organising and managing learning in relation to expected outcomes” (Demeuse & Strauven, 2006, p. 11). The resource developed by the COPIRELEM group is not an institutional one (even if it has been submitted to the institution). Therefore, it cannot be considered as a curriculum in the Demeuse and Strauven’s sense. But it has some common features with such curriculum, which justify the use of the framework developed by these researchers as a basis for our analyses. The “framework-document” terminology is hence used instead of “program” or “curriculum” to avoid any misunderstanding about the non-institutional status of the resource. Demeuse and Strauven (2006) emphasise how important for analysis is the context of a curriculum development. They identify two types of operations in curriculum developing: the drafting work (content and form) leading to one (or more) draft(s) and the *a priori* evaluation of this (these) draft(s). According to this research, we identify four main analysis points: preliminary decisions, development, *a priori* evaluation and testing. In this paper, we focus on the first three. We first highlight the main choices that led to the resource design. We then return to its conception and carry out an analysis. At last, we present and analyse the development process in relation to the *a priori* evaluation (op. cit.).

Preliminary decisions

Contextual factors

Demeuse and Strauven (2006) identify five contextual factors that interact to influence the delivery of a curriculum: the general (political, socio-economic, and cultural) context, the needs of the actors, the operations of the institution, the normative framework and social reference practices. The introductory part of this paper highlighted interactions between the general context and the normative framework (new recruitment examination, reform of pre- and in-service teacher training), but also the needs of the actors (trainers, trainees, designers of examination subjects), which led to the

¹ The COPIRELEM is a commission dedicated to the education to the primary school. It is stemming from the network of IREM (French institute of research on mathematical education).

development of the framework-document. Its design also considers all interacting factors. Thus, the aim of the framework-document is to train schoolteachers to teach mathematics in school and not to prepare students to pass a competitive examination, which, in the case of the written test in mathematics, focuses on disciplinary knowledge. Moreover, the framework-document does not attempt to correspond to a set of specifications defining training methods (number of hours, type of courses, etc.) within the national institutes of training education, but it could be a support point for trainers' teams in defining training and assessment content. In the next part, we highlight social reference practices of the COPIRELEM group on schoolteacher training.

Choices and schoolteacher training design of the COPIRELEM group

The COPIRELEM group advocates for future schoolteachers training fully organised and oriented towards the purpose of teaching mathematics to school pupils: mathematical contents must hence be revisited, deepened, enriched, consolidated, and restructured with a view to their teaching and learning by pupils. This is, for trainee schoolteachers, a new way of learning mathematics, which can only be done in close connection with didactical, historical, epistemological, and psychological fields of knowledge. Mathematical knowledge to be mastered must enable schoolteachers to help pupils acquire the main elements of mathematics that will enable them to be autonomous in their daily lives, to develop rational thinking, to enter a common culture, to understand the mathematics at stake in the other school subjects, and to build tools for scientific understanding of the world. It is therefore a question of teachers mastering a sufficient mathematical background in relation to the requirements of primary school. This mathematical background must be structured and organised: it goes beyond the simple mastery of the level above the one taught, contrary to what the recruitment examination programmes might suggest (MEN, 2021). According to the COPIRELEM group, schoolteacher training in mathematics should aim: to change the image of mathematics (reassure about mathematics and restore the taste for problem solving); to revisit, to consolidate and to take a step back from mathematics to be taught; to become aware of the complexity of naturalised notions (such as whole numbers numeration); to precisely identify the mathematical concepts studied; to acquire a more accurate mathematical language; to develop capacity for argumentation and reasoning; and to lead students to change their posture from students to teachers. Moreover, schoolteacher training must enable students to acquire professional skills that can be linked to the reference framework of professional skills for teaching and education professions (MEN, 2013).

Framework-document design

Theoretical elements for knowledge identification

Research has long pointed to the existence of different types of knowledge for teaching. Shulman (1986) propose a seven categories taxonomy that contains three elements, specific to the “discipline knowledge” (subject matter knowledge, pedagogical content knowledge and curricular knowledge). From the study of teaching practices, Ball, Thames and Phelps (2008) evolve this model in order to identify knowledge for teaching. They precise the Shulman's specific elements and reconfigure them in two domains (Subject Matter Knowledge and Pedagogical Content Knowledge (PCK)) subdivided into sub-domains. From this one, several frameworks have been developed. We present here the MTSK model (Carrillo et al., 2013) that focuses on mathematics teacher's specific knowledge and

beliefs. The first domain, renamed Mathematical Knowledge (MK), contains beliefs on maths and three sub-domains: Knowledge of Topics (KoT) includes knowledge of mathematical concepts, procedures and theoretical foundations; Knowledge of the Structure of Mathematics (KSM) corresponds to knowledge of the discipline, of connections between concepts and their evolution; Knowledge of the Practice of Mathematics (KPM) is related to mathematician work (producing, reasoning, communicating, testing...). The second domain (PCK) contains beliefs on maths teaching and learning and includes the three sub-domains of Ball's model by extending their significance: Knowledge of Features of Learning Mathematics (KFLM) is related to knowledge of students along with mathematics learning theories or models; Knowledge of Mathematics Teaching (KMT) corresponds to knowledge about teaching constrained by mathematical content; Knowledge of Mathematics Learning Standards (KMLS) includes in addition to institutional expectations, "objectives and measures of performance developed by external bodies" (op. cit., p.2991) (professional associations and researchers especially). Alongside to these works, Houdement and Kuzniak (1996) propose a framework for organising mathematical training practices for first-level teachers, on which we relied in developing the framework-document. They identify three types of knowledge (mathematical, pedagogical and didactical) that we explain in connection with MTSK model. "Mathematical knowledge corresponds to mathematics that a teacher needs to know in order to prepare, regulate and evaluate his lesson and his students" and "includes and specifies the content knowledge" (Houdement, 2013, p.12). It corresponds to KoT, KSM and KMP together. Didactical knowledge is linked to the mathematical content and fed by research in the field of mathematics didactics (op. cit.). It corresponds to analysis of teaching and learning phenomenon and to propositions of engineering. It allows linking a mathematical knowledge analysis, a possible development of the pupil and a possible organisation of the study (Perrin-Glorian, 2010). Therefore, it can be associated with KFLM, KLT and KMLS together. Pedagogical knowledge is characterised as "knowledge of experience" (Portugais, 1995) and is less dependent of the mathematical content than other types of knowledge. It is related to teaching and learning conceptions and to the organisation and management of the class. We believe that it includes beliefs on maths teaching. Moreover, these different types of knowledge needed for teaching go beyond static accumulation, through a recomposition in and through action (Schön, 1983), they are enriched and regulated by the feedback of practice. They are articulated in the teaching-learning situations teachers must design and implement in their classroom. Recent works of COPIRELEM group has highlighted the need to take these articulations into account in training situations (Masselot et al., 2016; Guille-Biel Winder & Tempier, 2017).

Framework-document presentation

The framework-document is divided into several parts. The preamble presents a link between the reference system of professional skills of the teaching profession and the contents to be taught; it also exposes designers' choices and conceptions on the contents of the schoolteacher training. In a second part, the document organisation is set out, together with guidance on how to read it. The other parts reflect a division of the contents into different areas taken from the school curriculum or specific to the training: "General issues in mathematics education"; "Numbers and Calculation", "Magnitudes and Measurements", "Space and Geometry" (primary school curriculum); "Digital data organisation

and management” (middle school curriculum); “Logic and Algorithmic” (includes a cross-curricular part related to the reasoning and logic involved in mathematical work). The “General issues in mathematics education” part is cross-cutting from the other categories that refer to content: items are essentially related to PCK and some of them cross-cut KMT, KFLM and beliefs on maths teaching and learning (adaptation of mathematics teaching to different audiences and contexts). The training content in the other (mathematical) areas is presented in a three-column table (Figure 1). The first column exposes didactical knowledge issues in mathematics teaching (KFLM and KLT essentially). The second one is dedicated to mathematical knowledge necessary for this teaching (KoT, KSM and KMP). We have chosen to make also appear in the title “epistemological knowledge”, *i.e.* the knowledge of the genesis, development, and mutual enrichment of concepts (included in KSM), that can help to understand the construction of pupils’ knowledge (in relation with KLMF). The third column, entitled “comments”, provides insights on the contents of the other two columns. It sometimes includes examples. It refers both to mathematical and pedagogical content knowledge (beliefs on mathematics and on maths teaching and learning included) that can be intertwined. The knowledge presentation follows, when possible, a chronology consistent with that to be followed in the teaching of mathematical concepts (in relation with KMT). Where themes are linked, the links are specified (sometimes the same knowledge is stated in several places in the document) (in relation with KSM).

Magnitudes and Measurements		
Didactic issues in the teaching of school mathematics	Mathematical and epistemological knowledge	Comments
<p>Basic sizes and their measurements</p> <p>• Understand that construction of the meaning of each size studied at school is based on classification and comparison activities (direct and then indirect) that define relations: "to have the same ... as ...", "to be more ... than ...", "to be less ... than ...". Implement activities that involve transitivity of relations (equivalence or order relation).</p> <p>• Understanding magnitude* and measurement* concepts (equivalence relation* ; additivity ; order relation* ; transitivity*) ; ordered magnitudes* ; measurable magnitudes*.</p> <p>Differences (and relationships) between physical or geometrical objects, magnitudes (equivalence classes of objects) and numbers (measures of magnitudes) must be highlighted : - a magnitude is the characteristic of an object ; several magnitudes may be related to a same object ;</p>		

Figure 1: Extract from the framework document (Eysseric *et al.*, 2022, p. 31)²

At the end of the presentation of each area, bibliographical resources are proposed. They are divided into four categories: “training situations” (involving the linking of different types of knowledge), “theoretical references” (research work), “resources for the classroom” proposing situations with *a priori* analysis (linked to the development of KMT and KFLM) and “institutional resources” (in relation with KMLS).

Framework-document analysis

According to Demeuse and Strauven (2006), a curriculum is articulated around three closely interacting poles: the targeted objectives (pole 1), the didactic processes and strategies implemented to achieve them (pole 2), and the situations that make it possible to assess their degree of mastery (pole 3). The framework-document is a benchmark listing the “essential knowledge” to be mastered in order to teach primary school mathematics (part 1) and providing guidelines for implementation in teacher training (part 2). The resource development did consider the general normative factor (the

² In the document, an asterisk is used to mark the knowledge that does not require too much theoretical development.

competency framework), but the framework-document does not provide a path for evaluation (absence of part 3). In connection with pole 1, Demeuse and Strauven (2006) distinguish three possible entry modes into a curriculum: by subject content (codified or formalised presentation of knowledge to be taught); by pedagogical objectives (“learning intentions formulated in terms of observable and measurable learner behaviours” (op. cit., p.65)); and by skills (which are constructed “through situations that are meaningful to the learner” (id., p.77)). The framework-document presents an entry by skills about the didactic issues in the teaching of school mathematics (Figure 1), closely linked to the mathematical contents that structures the different areas of the resource. Didactic processes and strategies are not highlighted in the framework-document: they only underlie references to training situations (particularly those developed by the COPIRELEM group), or to classroom situations, mentioned in the bibliography.

Development process and *a priori* evaluation

In order to ensure the framework-document relevance and “good reception” by trainers we made the choice of evolving the document until a consensus was reached within the trainers’ community. Various groups of professionals involved in schoolteacher training took part in writing the document. A first document (*version 0α*) has been written in the first quarter of 2021 by a small group of six COPIRELEM members, building on the work produced by the COPIRELEM group over many years. It was presented to the other members of the commission (14 members) and *a priori* analysed. This *internal evaluation* (Demeuse & Strauven, 2006) led to a revision of the document and resulted in a new version (*version 0β*). This document was then submitted to some thirty trainers participating in a conference organised by the COPIRELEM group (June 2021) to analysing and enriching it (Petitfour et al., 2022). These trainers had a wide range experience, various statuses and came from 12 different training institutes in France (out of 32). They were asked to note items that seemed superfluous to them, to indicate essential items that seemed to be missing, to note items for which they had encountered different interpretations or questions regarding their formulation. Each group, in charge of a part of the document, filled in a grid on these different points, adding comments and arguments. A group discussion highlighted the importance of the area “Knowledge that is transversal to the teaching of the different mathematical domains”, which makes it possible not to break down this knowledge into each of the themes in the different mathematical domains. It also reinforced the designers' view of the importance for such a framework-document to meet the needs of trainers. Based on this first *external evaluation* (Demeuse & Strauven, 2006), adjustments were made: some pieces have been completed, clarifications made, items reworded to remove interpretation ambiguities. Version 1 of the framework-document was then proposed. A further step was the testing of the document by these trainers in the development of their schoolteacher training programme in mathematics education. Feedback was used to produce a version 2 (March 2022), which was put online. New feedback led to a version 3 (March 2023)³ available to all trainers.

³ <https://www.copirelem.fr/ressources/pour-la-formation/>

Conclusion and perspectives

The visibility of knowledge for training in mathematics teaching and the need for harmonisation between trainers' practices are important and topical issues in France (they may not arise in the same way in other countries where training contexts are different). Based on orientations defended by the COPIRELEM group, the document design was guided by the Demeuse and Strauven's framework (2006): for example, the COPIRELEM group has sought to link mathematical and didactical knowledge in the document, just as it aims to link them in training situations (Guille-Biel-Winder & Tempier 2017). This research also allowed us to question the framework-document design. In this paper we do not address testing matters (Demeuse & Strauven, 2006). Studying the use of the framework-document by trainers in different training centres is a perspective of our research. We have already been able to perceive the interest shown by trainers during training sessions. Based on the criteria for evaluating the resource ergonomics (Tricot et al., 2003), we could continue this work according to different types of questions related to the uses of this resource. The study of its *usefulness* would make it possible to question the document's capacity to enable trainers to identify training issues and to implement a temporal organisation of training content. Looking at the *acceptability* of the document would lead us to question the way in which trainers “welcome” the document, while for *adaptability* it would involve analysing how trainers were able to adapt it to their local constraints. Finally, for its *usability*, it would be a question of questioning the capacity of trainers to use the resource, in terms of both content and form, in their training activities. Furthermore, it is important that this document does not remain fixed once and for all. It should continue to evolve, according to training developments and advances in mathematics education research.

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