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How arguments, in students interaction, might open up for reflecting on mathematics in action at primary school level

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The aim of this paper is to investigate how taking a perspective of critical mathematics education addressing a critical situation, such as climate change, in a mathematics classroom, enable reflection on mathematics in action. By reconstructing arguments, created in a group discussion at grade 6, imagine what they might open up for as part of a dialogic process, three possibilities were identified: (1) open up for experiencing two different perspectives; (2) open up for reflection on how mathematics can have an impact on how we perceive a situation; and, (3) open up for investigating the importance of recognizing what data is in use. The findings indicate that reflecting on mathematics in action is possible in a mathematics classroom at primary school level.

Keywords: Mathematics in action, arguments, dialogue, primary education.

Introduction

Mathematics plays a role in describing, predicting, and communicating critical situations such as the climate change, thus have an impact on how we perceive a situation and also how we act upon them (Barwell, 2013). Mathematical presentations can be misread (Skovsmose, 2021a) and mathematics can even be used deliberately to manipulate people (e.g., Hauge et al. 2019), and students need to become aware that there are choices made behind what is conveyed (e.g., Rubel et al. 2021). When taking a critical mathematical education perspective it is important that students get the opportunity to reflect on mathematics brought into action (Skovsmose, 2021b). However, it is not always clear what such education can look like. A lot of the research in critical mathematics education is theoretical (Andersson & Barwell, 2021), and there is a need to move towards more empirical studies (Jurdak, 2016). Some examples of studies looking at how critical mathematics education can allow students to experience mathematics in action are Steffensen and Kacerja (2021), and Stephan and colleagues (2021). Still, many of these studies are on older students. In addition, the focus is often on prescribing modelling (e.g., Steffensen & Kacerja, 2021), but there are few addressing the understanding that information presented as mathematics can be skewed (e.g, Stephan et al., 2021). Another dilemma is that when including controversial topics, such as climate change, into the mathematics classroom it might be seen as challenging by teachers (Abtahi et al., 2017). At the same time, crises due to climate change are something that society is facing, implying that mathematics education needs to address such issues (Barwell, 2013). The research question is: What possibilities for reflection on mathematics in action opens up while primary school students' discuss two different charts of CO₂ emission by the worlds regions?

Background

A concern in critical mathematics education (CME) perspective is the relation between mathematics and power, the importance of being critical with respect to mathematics and to question any form of the bringing of mathematics in action (Skovsmose, 2021b). Examples of bringing mathematics in

action could be in the use of mathematical models by experts in order to make a forecast of climate change, and when justifying decisions taken, including those taken by politicians. Decisions that can effect society, more or less directly. When addressing critical issues to the public mathematics is often brought into action, effecting peoples understanding, which can result in action taken and in its turn form for example the climate change (Barwell, 2013). There are examples of how mathematics is brought into action, but is often related to advanced mathematics (e.g., Skovsmose 2021b). However mathematics in actions also exist in the form of school mathematics (Skovsmose, 2021b). In this paper mathematics in action is related to how visualized data of CO₂ emission, by regions in the world, might have an impact on the how a situation is perceived. The mathematics is related to statistics as mathematics for primary education, where the ability to read a chart is a decired outcome. But in in the present study, reading is not the goal, reflections regarding the charts potential impact is here seen as a key factor, which is in line with CME.

For students to participate in reflections in a mathematics classroom, a dialogic process is a recourse (Skovsmose, 2021b). During a dialogic process, the participants are involved in a co-operative inquiry, an interaction where they want to figure out or clarify something (Alrø & Skovsmose, 2002). Different perspectives are important to explore in a dialogue and this could be done by arguments put forward, as a dialogical act of advocating. There are eight dialogical acts (1) getting in contact; (2) locating; (3) identifying; (4) advocating; (5) thinking aloud; (6) reformulating; (7) challenging; and, (8) evaluating. It is only in the description of advocating where arguments are explicit. At the same time, justification, which is here in the act of identifying, is according to Krummheuer (1995) an example of argument. An argument can also be challenged, and used for challenging, it can be reformulated, and expressed aloud. Therefore, it is reasonable to assume that argument play an important part in a dialogue. One common framework studying collaborative argumentation as dialogical processes is Toulmins model of argumentation (Toulmin, 2003) and it has been used within mathematics education when analysing interaction (e.g., Krummheuer, 1995). The model was used in the present study by reconstructing arguments created in the communication and how they play a role or could play a role in a dialogical process, reflecting on the impact of mathematics. This will described further down, also with an example of the recontruction of an argument.

Methods

The approach for this small scale study was inspired by critical research, attempting to change classroom practice and consider not only what take place but also imagine possible alternatives (Skovsmose & Borba, 2004). They present three different situations: the current, the imagined, and the arranged. The current situation in this study is a classroom practice that focuses on exercises where the students are working on their own in the textbook. This is the type of practice that the class was accustomed to, and something the teacher, who was new to the class, was trying to change. The different visions about alternatives for the current situation can lead to an imagined situation. The teacher and I imagined a situation where students are engaged in a dialogue, reflecting on issues related to climate change (CO₂ emission) and how mathematics might affect how we perceive a situation. With that in mind, we planned and prepared for a lesson. According to the model by Skovsmose and Borba (2004), the lesson that took place, conducted by the teacher, is the arranged situation, here the research lesson. The research lesson started with a recap from previous lesson,

which was not a mathematics class, about climate change and what could be causing it, with a focus on CO₂ emission. After providing the context, the teacher presented a scenario. The teacher told the class that two (fictive) students in another (fictive) class had a discussion regarding emission and which region emitted the most, while looking at two charts (see Figure 1). The two fictive students, Sigge and Charlie, had two different opinions.



Figure 1: Carbon dioxide emission by regions, cumulative and for 2017

The charts were in the form of treemaps where the sizes of rectangles, that were in different colours, are proportional to the values of the data points they represent. One chart represented the cumulative CO₂ emission 1751–2017 and the other represented the annual CO₂ emission in the world in year 2017. The teacher clarified that one student, Sigge, claimed that Asia emitted the most, and the other student, Charlie, claimed that North America and Europe together emitted the most. The class in the research lesson were divided into groups of three or four, provided with the same charts as the fictive students and the teacher asked which one of the (fictive) students in the other class was right. Both the claims by the fictive students, (visible on the board) as well as the question by the teacher was considered as initial contributions to the discussion. The group discussions were video recorded (e.g., Cohen et al., 2011) to capture what took place in the arranged situation and thereafter transcribed, including verbal expression as well as the use of gestures and visual representations. Data in this paper consists of one of the group discussions, a group consisting of four students.

In order to explore what happened, but also what could have happened (Skovsmose & Borba, 2004), imagine possibilities where students reflected on mathematics in action, arguments were reconstructed, here seen as possible part in a dialogue, for example as advocating (e.g., Alrø & Skovsmose, 2002). This was done by using Toulmin’s model of argumentation (Toulmin, 2003) and an example of a reconstruction will be shown further down. The core of an argument, according to the reduced model (see Krummheuer, 1995), consists of at three elements – claim/conclusion, data and warrant. The claim/conclusion is a statement that is grounded by data and warrant, where the warrant functions as a bridge between the data and the claim/conclusion. The data can be seen as answering the question “What have you got to go on?” if the claim where to be questioned and the warrant would answer “How do you know?” (Toulmin, 2003).

In the reconstruction of an argument, the first step is to identify claims in the transcript, and there after search for support for that claim, which is the data and the warrant (e.g., Nordin & Boistrup, 2018). The elements in an argument do not need to be expressed by the same person or occur in any

specific order in the communication and they can be visual, including gestures etcetera. Many studies include warrants that are implicit (Coffin & O’Halloran, 2008) which could be the case for arguments in this paper, since the interest is what supported claim/conclusion might open up for in a dialogue. When reconstructing arguments, it is important to take the context into consideration, since statements alone cannot be determined as a certain element in an argument (e.g., Nordin & Boistrup, 2018). Hence, the framework from Skovsmose and Borba (2004) function as a theoretical framing of the study, whereas Toulmin’s (2003) offer a detailed analysis of the group discussions.

In the following example of a reconstruction of an argument from the group discussion, it has been stated that Asia emits the most, which was identified as a claim since the task was about who, Sigge or Charlie, was right when discussing who emits the most. One student, here denoted Ali, had one of the two charts, the one for 2017, in front of him and continued:

Ali: Listen, 19 billions [pointing at the text for Asia]. Let us look for someone that has 19 billions. [Points with his finger at the text next to North America and Europe]. No one! These are the only ones [points to the information next to Asia].

After identifying a claim, a search for support for that claim, the data and thereafter warrant was done. Here, the written information for emission for the different regions, something that the students drew attention to by pointing, and the student’s verbal expression, “19 billion”, were identified as data, supporting the claim. Warrant was identified in the expression by Ali, “Let us look for someone that has 19 billions” including the comparison of quantities. By taking the context into account this is construed as if no one has more than 19 billion tonnes of CO₂ emission, it will be the most. The reconstructed argument is shown in Table 1:

Table 1: An reconstructed argument: Asia emits the most

Data	Warrant	Claim/conclusion:
The information in the chart: Asia emits 19 billion tonnes, Europe emits 6,1 billion tonnes and North America emits 6,5 billion tonnes.	By comparison of emission: no one else has 19 billion tonnes or more	Asia emits the most

Consider not only what took place but also what can take place (Skovsmose & Borba, 2004) one could imagine the other students agreeing “Yes, you are right” to Alis claim that Asia emits the most as well disagreeing “No, it is...”, as a possible continuation of the discussion. By reconstructing the argument, it becomes explicit that the statement – who emits the most CO₂, – meant that others had something to react upon. Providing arguments, prepared to examine it further or be challenged, could be an act of advocating.

Findings

The analysis generated three different possibilities for a reflection regarding the mathematics in action: (1) open up for experiencing two different perspectives; (2) open up for reflection on how mathematics can have an impact on how we perceive a situation; and, (3) open up for investigating the importance of recognizing what data is in use. For each possibility reconstructed argument/s will

be presented followed by an imagine continuation of an dialogue and how it could enable reflection on mathematics in action,

Open up for experiencing two different perspective

During the discussion, arguments for who emitted the most was provided. Tom for example, claimed it was Europe.

Tom: Listen, listen, listen. If one looks at Asia [points] it says 29 % global emission. Then after, when one look at Europe it says 33 % global emission [points] [turn both hands with the palm upwards as a mark]. Yes [in English], OK.

Table 2: The reconstructed argument: Europe emits the most

Data	Warrant	Claim/conclusion
The chart with the information of cumulative emission for 1751 - 2017.	$33 > 29$ (implicit)	Europe emits the most

Max on the other hand claimed it was Asia. Max had earlier stated that by looking at the emission of CO₂ by each region, one could compare them in order to see which region emitted the most. He added the percentage of emission, 18 % for North America, and 17 % for Europe in order to do a comparison with the percentage of emission for Asia.

Max: Ok, this is 35 [holds a finger on each field of North America and Europe] and this is 53 [points over Asia]. So [with emphasis], Asia emits more than these two together[points first over Asia and then over North America and Europe].

Table 3: The reconstructed argument: Asia emits the most

Data	Warrant	Claim/conclusion
The chart with the information of annual emission for 2017	$18 + 17 = 35 < 53$	Asia emits the most

In these arguments, the students supported their claim, by comparing the emission by regions within the chart each of them had in front of them. Since they looked at different charts, the claims about which region should be seen as the one emitting the most differed. The two different perspectives were identified, and the two different claims, both seemingly grounded, is conflicting and thereby challenging each other. Two challenging claims could be part of a dialogical process. One imagined utterance could then be, “How strange that both seems to be correct, how could that be?”. The two arguments could be seen as putting the students in a situation where they experience how mathematics in action (at a primary school level) lead to different perspectives regarding who emitted the most.

Open up for reflection on how mathematics can have an impact on how we perceive a situation

The second possibility is about the impact mathematics might have. Here, it is illustrated by a student, Ali, who expressed an argument as to how the fictive students, Charlie and Sigge could provide different claims:

Ali: Wait, wait, wait guys. I think, I think that, because. We say like this, Charlie got this one [points at the chart were Asia emits the most] and Sigge got this one [points at the chart were North America and Europ together emit the most] and then they checked which one is the biggest, so that's what they thought...for sure.

Table 4: The reconstructed argument: They were looking at different charts

Data	Warrant	Claim/conclusion
The two charts with information of emission. The statements by Sigge and Charlie	Asia emits most in one chart and North America and Europe emits most in the other chart.	They were looking at different charts.

Ali is advocating, putting forward argument for an idea how both claims make sense by taking both charts into consideration and identified that there were support for both claims in respectively chart, suggesting that Charlie and Sigge had been looking at different charts. This is a suggestion and can be seen as having an invitational function for the others to react upon. We can imagine how they would continue the dialogue, how the two different charts (as mathematics in primary school) can have an impact on how a situation (here CO₂ emission and in extension who is causing climate change) may be perceived, “So it depends on which one we are looking at” but put in their words,

Open up for investigating the importance of recognizing what data is in use

At the end of the discussion, Max (earlier claiming that Asia emitted the most) started to take notice to what Tom was expressing (claiming that Europe emitted the most) and realized that the region/s emitted the most in the two charts differed, and the charts did not show the same information. This made him wonder how they could determine who emits the most (and who of the two fictive students being correct).

Max: But here [points hastily over North America and Europe in his chart] it is not the same [points over North America and Europe in Tom’s chart], so how should we...It is more, here [points at Asia in his chart] but here [points at North America and Europe in Tom’s chart] it is more.

Table 5: The reconstructed argument: It is not possible to decide, they convey different information.

Data	Warrant	Claim/conclusion
The two charts with information of emission. The statements by Sigge and Charlie	The charts convey different things	It is not possible to decide which one emits the most.

Max was also considering both charts, and while identifying they did not show the same information, he started questioning how they were to decide which region/s emit/s the most. This was construed as it is not possible to determine which region/s emits the most since the charts don't share the same information. It is possible to imagine a continuation of the discussion, looking into what actually can be claimed by the information in the two different charts. Maybe something like: "You can't say who emits the most by looking at these", "You can say North America and Europe emitted the most during 1750 to 2017 and Asia emitted the most in year 2017".

Discussion

This is an empirical study taking a perspective of Critical Mathematics Education, thus a contribution to the research field of CME which to a large extent consists of theoretical research (Andersson & Barwell, 2021). Here, we applied, the model by Skovsmose and Borba (2004) which meant that the focus is not just on what happened but also what could happen, the imagined situation. As such, it offers a comparison between the actual dialogical process, seen as a resource (e.g., Skovsmose 2021b), and the dialogue that could happen. The latter should also be seen as a resource according to Skovsmose and Borba (2004) since it offers a glimpse of what is possible.

Recognizing the limitation of the study, being a small scale study consisting of only one lesson, the findings nevertheless indicate that argument provided in a dialogue could open up for further investigation and reflection of the mathematics brought into action in a primary mathematics classroom. There were three different possibilities presented: (1) open up for experiencing two different perspectives; (2) open up for reflection on how mathematics can have an impact on how we perceive a situation; and, (3) open up for investigating the importance of recognizing what data is in use. These three possibilities should not be seen as isolated categories. Rather, they should be seen as contribution to a mathematics education aiming to illuminate there are choices behind what is conveyed (e.g., Rubel, et al. 2021). During the dialogical process, the participants were involved in a co-operative inquiry (e.g., Alrø & Skovsmose, 2002), expressing a variety of arguments that had different claims and warrants (e.g., Krummheuer, 1995). As such, the results from the present study is a contribution to previous research aiming to stimulate students' experiences of mathematics in action (e.g., Steffensen & Kacerja, 2021; Stephan et al., 2021). It is not clear though what in the design of the task, including the entire staging and contribution by the teacher, that makes it adequate for stimulating such experiences. Further studies are therefore suggested.

References

- Abtahi, Y., Götze, P., Steffensen, L., Hauge, K. H., & Barwell, R. (2017). Teaching climate change in mathematics classroom: An ethical responsibility. *Philosophy of Mathematics Education Journal*, 32, 1–18.
- Alrø, H., & Skovsmose, O. (2002). *Dialogue and learning in mathematics education: intention, reflection, critique*. Kluwer Academic Publisher.
- Andersson, A., & Barwell, R. (2021). Applying critical mathematics education: an introduction. In A. Andersson & R. Barwell (Eds.), *Applying critical mathematics education*. Brill.

- Barwell, R. (2013). The mathematical formatting of climate change: critical mathematics education and post-normal science. *Research in Mathematics Education*, 15(1), 1–16. <https://doi.org/10.1080/14794802.2012.756633>
- Cohen, L., Manion, L., & Morrison, K. (2011). *Research methods in education* (7th ed.). Routledge.
- Coffin, C., & O’Halloran, K. A. (2008). Researching argumentation in educational contexts: new directions, new methods. *International Journal of Research & Method in Education*, 31(3), 219–227. <https://doi.org/10.1080/17437270802416582>
- Hauge, K. H., Kacerja, S., & Lilland, I. E. (2019) Xenophobia and numbers in the media. *Philosophy of Mathematics Education Journal*, 35.
- Jurdak, M. (2016). *Learning and teaching real world problem solving in school mathematics: A multiple-perspective framework for crossing boundary*. Springer.
- Krummheuer, G. (1995). The ethnography of argumentation, In P. Cobb & H. Bauersfeld (Eds.), *The emergence of mathematical meaning: Interaction in classroom cultures*, (pp. 229–270). Lawrence Erlbaum.
- Nordin, A.- K., & Boistrup, L. B. (2018). A framework for identifying mathematical arguments as supported claims created in day-to-day classroom interactions. *Journal of Mathematical Behavior*, 51, 15–27. <https://doi.org/10.1016/j.jmathb.2018.06.005>
- Rubel, L. H., Nicol, C., & Chronaki, A. (2021). A critical mathematics perspective on reading data visualizations: reimagining through reformatting, reframing, and renarrating. *Educational Studies in Mathematics*, 108, 249–268. <https://doi-org.ezp.sub.su.se/10.1007/s10649-021-10087-4>
- Skovsmose, O., & Borba, M. (2004). Research methodology and critical mathematical education. In P. Valero & R. Zevenberger (Eds.), *Researching the socio-political dimensions of mathematics education: Issues of power in theory and methodology*, (pp. 207–226). Kluwer Academic Publisher.
- Skovsmose, O. (2021a). Mathematics and crises. *Educational Studies in Mathematics*, 108, 369–383. <https://doi.org/10.1007/s10649-021-10037-0>
- Skovsmose, O. (2021b). A philosophy of critical mathematics education. *Philosophy of Mathematics Education Journal*, 37. https://doi.org/10.1007/978-3-031-26242-5_18
- Steffensen, L., Kacerja, S. (2021). Carbon Footprints Calculators and Climate Change. In F. K. S. Leung, G. A. Stillman, G. Kaiser, & K. L Wong (Eds.), *Mathematical Modelling Education in East and West*. (pp. 518–523). Springer.
- Stephan, M., Register, J., Reinke, L., Robinson, C., Pugalenti, P., & Pugalee, D. (2021). People use math as a weapon: critical mathematics consciousness in the time of COVID-19. *Educational Studies in Mathematics* 108, 513–532. <https://doi.org/10.1007/s10649-021-10062-z>
- Toulmin. S. (2003). *The uses of argument* (updated ed.). Cambridge University Press.