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In-service Italian primary mathematics teachers' knowledge and beliefs about possible students' mistakes in mathematics large-scale tests

Federica Ferretti¹, Francesca Martignone² and Giada Viola¹

¹University of Ferrara, Italy; federica.ferretti@unife.it

²University of Eastern Piedmont, Italy

This paper presents part of a larger research that is still under development. We discuss some results of a questionnaire administered to a sample of in-service Italian primary teachers. We show teachers responses about student answers in large-scale tests administered for Italian national assessments. The data collected show some aspects of teacher pedagogical content knowledge and beliefs. The analysis carried out in our study can become the starting point to design teacher educational program in which data from large-scale tests are used to reflect on student possible difficulties and mistakes.

Keywords: In-service primary school teachers, Pedagogical Content Knowledge, MTSK model, student mistakes, large-scale tests.

Rationale

This contribution presents a project conducted by a national research group, named the INVALSI-Didactics Disciplinary Group, which is part of the observatory of the S.I.R.D. - Italian Society for Educational Research (<https://www.sird.it/en/home-en/>). In Italy, the INVALSI national Institute is responsible of the standardized mathematics assessment (INVALSI large-scale tests). The INVALSI tests are administered at a census nationally from grade 2 to grade 13. Each year the results of the national survey, elaborated on a valid statistical sample, are returned to the schools and publicly discussed. The theoretical framework of the INVALSI tests is aligned with the research results in mathematics education and with the Italian National Guidelines. Our research group is composed of experts in mathematics education, computer science education and pedagogy, and primary in-service teachers. The overall purpose of the research is to promote dialogue and collaboration among school teachers, researchers from pedagogical and docimological field, and mathematics education researchers. The final aim of our research project is to outline mathematics teaching educational courses based on a conscious use of INVALSI large-scale national mathematics tests, which can be deployed in both pre-service and in-service teacher education. The starting point consisted of collecting and analysing information on primary school teacher knowledge, beliefs and on their teaching uses of INVALSI tests in schools. We wanted to reflect on the educational needs of teachers: in particular, we focus on how to support them in reflecting on student mistakes and becoming aware of their own teaching. We ground our research on studies that show the potentially fruitful role of large-scale assessments in teacher education (Campbell, & Levin, 2009, De Lange, 2007). In particular, in the Italian context several research show how an informed and conscious use of the Italian INVALSI tests could have a positive impact (Di Martino, & Baccaglini-Frank, 2019; Ferretti, Gambini, & Santi, 2020; Ferretti, Martignone, & Santi, 2022; Martignone, 2016) on teachers' professional development. Our project is still in progress. We collected data on what teachers think

about INVALSI assessment in Mathematics and the educational use of INVALSI data in classroom activities.

The empirical research design fine-tuned within the Project aimed at knowing, describing, interpreting-through data surveys-teachers' knowledge, beliefs and statements of practices towards INVALSI assessment: both of INVALSI tests purposes and methods, and of the INVALSI Mathematics tests and their effects on teaching practices. A first survey was designed to investigate primary teachers' knowledge and beliefs about INVALSI tests results. In this paper we analyse data concerning the mathematics education section of this survey. We discuss some aspects that emerged from a questionnaire administered to 516 in-service primary teachers. In this contribution we aim to open fresh insights that can provide new elements for planning teacher education programs centred on the development of pedagogical content knowledge that make use of analyses of INVALSI mathematics tests.

Specifically, we will show how teachers' responses to our questionnaire can highlight some aspects of their pedagogical content knowledge, detailed using the interpretive lenses from the MTSK model (Carrillo-Yañez et al, 2018).

The specialized knowledge of mathematics teachers

In mathematics education, many research on Mathematics Teacher Knowledge are based on the foundational studies of Shulman (1986). He defined two distinct but strongly interrelated components of teacher knowledge: Subject Matter Knowledge and Pedagogical Content Knowledge. Within this line of research, numerous works in recent years have addressed various aspects concerning the so called teachers' specialized knowledge, in particular Carrillo-Yañez and colleagues (Carrillo-Yañez et al, 2018) introduced the *Mathematics Teacher Specialized Knowledge (MTSK) model*. The MTSK coordinates two broad areas of knowledge, *Mathematics Knowledge (MK)* and *Pedagogical Content Knowledge (PCK)* that meet and intersect in the teacher's belief system. One of the distinctive features of this model is to emphasize that teachers' actions are strongly related not only to what they know about mathematics, but also to their conceptions and beliefs (Thomson, 1992) about mathematics, how it is learned and how it should be taught. Thus a more or less consistent set of beliefs permeates a teacher's knowledge in each of the subdomains of specialized knowledge (Carrillo-Yañez et al, 2018). In the model, MK and PCK are categorized into different subdomains. In this paper we deal with the PCK subdomains: KMT - *Knowledge of Mathematics Teaching* (e.g. knowledge of theories of mathematics teaching or knowledge of teaching resources, materials and technologies, but also knowledge of strategies for introducing and representing contents and concepts, etc); KFLM - *Knowledge of Mathematics Learning Features* (knowledge of theories of mathematics learning or knowledge of the way in which pupils interact with mathematics); and KMLS - *Knowledge of Mathematics Learning Standards* (knowledge of expected learning outcomes and teaching goals in different school segments). One aspect that characterizes the MTSK model is the centrality of teachers' beliefs about mathematics and mathematics teaching and learning. Beliefs are located at the "centre" of the model precisely to emphasize the reciprocity between beliefs and all knowledge domains (Carrillo-Yañez et al., 2018). The picture that frames this knowledge and belief requires ongoing interactions between mathematics as a discipline and pedagogical knowledge. Within this

holistic view of the knowledge required for and in teaching, we based the design of the first phase of our investigation on teacher knowledge, beliefs, and reflections about INVALSI assessment.

Design and administration of the questionnaire

In the early stages of the project, the research team engaged in interdisciplinary design work of the survey instrument. We carried out large-sample observational research for investigating teachers' knowledge and beliefs on INVALSI assessment. The questionnaire consists of two domains of variables, one specifically of Mathematics Education and one related to aspects of General Didactics (teaching and assessment). These two domains are broken down into 51 questions (mostly closed-ended and with ranking scales) divided into three main sections: a first section on Mathematics Education, a second section on teachers' beliefs toward INVALSI tests and toward the use of assessment with a diagnostic-formative function, and a third section collecting personal data and information on educational and professional backgrounds. In detail, the Mathematics Education variables are aimed at detecting the extent to which the mathematical content and skills measured by INVALSI tests are believed: to be in tune with teacher personal teaching experience and daily practices; to be consistent/inconsistent with the National Guidelines; to be useful in influencing/innovating personal teaching practice. In particular, the research question we want to answer in our study is: Which aspects of teacher pedagogical content knowledge could emerge from teachers' responses about possible students' mistakes in particular large-scale test tasks?

The first section of the questionnaire proposes seven items from the INVALSI tests in grade 5 and grade 6 mathematics. In this paper we focus on teachers' answers to two questions belonging to this Mathematics Education section. We show the quantitative data collected and then we carry out a qualitative analysis of teachers' responses to identify which aspects of teacher pedagogical content knowledge could be the subject of further reflection on their pedagogical content knowledge.

The questionnaire was administered in 2020 via online mode to 516 primary teachers. The participants (all volunteers) were teachers from Italy distributed across the country and do not constitute a representative sample from a statistical point of view. The collected data were coded and analysed using IBM® SPSS® Statistics 27 statistical software.

The initial results obtained by the research team have been the subject of national and international research papers (e.g. Arzarello & Ferretti, 2021; Faggiano, Ferretti, & Arzarello, 2022). In this contribution we will present an analysis of the results obtained from two questions of the first section of the questionnaire.

Results and analysis

In this paragraph we show teachers' answers to questions that are part of the section of the questionnaire concerning Mathematics Education. The focus is on the possible main causes of students' answers to the INVALSI tasks and the, closely related, perception of the difficulty of the

tasks. The INVALSI task (Figure 1) is reported in the first question: teachers are asked to estimate the students' difficulty in solving the INVALSI task.

Figure 1 shows a Multiple Choice task for grade 5 students administered by INVALSI in the s.y.2008-2009. There are four choices and only one is correct.

10. To which number do they correspond "12 tens, 7 tenths and 2 thousandths"?	
<input type="checkbox"/>	A. 12,702.
<input type="checkbox"/>	B. 120,702.
<input type="checkbox"/>	C. 12,72.
<input type="checkbox"/>	D. 120,72.

Figure 1. Task 10, Grade 5 Mathematics INVALSI test 2009

The task requires a *conversion* (Duval, 2006), the semiotic transformation from a representation in one register (in this case, the verbal register) into another representation in another register (the positional numerical register). As we can see in the Figure 2, at national level, the percentage of correct answers (option B) is the 33%. Instead, the most chosen option is option A, which in the national sample is chosen by 44% of students.

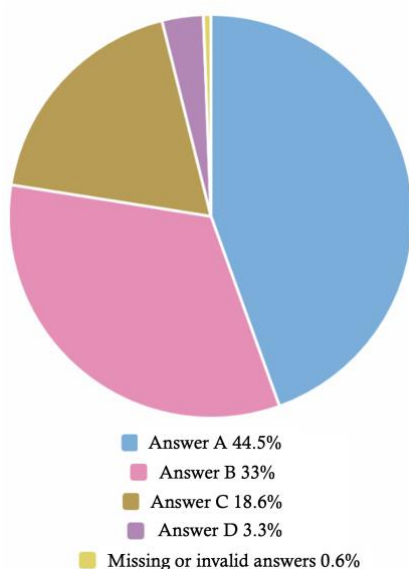


Figure 2. Percentage of choice of each option (answer chosen by Italian students), Task 10, Grade 05 Mathematics INVALSI test 2009, www.gestinv.it

Teachers of our sample did not know the percentage of choice for each option collected by INVALSI on national sample (Figure 2). Teachers are asked to estimate the degree of difficulty of the task (asking them to rate the degree of difficulty on a scale of 1 to 10), in terms of "How difficult do you think this task is for students at the end of grade 5?".

In the following table (Tab. 1) we can find the results with reference to 516 collected responses. (1 = very easy, 10 = very difficult).

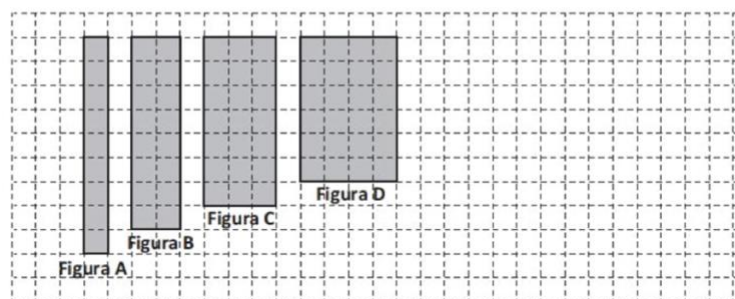
Table 1. Rate of choice of teachers' questionnaire question on Task 10 (Figure 1)

Value	Answers
1	75
2	143
3	86
4	39
5	67
6	29
7	37
8	30
9	7
10	3

The perception of the difficulty of the task by teachers (Table 1) does not match the success rate on the national sample (Figure 2): looking at the cumulative probability we note that 410 teachers out of 516, almost the 80%, gave an answer from 1 to 5. Almost half of the teachers, the 42%, choose the first two values. We can notice that teacher's perceptions about student difficulties are not in tune with INVALSI test results. These data have already been analysed in a research conducted by Arzarello and Ferretti (2021) and this constitutes the first fold of what the authors have defined as three-fold meta-didactical conflict. In analogy to Anna Sfard's *incommensurable discourse* (2008), this conflict is generated by incommensurable language between teachers and large-scale assessments. This conflict is meta-didactic since it concerns reflections about assessment, students' competencies and mistakes, etc. In this study we want to review questionnaire results in light of the MTSK model and then we focus on identifying aspects of teachers' pedagogical knowledge that could influenced teachers' responses. As highlighted in Arzarello and Ferretti (2021), based on answers given to other questions in the questionnaire which for brevity of exposition are not given in the text, teachers know that the task (Figure 1) is definitely in line with the demands of the National Guidelines (KMLS - *Knowledge of Mathematics Learning Standards*) and also it is familiar to students and teachers because similar tasks are featured in the textbooks. Their knowledge of student difficulties and typical mistakes, also studied in literature, do not seem to have guided most of the teachers' answers. In a teacher education program, we could start from these results to reflect on and develop the *Knowledge of Mathematics Learning Features* and then teacher *Knowledge of Mathematics Teaching*, in particular by discussing with teachers the results from mathematics education literature.

In another question of the questionnaire the INVALSI task and the percentages of correct answers are given. In this question, teachers were asked to indicate which, among some proposed options, could be the main cause of the results collected by INVALSI about the task shown in Figure 3.

D3. Observe the following sequence of figures



b. Which of the following statements is true?

- A. The areas of the figures always remain the same
- B. The areas of the figures double with each pass
- C. The perimeters of the figures always remain the same
- D. The perimeters of the figures increase with each passage

Figure 3: Task 2b, Grade 6 Mathematics INVALSI test 2012 (www.gestinv.it)

35.7% of the students correctly answered to the task, option C. 35.5% chose option D. 22.4% chose option B. 4.4 % chose option A. 2% . are missing or invalid answers.

In this case, the percentage of correct answer and various possible causes of students' mistakes were reported in the questionnaire for teachers. Teachers were asked to indicate which, by their opinion, was the main cause of students' difficulties.

The options are:

- *because the students didn't read the text carefully;*
- *because the students are not familiar with the formulas for the area and the perimeter;*
- *because the students are deceived by the figure;*
- *because the students think that if the area increases, the perimeter also increases;*
- *other (specify).*

In reference to this question, 516 responses were collected. It is interesting that the teachers attribute the mistakes of the students, for more than 60%, to "generic" factors such as *because students do not read the text carefully* (180 teachers out of 526, the 34%) and *because students are deceived by the figure* (172 teachers out of 526, the 33%). Only 113 teachers out of 526 (just over the 20%) attribute the mistake to because students think that if the area increases the perimeter also increases. In this case we can interpret the different response options as related to PCK. Specifically, to answer the question teachers can lean on their *Knowledge of Mathematics Learning Features*. This knowledge is developed not only through experience but also through contact with research results. For example, the behaviour of students when faced with this task can be traced back to Dina Tirosh's theory of intuitive rules (Stavy & Tirosh, 1996). according to which if there are two relationships with some mutual connection, the student tries to apply the following "law of conservation": if this thing grows,

the other thing related to it grows as well (and vice versa). In this case, the two “things” are area and perimeter.

Conclusion

This paper deals with the first phase of a project focused on Mathematics Education and INVALSI tests. We show some first results from a questionnaire administered to a sample of 516 primary Italian teachers. We analyse teachers’ answers about INVALSI results focusing on teacher *Pedagogical Content Knowledge*, declined in the MTSK model subdomains. We analysed teachers’ answers to two questions focusing on students’ possible difficulties and mistakes. These answers show that there are some student responses to INVALSI tasks that teachers expect to be different or consider to be influenced by various factors. Teachers’ choices expose their *Knowledge of Mathematics Learning Features* and might have been driven by their *Knowledge of Mathematics Learning Standards*.

The results obtained by means of our questionnaire can be the starting point for reflections to be developed in teacher education programmes. Activities can focus on teacher knowledge and beliefs about possible students’ mistakes that emerged from INVALSI assessment and link them to the results of mathematics education research. In particular, as emerged from the discussion during the TWG20 small group work, it would be interesting: to ask students to justify their answers/explain the solution procedures they used to solve INVALSI tasks; and to ask teachers to interpret students’ justifications and explanations in the light of the theoretical lenses shared in professional development paths. Another interesting line of development emerged is the possibility of administering the same items to teachers from other European countries in order to carry out a comparative cultural study on teachers’ beliefs and knowledge.

In our project the analysis and interpretation of the data is still in progress. Experts from different educational fields will give a holistic view of the information collected, which is proving to be effective from the point of view of designing guidelines expendable in teachers’ professional development courses aimed at supporting teachers in their daily activities of managing mathematics teaching and learning processes.

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