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► **To cite this version:**

Nuria Climent, Ma Isabel Pascual, Juan P. Martín, Myriam Codes, Luis C. Contreras. Analysis of the implementation of a professional task designed around the MTSK model. Thirteenth Congress of the European Society for Research in Mathematics Education (CERME13), Alfred Rényi Institute of Mathematics; Eötvös Loránd University of Budapest, Jul 2023, Budapest, Hungary. hal-04407318

HAL Id: hal-04407318

<https://hal.science/hal-04407318>

Submitted on 20 Jan 2024

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Analysis of the implementation of a professional task designed around the MTSK model

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Professional tasks are a key aspect of primary teacher education. The aim of this paper is to analyse the design and implementation of a professional task for prospective teachers based on analysis of a lesson recording. Following a design-based research methodology, we present our findings with regard to the targeted professional knowledge and that mobilised during the training session. The analysis revealed both benefits and limitations in the design, to be taken into consideration in the implementation of successive experimentation cycles. Among the benefits offered by this task design, we foreground the richness of the connections between different subdomains revealed by the use of the Mathematics Teachers' Specialised Knowledge (MTSK) model as the theoretical underpinnings of the task, and the role of video recordings as a central structuring element due to their potential in contextualising classroom practice.

Keywords: Professional tasks, MTSK, video analysis, initial training, design-based research.

Introduction

Initial teacher education programmes typically cover interrelated content areas concerned with knowledge, skills and/or professional identity (Ponte, 2011). Preservice primary teachers enrolled in training courses which include pedagogical content knowledge and mathematical knowledge feel more secure and motivated to teach mathematics as a result of the connections being reinforced between these two knowledge domains (Cardetti & Truxaw, 2014). The aim of this study is to explore the scope of a professional task which brings together different elements of professional knowledge from the starting point of a video-recording of a primary lesson. This will provide us with information for the redesign of effective training tasks.

Theoretical foundations

Professional knowledge in initial primary teacher training

One possible framework for structuring the professional knowledge to be developed in teacher training is the Mathematics Teacher's Specialised Knowledge model (Carrillo *et al.*, 2018) (henceforth MTSK). The model enables mathematics teacher educators to approach teacher education addressing different subdomains of professional knowledge (Policastro *et al.*, 2019). In keeping with the theoretical foundations of the MTSK model, though essentially holistic in nature, for the purpose of analysis, professional knowledge can be usefully divided into the separate domains of knowledge of mathematics, pedagogical content knowledge, and beliefs about mathematics and how it is taught and learnt. Likewise, within these domains, distinct subdomains and knowledge categories can be distinguished, which represent useful analytical tools for research into the specialised knowledge brought into play at different educational levels (Zakaryan & Ribeiro, 2019), and which have also been used to shape different tasks for teacher education (Semanišínová, 2021).

The professional task we analyse in this study was designed so as to approach different subdomains in the MTSK model, and the analysis we carry out on its scope is organised in accordance with these (Carrillo *et al.*, 2018). On the one hand, with respect to Mathematics Knowledge (MK), the subdomains Knowledge of Topics (KoT) and Knowledge of the Practice of Mathematics (KPM) were considered. These subdomains cover substantive knowledge and syntactic knowledge of mathematics (e.g. knowledge of a definition of polygon, KoT, and how must be a mathematical definition, KPM). The task also concerned Pedagogical Content Knowledge (PCK), including Knowledge of Mathematics Teaching (KMT) (e.g. resources to teach the polygons), Knowledge of the Features of Learning Mathematics (KFLM) (e.g. usual students' images of polygons), and Knowledge of Mathematics Learning Standards (KMLS) (e.g. how is sequenced polygons in some standards).

Tasks for teaching mathematics in initial primary teacher education

The term task can be understood broadly as a lesson segment, context or medium, which configures a teaching unit so as to develop an idea or disciplinary practice (Stein *et al.*, 2009). We will refer to any task proposed in teacher education as a training task. Some training tasks are explicitly oriented towards professionalization (because they raise issues related to teacher practices, for example); we will call them professional tasks. The learning of preservice primary teachers (PTs) depends in large part on the tasks they are given, in which design must be considered their function, their form, and their focus (Grevholm *et al.*, 2009). In the task we analyse, the function concerns the construction of specialised knowledge, the form consists of the analysis of a video recording and the subsequent exploration of key concepts, and the focus is the construction of the definition of polygon.

The use of tasks based on video analysis enables PTs to become aware of why their knowledge of mathematics needs to be deep and well founded (Millman *et al.*, 2009), especially with regards to the potential requirements of their future pupils. In general, the use of video analysis in teacher education aims to complement the development of professional knowledge and/or the assimilation of professional practices (van Es *et al.*, 2020). In our case, the emphasis is on the former aspect, in the context of an authentic lesson. As PTs do not yet have a highly developed capacity to analyse unaided the classroom practice of other teachers (Star & Strickland, 2007), particular emphasis should be given to the production of appropriate guidelines for directing PTs' observation.

To explore key concepts, the second part of the task has been designed so that the PTs work with constructive definitions (De Villiers, 1998), analysing variation dimensions of the polygon concept (Kullberg *et al.*, 2017) and evaluating the didactic potential of the examples (Bills *et al.*, 2006).

Methodology

The methodology followed is that of design-based research (Cobb *et al.*, 2003). The researchers (authors of this paper) are the educators who implement the design in a group in the last year of prospective primary teachers who had taken mathematics teaching subjects previously (more nothing about polygons). This paper focuses on a training task with two parts: the first part of the task (henceforth t1) is the analysis of a video-recording of a lesson in primary education; the second part of the task (t2) is showed in Figure 2. Both parts were implemented by an educator with more than 25 experience years, with degree in Mathematics and doctoral dissertation in Mathematics Education.

The video was of a 5th year primary education class on shapes. The teacher asked pupils to come to the board in turn, and to pull out at random unseen geometric shapes from a bag, which they then had to assign to one of two groups, without any indications of what criterium to employ. The teacher places a first shape on the board and the pupils, one by one, must draw a shape at random and decide freely whether to place it in one of the groups already formed or in another, justifying why. Once all the shapes had been assigned to one of the groups, as shown in Figure 1, the whole class worked together to identify the essential features of a polygon and reach a definition. The video is a recording to which the researchers had access, without the classroom teacher participating in this design-based research, rather than authorising the use of the video in teacher education. The Primary lesson observed was designed and implemented by the Primary classroom teacher.

In t1 the PTs watched two excerpts of the recording and made notes in a pre-prepared table as they did so. The table consisted of six columns: Thinking strategies and pupils' ideas, intuitive ideas | Lesson content and placement of emphasis | Activity type | Resources: potential, limitations and use | Examples used, representations of content | Fit with curriculum.

1.- Use the table below to examine the set of examples selected by the teacher in the video to shape the teaching of the polygon concept. Justify your ratings for each item, reflecting afterwards on why you made your assessment (explain as much as you can about your assessment).



Figure 1. Final classification of the shapes in the primary lesson.
(fromMontes et al., 2022)

INDICATOR	RATING (from 1 to 5)
Variability in positions: how the set of examples contemplates or does not contemplate the different supports of the figures, avoiding prototypical positions.	
Variability of characteristics: ability of the set of examples to highlight the elements that can distinguish one figure from another.	
Potential to generate groups describing common properties: the degree to which the proposed figures allow classifications into different subgroups.	
Variability in the elements of the same groups: ability of the set of examples to show multiple groupings in which different properties converge.	
Ability to cover extreme examples: the degree to which the set of examples allows the construction of figures whose characteristics are not obvious at first sight.	

Figure 2. Second part of the task (t2). (Authors' own elaboration)

The two video excerpts selected for viewing showed first the teacher setting up the task and the pupils sifting the shapes into the two groups, and then the class together determining the definition of a polygon. In t2, based on their reading of a text on the use of examples in mathematics teaching (presented within the framework of the “variation theory” of learning, Kullberg et al., 2017), the PTs were asked to assess the set of examples used in the lesson (Figure 2). The aim was to encourage PTs to begin the process of amassing the specialised knowledge detailed in Table 1 (obtained from a pre-implementation analysis of t1 and t2 using MTSK model). The aim with the design of t1 and t2 was

to address specialised knowledge related to the formative potential of the Primary lesson, without seeming overly forced (hence we did not include KSM). Beliefs are excluded from the paper for reasons of space, hence they are not in Table 1.

Table 1: Knowledge targeted in t1 and t2 (Authors' own elaboration based on MTSK model)

KoT	Definitions, properties and foundations (t1 & t2): 1. Characteristics and elements of plane shapes 2. Examples and non-examples of polygons 3. PTs own image and definition of a polygon	KMT	Strategies, techniques, tasks and examples: 9. Learning tasks for the mathematical practice of defining and the concept of polygon (t1) 10. Examples for guiding the construction of the definition of polygon (t1 and t2)
KPM	4. The practice of classifying, identification of criteria (t2) 5. Inclusive and disjunctive classification systems, advantages and disadvantages for mathematics (t2)		Teaching theories: 11. Use of exemplification in teaching, dimensions of variation and transparency of examples (t2) Resources: 12. Teaching resources for the mathematical practice of defining and the concept of polygon (t1)
KFLM	Learning theories: 6. Mental image and conceptual definition of a geometric object (t2) Strengths and difficulties: 7. Difficulties associated with learning the concept of polygon (t1 and t2) 8. Ways of interacting with mathematical content: Characteristics of primary pupils' mental images of polygons (t1 & t2)	KMLS	Expectations: 13. Expected learning outcomes for 5th year primary education with respect to the definition of a polygon (t1)

The research data consist of the training sessions recordings in which t1 and t2 were carried out, respectively, and the written texts produced by the PTs in small groups (related to t2). A content analysis was carried out on both the transcript of the lesson recordings and the PTs' texts, in respect of the MTSK mobilised in comparison with the targeted knowledge (Table 1). This was first done individually by each author, who then met to agree a definitive analysis.

Results

Regarding to t1, PTs' contributions to the discussion provide evidence of some MTSK subdomains. The debate starts with a reflection by a PT about a moment of doubt involving one of the pupils:

- PT1: I'm going to talk about the first part. The second pupil who goes up takes a triangle and doesn't know which side to put it on.
 Educator: Why do you think he isn't sure?
 PT1: I think it's because he doesn't know if he should place it directly with the regular polygons or with the irregular polygons.

The video-recording provides the PTs with an opportunity to reflect on how polygons are classified, with reference to the frequent classification of regular-irregular, and further, that this classification was known by the children at this educational level (KMLS, *level of expected conceptual and procedural development* [13] in Table 1). PT1 attributes the pupils' hesitation to his having to classify the shape according to an unfamiliar criterion (KFLM, *ways of interacting with mathematical content*). At the same time, the definition of polygon allows them to choose where to place the right-angled triangle (KoT, *definitions, properties and foundations* [1]). There is a question-mark over PT1's knowledge of regularity in polygons, as they appear to suggest that the criterion being employed to classify is regularity (while in fact there are irregular shapes on both sides of the board).

At the end of the session, the educator sets the PTs to solve the task:

- Educator: Do any of the shapes on the left strike anybody as odd? If you'd been in this five grade class would you have placed any shape which is currently on the left outside this area? For you, is everything on the left a polygon?
- PT2: I'd probably have put the star on the right.
- Educator: That's understandable [...]Why is it to be expected?
- PT3: Because of the examples in the textbooks.

PT3's reply indicates recognition of errors induced by textbooks, which frequently offer prototypical examples (KMT, *strategies, techniques, tasks and examples* [10]). This means that the mental image of polygons in the minds of primary pupils can be constrained by examples which are repetitive and always positioned in the same way (KFLM, *learning theories and learning difficulties* [6, 7]). Once again, content knowledge of polygons, in this instance *definitions, properties and foundations*, would seem to act as an aid to bring into play pedagogical content knowledge.

The next episode arises when, in the video recording, the teacher takes out a box of chalk in order to add elements to the definition of polygon which distinguish flat shapes from three-dimensional ones:

- PT4: [...] when he talks about edges the teacher doesn't emphasize that this is for three-dimensional shapes, and they are only doing flat shapes.
- PT5: The thing about the edges, that comes up when the teacher takes out the boxes of chalk, doesn't it? So, they have done it, well, maybe not done it, but they have talked about the three-dimensional shapes.

PT4 mobilises content knowledge when introducing the notion of edge linked to 3D shapes (KoT, *definitions, properties and their fundamentals* [1]). There is also evidence that PT5 mobilises didactic content knowledge related to the use of examples (KMT, *strategies, techniques, tasks and examples* [10]), when it reflects on the chalk box as a non-example of a polygon.

Finally, on being asked about the set of examples provided, PT4 says:

- PT4: Also, the children might think that those are all the examples there are (gives the example of a multi-sided polygon which looks like a circle).

On the one hand, it can be seen that PT4 is aware that the wider the range of examples offered, the more varied the pupils' mental images of polygons. On the other hand, he calls into question the set of examples offered, and demonstrates that he is able to come up with a further example which focuses on the number of sides and on the similarity of a multi-sided polygon to a circle. The knowledge underlying this intervention concerns the use of examples, and possible examples which can boost the pupils' example space (KMT [10, 11]; KFLM [8]). In addition, the example he suggests (a multi-sided polygon which looks like a circle), implies knowledge of the similarity between a circle and a multi-sided polygon (KoT [1]). Further, he knows that giving this particular example can be especially relevant, not only in boosting the example space, but also in revealing potentially erroneous conceptions on the part of the pupils as a result of confusing the two shapes (KFLM [7]).

With respect to t2, while we found only the mobilisation of content knowledge (KoT) by PTs during the recording, we were able to extract elements of PCK from their written answers. Attending to the first variable rated by the PTs (*variability in positions*, Figure 2), they state that the positions in which the shapes are shown "are variable, as they are mobile", allowing for "alternation between a conventional and non-conventional placement of the shapes" (KMT [12]). This fact, as one PT notes,

“...contributes to the process of constructing knowledge by enabling recognition of the shapes irrespective of their position.” This argument also taps into an understanding of the pupils’ mental image and how this is constructed (KFLM [7]).

The responses to *variability of characteristics*, again reveal knowledge associated with KMT ([12]). The PTs comment that “the shapes chosen by the teacher feature all the characteristics needed to determine what a polygon is”. In addition to demonstrating knowledge of the teaching resource, another PT also mobilises knowledge of the topic in question, noting properties of the shapes and criteria for classifying them (such as the number of sides, axes of symmetry and diagonals) (KoT [1]).

In the same vein, with respect to the variable *potential to generate groups describing common properties*, we find statements such as “all the shapes have at least one property in common with the others, so they could be organised into subgroups.” Here again, we can surmise that the PT is aware of characteristics and elements of plane shapes (KoT [2]). Another PT adds that it would have been interesting to include a few extreme examples of polygons as this might have given rise to another classification. This argument goes further to give a critical assessment of the example space, demonstrating knowledge of the existence of extreme examples, and also of how adding examples can add new dimensions of variation by giving rise to a new type of classification (KMT [10, 11]). There is an implicit knowledge associated with KFLM regarding the pupils’ mental image ([6]), in that by adding extreme examples, the pupils might arrive at a different kind of classification.

In terms of *ability to cover extreme examples*, some PTs argue that the star and the blot represent extreme examples within the set, while others question the need to include new shapes on the grounds that, although “it is true that the shapes used are not restricted to those found in textbooks, these don’t feature extreme examples which demand a deep understanding of each concept.” In this instance, the PT’s knowledge about the use of examples, their transparency and their usage in textbooks (KMT [10, 11]), underpins their reflection on the potential characteristics of the pupils’ mental image if they work within a limited example space (KFLM [6]).

Conclusions

The results illustrate the potential of this task for mobilising specialised knowledge, highlighting the distinct nature of the interconnected knowledge. At one level, we can see how Knowledge of Topics (KoT) underpinned the knowledge mobilised in different subdomains of PCK, representing an example of MK-PCK interconnections. At another level, connections were identified in which the underlying knowledge, alongside the Knowledge of Topics (KoT), derived from PCK itself, representing an example of interconnections within the same subdomain. This highlights how a task starting from a real teaching scenario allows for the mobilisation of interconnected knowledge. At all times we have referred to mobilised knowledge, as we cannot document the knowledge development of individual preservice teachers.

Also worthy of note is that the use of the MTSK model in the task design proved to be useful in terms of dealing with the complexity of the analysis, as its epistemological foundation provided a means of articulating the emergence of interrelated specialised knowledge from the analysis of its parts. The intersection of the evidence of knowledge mobilised in the session with the knowledge targeted by the task reflects the effectiveness of the design as well as the management of the educator.

Notwithstanding, some divergence from the anticipated knowledge can be noted. For example, there appears to be little mobilisation of KPM on the part of the PTs, even though the video-recording of the lesson they watch concerns the construction of the definition of a polygon. Further, rather curiously, the PCK targeted in t2 can be noted solely in the PTs' written answers, and not in their observations during the session. The lack of KPM underlines the need to create tasks specifically designed to tackle the construction of knowledge in this subdomain, given that the PTs are focussed less on the practice of mathematics than on the practice of teaching. Moreover, future research must include analysis of the educator's management in carrying out the professional tasks, since this management could be influencing the task results. At another level, there is targeted knowledge which we believe might actually have been mobilised by the PTs, but which was difficult to register in the data collected. This knowledge concerns the task recorded in the video as a model of how to teach primary education content, and the enrichment of the PTs' image and conceptual definition of a polygon.

The MTSK model has been used in the design of the task (to determine the potential of the video, to propose to PTs categories for the analysis of this video and to determine the focus of the second part of the task) and in analysing its implementation, providing lenses with which to systematise teacher education. It would also be interesting to study their possible role in professional task management.

Acknowledgment

This study was conducted within the framework of the project PID2021-122180OB-I00 under the Ministry of Science and Innovation of the Government of Spain, at the COIDESO research centre by the research group DESYM (HUM-168) and the MTSK network, under the sponsorship of the AUIP.

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