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► **To cite this version:**

Judit Chico, Juan Pedro Martín-Díaz, Miguel Montes, Edelmira Badillo. How do prospective primary teachers interpret ‘to promote understanding’?. Thirteenth Congress of the European Society for Research in Mathematics Education (CERME13), Alfréd Rényi Institute of Mathematics; Eötvös Loránd University of Budapest, Jul 2023, Budapest, Hungary. hal-04407308

**HAL Id: hal-04407308**

**<https://hal.science/hal-04407308>**

Submitted on 20 Jan 2024

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# How do prospective primary teachers interpret ‘to promote understanding’?

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*We present an exploratory study that aims to categorize the didactic intention of prospective primary teachers (PPT) when they transform multiplicative problems under the prompt ‘to promote the understanding of mathematical contents’. Constant comparison and inductive methods were applied to examine the justifications of the transformations performed by PPT, in order to identify the didactic intention, meaning their interpretations of the given prompt. Four main categories emerge from the data: focusing on content, varying complexity of the problem, facilitating understanding of the word problem, and evaluating understanding. We illustrate and describe in detail the nature of each category and conclude reflecting on the PPT’s interpretations of the prompts as a variable to take into account in the analysis and data collection.*

*Keywords: Didactic intention, understanding, interpretation, teacher education, problem posing.*

## Introduction

Research on the mathematics teacher has been addressing different foci for several decades. In particular, there is a variety of research that focuses on the teacher's knowledge about students' learning (e.g. Shulman, 1986; Carrillo et al., 2018; Jakobsen et al., 2022), as well as their ability to notice students' understanding (e.g. Callejo & Zapatera, 2017; Ullusoy & Çakıroğlu, 2021), or their beliefs about teaching and learning mathematics (Ponte, 1999). Thus, teachers' pondering of the elements that influence their students' learning encompass cognitive, affective, or socio contextual aspects. When a teacher makes decisions, particularly to promote student learning, he or she does so with a set of resources, goals, and orientations (Schoenfeld, 2010). Resources refer to the cognitive elements that underpin decision making, while goals define the teacher's ultimate objective. Orientations comprise elements such as beliefs, which shape goals and the articulation of cognitive aspects, determining the characteristics of the goal and the way to approach it. The choosing of a goal for the task determines the didactic intention of a teacher.

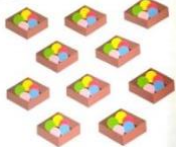
Intentions have been the subject of research in general approaches (Baxandall, 1985; Duranti, 2006), and in others specific to Mathematics Education (Sensevy, 2012; Brousseau, 1997). The intention in an action is given by the final goal and the orientations that concrete it. Thus, the idea of intention can be considered at various levels of specificity (Sensevy, 2012), from the general orientation and goal of an action, to the particularities that lead to its final form. Particularly, as didactic intentions we refer to the intentions that a teacher has to teach certain mathematical contents to their students (Brousseau, 1997). In this regard, tasks in initial teacher training usually impose a focus (Grevholm et al., 2009), assuming that there is a didactic contract between trainer and future teacher. Concretely,

in tasks for teacher education with focus on problem posing, future teachers transform a problem with a didactic intention determined by their interpretation of the prompt of the task. However, the didactic intentions of future teachers haven't been broadly addressed in mathematics education research concerning their problem posing skills.

This paper is part of a broader study focused on characterizing the teachers' professional competence to pose and transform mathematical problems in elementary education. Specifically, this study characterizes the didactic intentions of PPT when transforming problems, answering the research question: How do PPT interpret the prompt 'to promote understanding' when transforming problems? The PPTs were asked to transform a problem 'to promote understanding'. This prompt was interpreted by PPTs from different perspectives, reflecting different didactic intentions. Since this approach is in line with methodologies of Grounded Theory (Strauss & Corbin, 2002), lacking explicit theoretical references about didactic intentions, this paper is not based on a specific theoretical framework.

## Methods

This research is an exploratory study in which the analysis is developed from an interpretative approach (Scheiner, 2019). We design a task addressed to 159 PPT at the end of their training of the second course of the academic year 2020-2021 in the University of Huelva (Spain). At this point of the degree, the PPTs have passed one subject focused on problem solving and another focused on numbers, arithmetic and its teaching. The task demands to transform a multiplicative problem with the structure of isomorphisms of measures, with the prompt 'to promote the understanding of the mathematical contents' identified in the word problem (Figure 1).

Modify the following word problem to promote the understanding of the mathematical content you identify. For each modification, indicate in the table: the type of transformation; the reason why you perform it; and the contribution to the students' mathematical learning.
Problem: Bruno has 10 boxes with marbles. Each box contains 5 marbles. In total, how many marbles has Bruno?


**Figure 1: Word problem of the task**

A total of 233 transformations with their corresponding written justifications were collected. We began an inductive coding process based on constant comparison methods (Strauss & Corbin, 2002) with the aim of characterizing the didactic intentions in the justifications of the transformations performed by PPT. This process was carried out in three cycles of analysis. In the first cycle, two independent coders begin a process of open coding of the written justifications and compare their codification in order to negotiate interpretations and meanings. From this cycle, a first set of codes emerges: wording, simplifying, context, among others. In the second and third cycle, the initial codification was refined and expanded with new emerging codes following the previous process. This process involves the negotiation of a nomenclature common to both coders, which at the end of the third cycle is discussed with two other researchers who triangulate the coding. In this third cycle, relationships and hierarchies between codes are also established, outlining the nature of each category

in memoranda. The second and third cycles are repeated until new codes don't emerge and the relationships between codes become stable. Once the four non-exclusive categories have been established, we count the frequency of each category and calculate the percentage of each with respect to the total number of transformations in which intention was identified. In the next section, we illustrate the four emerging categories along with their frequency.

## Categories for the didactic intentions of PPT

We identify at least one didactic intention in 94% of the written justifications analyzed, classifying 97% of these intentions in four main categories: i) focusing on content; ii) varying the complexity of the problem; iii) facilitating the understanding of the word problem; and, iv) evaluating understanding. Below, we describe the nature of each intention and its frequency in the data set.

### Focusing the problem on specific contents

Focusing the problem on specific contents is the most frequent interpretation of the prompt 'to promote the understanding', appearing in 66% of the responses of the PPTs. This category includes transformations with the intention of focusing the problem on: i) division (24%); ii) multiplication (19%); iii) other content such as addition, probability or percentages (12%); and, iv) the combination of various operations, mostly multiplication or division with addition or subtraction (11%). For example, PPT16 proposes to add a question to the word problem to focus the transformation on the commutative property (Figure 2). Thus, most of the PPT transform the problem to address meanings, properties or the calculation of multiplication or division. In many cases, PPTs transform the multiplicative rate problem to a quotative or partitive division problem by exchanging the given and demanded information in the original word problem (Figure 4). Also, PPTs intend to focus on division with remainder or certain times tables (Figure 5).

Transformation: Add the following question, "If his sister Laura has 5 boxes with 10 marbles each, who has more marbles?"
Justification: Add a new question and interchange the numbers involved in the operation. To practice the commutative property. For students who are learning to multiply, to practice the properties of operations helps them to understand better and strengthen their learning.

**Figure 2: Problem posed and justification of PPT16**

We also found transformations that intend to promote the understanding of different meanings of multiplication or to add questions so that potential solvers have to combine the use of multiplication or division with an addition or a subtraction in their solutions. For example, PPT33 intends to facilitate the understanding of multiplication as a repeated addition and introduces a subtraction in the solution of the problem (Figure 5). Finally, and less frequently, PPTs modify or eliminate the image to promote the use of a particular problem-solving strategy. For instance, PPT119 proposes changing the image and showing a single box with five marbles (the multiplicative unit) "*so that the children do not just count all the marbles in the boxes (...) and conclude that they must multiply rather than add the marbles in each box*".

## Varying the complexity of the problem

Varying the complexity of the given problem is the second most frequent didactic intention when prospective teachers transform it under the instruction to 'promote understanding', appearing in 35% of the responses analyzed. In this category, we find some transformations with the intention of increasing the complexity of the problem (22%) and others of decreasing it (13%). It is worth mentioning the great diversity of strategies used by the PPTs to transform the problem with this didactic intention. We found PPTs that vary the data of the problem so that future solvers practice more complex times tables than ten- and five-times tables (Figure 6). This strategy is also widely used to decrease the complexity of the given problem. For instance, PPT46 uses smaller quantities to decrease the complexity of the problem solving process and to promote the understanding of repeated addition and multiplication (Figure 3).

Transformation: Bruno has 5 boxes of marbles. Each box has 2 marbles. How many marbles does Bruno have in total?
Justification: To reduce the difficulty of the problem-solving process, because if the student does not know how to multiply, he/she can do a shorter repeated addition with less difficulty. To promote the understanding of mathematical concepts, in this case, multiplication and repeated addition.

**Figure 3: Problem posed and justification of PPT46**

In other cases, the PPTs intend to increase the complexity of the problem by: introducing irrelevant data in the word problem, adding steps to the problem to involve more than one operation in the solution, modifying the image to avoid counting marbles, or varying the contents of the problem. For example, PPT109 transforms the given problem into a partitive division problem because it is considered a more complex content for elementary school students (Figure 4).

Transformation: The proposed change would be: Bruno has 10 boxes with marbles. In total he has 50 marbles. How many marbles does he have in each box?
Justification: The proposed change would be to change from multiplication rate to division partition rate. As in the proposed problem we do a multiplication to solve it, I have thought about changing the word problem to have to do a division when solving it, and make it more complex since most of the students find divisions more difficult. The objective is to encourage students to use division, since it is more complex than a multiplication when solving the problem.

**Figure 4: Problem posed and justification of PPT109**

## Facilitating the immersion in the word problem

Transformations with the intention of facilitating the understanding of the word problem represent 20% of the written justifications analyzed. This category includes those transformations that modify: i) the context, proposing a situation closer to elementary school students' experiences to encourage their immersion in the problem (14%); ii) the word problem, introducing connectors or changing the

order of the information provided in the wording or modifying the colours or size of the image to facilitate the interpretation of the information given (8%). Occasionally, PPTs combine several changes to facilitate the understanding of the word problem as illustrated in Figure 5, in which PPT 33 performed three different modifications. First, PPT33 modifies the main subject of the word problem by placing the reader as the main character. Second, he/she changes the order of the information given in the word problem, presenting first the information about the amount of marbles in one box. And last, PPT33 numbers the boxes in the image of the word problem. This combination of modifications is not only intended to facilitate the understanding of the word problem, but also to promote the meaning of multiplication as a repeated addition. Furthermore, PPT33 adds a new step in the problem to introduce a subtraction in the solution of the problem. This example highlights that PPTS can combine several modifications to address one or more intentions.

Transformation: Today is your birthday, and your mother gave you a box containing 5 marbles. How many marbles would you have if you were given 10 boxes in total? If you lose 10 marbles, how many marbles do you have left? (I would number each box in the picture, i.e., write the numbers 1 to 10 on top of each box).
Justification: In the first place, I consider that, by placing them as the protagonists of the word problem, they understand the context better, and therefore, they are able to solve it better. On the other hand, I have presented first the marbles in one box, to ask how many there would be in the 10 boxes, since I believe that to facilitate their understanding it is better for them to understand the problem from the smallest to the largest details.

**Figure 5: Problem posed and justification of PPT33**

## Evaluating the comprehension of contents

Finally, the didactic intention of evaluating the understanding appears in 20% of the responses. This category includes transformations with the intention of using the problem to: i) review certain contents while elementary school students solve the new problem (18%) and, ii) check the students' understanding of the contents involved in the new problem (2%). For example, PPT44 (Figure 6) has added a step to the problem with the intention of reinforcing the subtraction operation. He/she also modifies the data in the problem to allow students to practice more complex multiplications.

Transformation: My first change would be to modify the data of the problem by using more complicated numbers such as 29 boxes with 8 marbles each. My second change, I would add another question: If two boxes are taken away from Bruno, how many marbles does he have left in total?
Justification: I would perform this change to make the problem-solving process a little more complicated since I think that for the students, the 8 times table is more complicated than the 5 times table. They would learn through practice, to further develop the skills of multiplication. I would make this change so that students would continue to practice multiplication and reinforce the skills of subtraction.

**Figure 6: Problem posed and justification of PPT44**

It is interesting that PPTs perform the same type of modification to address different intentions. For example, adding an additive procedure to the problem-solving process may respond to: the intention of focusing the problem on the combination of several operations, increasing the complexity of the problem, or reinforcing the learning of content related to addition or subtraction.

## Discussion and conclusions

Decision-making during the formulation of problems involves the consideration of goals (Cruz, 2006), as well as the teachers' beliefs. It is known that there is a need to characterize different dimensions of problem posing (Cai, et al., 2022) This paper proposes a first characterization of the didactic intentions for the transformation of problems, according to the goals that prospective teachers assume. Thus, these transformations aim to focus the problem on different contents to be practiced by future solvers, to vary the complexity of the given problem, to facilitate the understanding that the solvers might have of the word problem, or to guide the resolution of the problem towards an evaluation task in order to review certain contents. As a summary, Table 1 shows the percentages of each didactic intention and its variants identified in the responses of the PPTs.

**Table 1. Percentages of each didactic intention**

Focusing the problem on specific contents			Varying the complexity		Facilitating the immersion in the word problem		Evaluating the comprehension	
Multiplication/ Division	Others	Combining operations	Increase	Decrease	Context	Word problem	Reviewing	Checking
43	12	11	22	13	14	8	18	2
66			35		20		20	

These results suggest that the PPTs mostly interpret the prompt 'promote understanding' as focusing the problem on content related to multiplicative thinking, increasing the complexity of the problem, reinforcing the understanding of certain content, or bringing the context of the problem closer to the experiences of elementary school students to facilitate its understanding. Interestingly, this category system, although disjoint, does not imply that two or more didactic intentions cannot coexist in the same transformation as illustrated in various PPTs responses discussed previously. Although a single interpretation of the prompt to 'promote understanding' predominates, there is a significant percentage, 34% of the cases (Table 2), that concretizes the given prompt as a combination of various didactic intentions that lead to modifying different aspects of the given problem.

**Table 2. Combinations of didactic intentions**

# intentions	0	1	2	3	4
Percentage	8	58	31	2	1

The contribution of this study is a first emerging categorization of the declared didactic intentions for the formulation of problems. However, it also provides a reflection about the data collection instruments used in the research, and in particular, the prompts we use in them, as well as the analysis tools used (Fauskanger & Mosvold, 2015). In this research, a prompt oriented to transform problems to "promote understanding" was chosen, highlighting that students assumed this prompt from four different views. Thus, in the design of research instruments, one must be aware not only of what one intends to investigate, but also take as a variable the possible interpretations that the subjects with whom one is investigating the question might assume.

This study has as natural line of continuity to relate the characteristics of the transformations of the problems (Chico et al., 2022) with the different didactic intentions that the PPTs assume during the transformation of the problems, in order to deepen the processes of problem formulation (Baumans & Rott, 2022). On the other hand, given that decision-making also involves the mobilization of cognitive resources (Schoenfeld, 2010), we think it is necessary to ask which specific professional underpins teachers' decision-making when transforming problems, using models of mathematics teacher knowledge analysis (e.g. Carrillo et al., 2018).

## **Acknowledgment**

To the teams of the research groups GIPEAM-2021-SGR-00159 and DESYM (HUM-168), the projects PID2019-104964GB-I00 and PID2021-122180OB-I00 of the Ministry of Science, Innovation and Universities of the Spanish Government, and to the Research Centre COIDESO of the University of Huelva. To the Red MTSK, funded by AUIP.

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