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Students' strategies for presenting the problem

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Students are, to various degrees, expected to present a context or explain what problem they are working on when they report on their problem-solving work. This expectation rarely comes with instructions on the best way to do this, and, as with most writing in mathematics, research on the issue or curricular standards is limited. This study uses multimodal analysis to investigate students' strategies for presenting the problem, premises, and facts. The students are part of a research project focused on developing their ability to design written accounts of problem solving by addressing the efficiency and clarity of their writing. Findings suggest students use one of two strategies: integrating the problem and its premises with the calculations or separating the information from the rest of the account. Within these strategies, students approach demands for effective and clear communication in different ways using different semiotic resources.

Keywords: Problem solving, writing, problem presentation, strategies.

Written accounts of problem solving

Commonly, people will want pen and paper when solving a mathematical problem. The opportunity to write down numbers and calculations or draw a picture reduces the strain on your working memory and helps you organise and structure your ideas. Along with being cognitive, this type of writing is also personal, as the person you communicate with is yourself. Communicating with someone else requires an entirely different kind of communication where you must consider several issues regarding how and what to share and with whom you are speaking. In school mathematics, students are often asked to solve a mathematical problem and hand in a written account of their process, where they show their thinking without separating the two processes of (i) solving the problem and (ii) writing about it. Given that students' mathematical writing has been used for centuries to assess their mathematical ability, Morgan (2001) argues that this writing is important enough to merit an interest beyond its indiscernible part of the problem-solving process, thus suggesting that the ability to communicate through writing in mathematics is an ability that can be separated from other mathematical skills. Communication competence is also an essential part of competence frameworks for mathematics, such as, for example, the Danish KOM project. This importance, however, is not reflected in the level of attention that writing is given in school mathematics, where teachers seem to have few ideas on how to improve their students' writing beyond instructions to use mathematical notation (Teledahl, 2015). Research also provides little guidance on the quality of mathematical writing or how it can be taught (Powell et al., 2017). My colleagues and I developed a teaching model for a project about students' writing in mathematics. In this project, presented below, students and teachers have discussed students' writing with quality aspects such as clarity and efficiency in mind. This paper investigates how students approach these aspects when they report on their problem solving and present the problem and its premises and facts. The terms clarity and efficiency will be further elaborated below. The research question guiding the analysis is: *What semiotic resources are employed to achieve efficiency and clarity?*

Conventions and rules for presenting the problem and its premises and facts

In research-based phase models that describe students' problem-solving processes, it is common to include a phase in which students understand and make sense of the problem and its conditions (Rott et al., 2021). How much and in what way this phase should be documented in students' written accounts depends on how mathematical writing is viewed. Barwell (2018) suggests that mathematical writing can be viewed as a rule-based system that is fixed and stable and where the rules are something that students need to learn to succeed in mathematics. Unlike the phases of the problem-solving process that contain calculations and explorations of the problem and where there are certain universal conventions regarding the writing, there are few conventions for the understanding-the-problem phase. This creates a problem for students uncertain about what to write and how and for teachers who may be unsure about what they should demand. The second view of mathematical writing that Barwell (2018) suggests is the discursive model in which writing is understood as a social practice where the meaning-making is shaped by the social structure of, for example, a mathematics classroom. In a classroom where writing is viewed as a social practice, students and teachers must continually negotiate local rules, and it may be challenging to discern progression in students' ability to make clear and concise arguments and to have their text make sense outside the classroom context. In our project, we argue that mathematical writing in the context of school mathematics has elements that are socially negotiated but also elements that are guided by universal rules, conventions, and norms. With this idea, we designed a model for teaching mathematical writing, particularly the type of writing students engage in to report on their problem solving. This design will be described below to give an idea of the conditions in which data was collected for the study presented here.

The teaching design

The data for this study is collected from a research project in which the two processes of (i) solving a mathematical problem and (ii) communicating a solution to the problem in writing to someone else are separated. This creates an opportunity for teachers and students to have discussions focused exclusively on the communicational quality of students' writing.

The first part of our design is a problem-solving lesson. Students are presented with and work on a mathematical problem. This part of our design is enacted differently depending on what students and teachers are used to and comfortable with. The writing that takes place in this phase is the type of writing that assists students in their explorations, calculations, and discussions. Reporting is optional beyond writing on a whiteboard or using other means to help fruitful strategy discussions. When the exploration phase is over, some whole-class discussion should occur during which the students will, ideally, present and discuss different strategies for solving the problem. At the end of this, most students will have grasped the problem and at least one method for solving it.

Following the discussion, we ask all students to write down a formal mathematical text that reports on how the problem can be solved. This part of the design differs from the way writing is typically handled in school mathematics. In our lesson, students are not expected to describe their thinking but rather to describe and justify one of the ways the problem can be solved. This means that individual students do not have to report on the strategy they used in solving the problem; they can choose the strategy of a peer or a strategy discussed in class. In our design, this first part typically represents one

mathematics lesson, and this lesson is thus concluded with the teacher collecting students' formal mathematical texts.

The second part of our design, which mostly coincided with a second lesson separated from the first, aimed to discuss the texts the students produced in the previous lesson. In this part, we focus exclusively on the students' texts as products that can be assessed based on their communicational merits. The teachers in the project prepared the lesson by reading the students' texts and selecting a few to discuss with the class. In trying out the design, we, as researchers, discussed the students' texts with the teachers to identify interesting differences in quality. Using the limited research on writing in mathematics that exists (see, for example, King et al., 2016; Kosko & Zimmerman, 2019) together with research on communicative competence in general (Rickheit et al., 2008), we designed a framework that the teachers in our project could use as a tool for conducting discussions on the communicational quality of their students' mathematical texts.

The framework describes elements that should be featured in students' mathematical texts along with quality aspects. In this study, we will focus on a limited version of the framework in which we focus on the qualities of *efficiency* and *clarity* (see Table 1). Efficient communication can be described as achieving its goal with as little effort or as few representations as possible. Mathematics notation can be seen as the ultimate example of efficiency as its symbols result from increased standardisation and narrowing of linguistic options throughout the last two centuries (Solomon & O'Neill, 1998). Clarity is about choosing the linguistic or semiotic options best suited to guide the expected reader through the text, answering whether it is *understandable*.

Table 1: A framework for discussing students' mathematical texts

Elements \ Quality	Problem description	Calculations	Arguments, justifications	Conclusion
Efficiency				
Clarity				
Notation				

Method

The students' texts were analysed using multimodal discourse analysis (Jewitt, 2011), building on the assumption that communication draws on multiple modes, all of which may contribute equally to meaning. Multimodal discourse analysis examines the semiotic potentials and resources available to text makers in a particular context and aims to describe the choices taken (Jewitt, 2011). The analysis is thus focused on the role the text elements play (for an example, see Alshwaikh, 2016). A differentiation between conventions that may be deemed universal (see, for example, Lew & Mejía-Ramos, 2019) or conventions that are locally negotiated was not identified through the whole-class discussions on the merits of the texts. The analysis was focused exclusively on the texts to identify different practices.

Data

The data for the study consists of 38 mathematical texts from two different problem-solving cycles in grade 6 (12-year-old students) and 27 from one problem-solving cycle in a first-grade upper secondary school (16-year-old students). Both classes were part of the research project, and the texts were derived from lessons in which the students had experience with the demand to include a problem statement to make the problem and its premises and facts clear, along with the expectation of efficiency and clarity.

Analytic procedure

The texts' analysis focused on examining the semiotic resources used by students to create meaning in relation to the problem and its facts and premises. The analysis started with a careful reading and interpretation of the three problems that the students were reporting on. After this, the analytic questions focused on identifying how different semiotic resources were combined to create efficient and clear communication of the problem. In this analysis, resources such as, for example, separation of elements, the use of space, sequencing, highlighting, colouring, and underlining were accounted for, as well as the use of words, symbols, and drawings. Efficiency was interpreted as the reduction of non-essential elements and efforts to have elements simultaneously serve different functions. Clarity was interpreted as an effort to create a narrative or a logical chain of information without missing essential steps.

Findings

The findings are presented with respect to the different strategies used to achieve efficiency and clarity. All figures contain texts translated and faithfully reproduced by the author to show something as similar as possible to the original.

Clarity

The most prominent semiotic resource used to create clarity was separation. This approach is used on two different levels. It is a common strategy across both student groups to separate the stating of the problem and its premises and facts from the rest of the text, where they report their calculations, arguments, and conclusions. Some students also separate their representation of the problem or question and the premises for solving it. The 6th graders demonstrate different ways of achieving this separation. They use semiotic resources such as lines, space, and naming the two elements. Naming the elements was the most common strategy in problem cycle 4 (see Figure 1). Integrating the problem statement and the facts with the calculations and arguments is a far less common strategy.

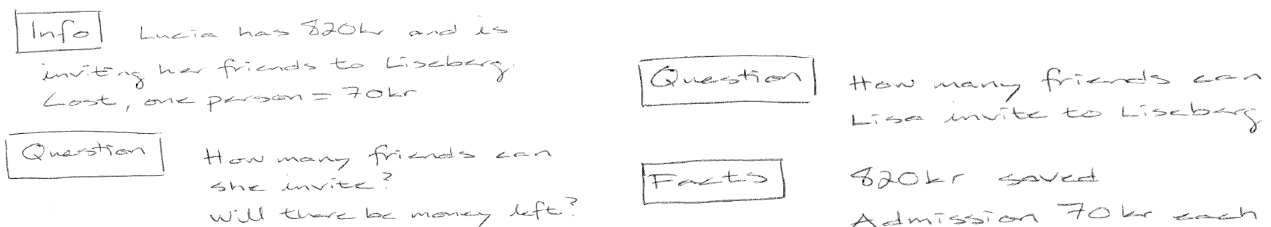


Figure 1: Two examples of how students separate and name the different elements

Representing the problem or question at the beginning of the text is common, mainly using words only. This strategy includes repeating the entire problem word for word, although this is rare. The most common strategy is to interpret the problem and use words to present an abbreviated version of it that still contains the necessary facts. It is worth noticing that it is relatively common among the students who abbreviate the problem to still include non-essential facts, for example, the name of the person building paddocks. Presenting the problem at the beginning of the text ensures that the question and premises for solving the problem are stated before anything else, enhancing the clarity of the rest of the text. It is very common, however, for students who present the facts using words to repeat the same facts using a drawing, a visual representation of the premises (see Figure 2). In this way, redundancy is accepted to enhance clarity.

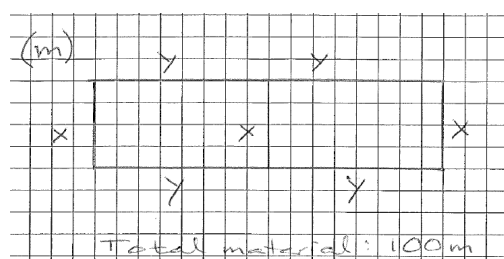


Figure 2: Presenting a visual representation of the same facts that are presented with words

Efficiency

The most common strategy to achieve efficiency when separating the problem and its premises and facts from the rest of the text is abbreviating the problem and combining resources such as words, symbols, numerals, and drawings by letting them play multiple roles in communication. This last strategy is only sometimes carried out so that something necessary is presented only once through one form of representation; there are instances where different forms of representation are used to say the same thing. This reflects a consideration of the balance between clarity and efficiency where students come to different conclusions.

Some students who use words to state the problem and the facts consider efficiency by presenting only the facts and premises necessary for understanding the problem. However, they still carefully show them in an order that enhances clarity (see Figure 3). The students that present the problem and the premises and facts together could be placed on a continuum ranging from *giving every fact* to *offering only the essential facts*.

- Build 2 rectangular paddocks of equal size
- Total 100m fence
- One side common
- How large can the paddocks be?

Figure 3: Presenting only the necessary facts

Some examples of texts employ an integration of the facts with the calculations, i.e., the numbers are not part of the presentation of the problem (see Figure 4). This is a way of avoiding redundancy that favours efficiency over clarity. The 6th graders are far more likely to separate the presentation of the problem and its premises and facts, thus repeating numbers at least once.

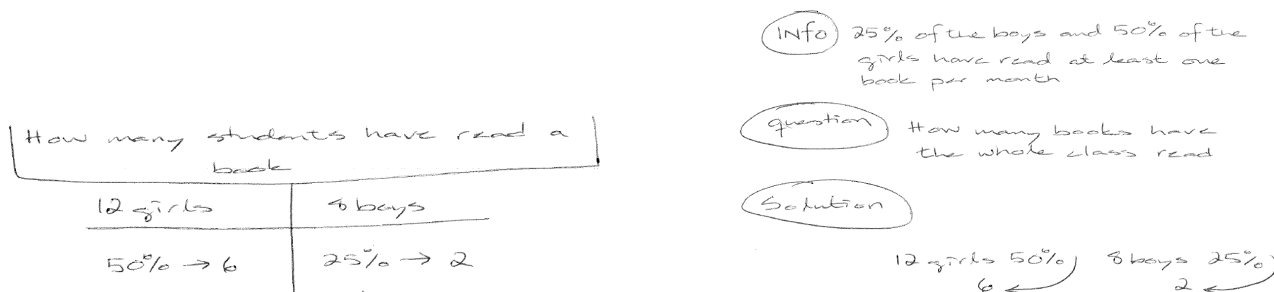


Figure 4: Two examples of integration of facts and calculations

Summary of the results

On an overarching level, students approach efficiency and clarity by separating the problem's presentation and its premises and facts from their calculations, justifications, and conclusions or by integrating them. Once they have chosen separation or integration, they employ different semiotic resources. The balance between efficiency and clarity in texts that have separated the problem statement from the rest of the text, which is the most common strategy, is approached using strategies such as abbreviations of the problem, combining different semiotic resources and using order to enhance understanding.

Discussion

The two conceptualisations of mathematical writing, a rule-based system and a socially negotiated discourse, conflict regarding how students should be taught to write. The first conceptualisation builds on the idea that there are already rules for all parts of the writing, while the second suggests that there are few if any, rules. The findings in this study may serve as an illustration of this potential conflict since the existence of locally negotiated practices, such as the careful separation of the facts and the question (see Figure 1), are visible alongside universal methods, such as visually representing the problem (see Figure 2). Outside of individual classrooms, mathematics education does not prescribe distinct rules for how students should state the problem and present the premises and facts when accounting for their problem solving, but neither does it suggest that all the different ways to do this are equally good. When the two conceptualisations of writing coincide, like two sides of a coin, teaching students how to write “better texts” becomes challenging in classrooms. Research has also pointed out that teachers find it difficult to attend to progression in students’ mathematical writing, claiming they lack the tools and the terminology (Morgan, 2001; Teledahl, 2015). In our project, we aim to view the two conceptualisations of writing in mathematics as complementary rather than dichotomous, in line with Stylianides’ (2007) ideas on proof in primary school mathematics. Stylianides decomposes the conventional requirements of mathematical proof and transforms them to function in a school context that includes young children. In this way, he shows that it is reasonable for third graders to produce understandable proofs for their peers and to have a clear connection to the general and universal requirements for mathematical proofs.

In the two classes in this study, the students and their teachers have discussed different aspects of quality in students' mathematical writing, especially the two aspects of *efficiency* and *clarity*. These are complementary because clarity can be achieved at the expense of efficiency and vice versa. As the findings suggest, there are different ways to approach the balance between efficiency and clarity, and some of these differences are likely dependent on different local classroom practices. In contrast, others are at least partly dependent on mathematical conventions. What is considered good mathematical writing depends on context. If we want to keep individual differences and creativity but also discuss what constitutes good communication among students, we need exemplars to discuss. We also need to have ideas on what constitutes quality. What good is a discussion on something so insignificant as how to present a problem? It is perhaps not the practice itself but the opportunity to discuss different ways to achieve precise and effective communication that is important here. A discussion on efficiency and clarity has the potential to turn the discussion into mathematical notation, an effective form of communication that humanity has perfected for the last 200 hundred years (Solomon & O'Neill, 1998). Such a discussion, while considering ways to achieve efficiency, would offer opportunities for students to appreciate notation in a way they otherwise would not. It can also introduce a discussion on the necessary facts and various forms of representing them. This discussion can develop students' ability to distinguish essential and non-essential facts in future problems. The exemplars used in the study derive from students' work, and they have lively discussions where they question what is necessary and what is understandable. The different ways of presenting a mathematical problem as part of an account of problem solving that are presented here can serve as a starting point for a discussion on quality in students' mathematical writing. Such a discussion provides opportunities for students to develop a meta perspective on their writing, something we believe will benefit them not only in their future writing in mathematics but also in their problem solving and their writing in other subjects.

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