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► **To cite this version:**

Núria Planas, José María Alfonso. Practices of noticing important aspects of mathematics teaching talk (about linear equations) with secondary-school mathematics teachers. Thirteenth Congress of the European Society for Research in Mathematics Education (CERME13), Alfréd Rényi Institute of Mathematics; Eötvös Loránd University of Budapest, Jul 2023, Budapest, Hungary. hal-04406752

HAL Id: hal-04406752

<https://hal.science/hal-04406752>

Submitted on 19 Jan 2024

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Practices of noticing important aspects of mathematics teaching talk (about linear equations) with secondary-school mathematics teachers

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Our study explores the potential of tool-supported discussions with a group of seven secondary-school mathematics teachers in helping them to notice important and critical aspects of mathematics teaching talk. We take the one-day workshop around linear equations to illustrate our guiding tools: a framework of mathematical practices of naming and explaining in mathematics teaching talk, and an associated framework of students' mathematical learning challenges. The analyses of discourse moves in the collective discussions during the workshop reveal that the two tools interacted between them, and in connection with discourses of the teachers on teaching and learning to guide the identification, interpretation and elaboration of mathematical naming and explaining in content teaching talk aimed at supporting the students' understanding and learning of linear equations.

Keywords: Noticing work, secondary-school mathematics teachers, mathematics teaching talk, mathematical naming and explaining, tool-supported discussions.

Introduction

The current study is part of a project in which we work with secondary-school mathematics teachers on mathematics teaching talk aimed at supporting students' learning of specific mathematical contents. In the context of task-based workshops, we present instances of mathematics teaching talk to the teachers, from classroom lessons of completed projects of the team or from our own teaching experiences, and we draw on the noticing approach of Mason (2002) to support teachers in what to tackle and how to interpret it. For helping the groups of teachers to engage in the social practice of noticing important and critical aspects of mathematics teaching talk, we use tools with the potential to guide focused discussions. The multiplicity of aspects involved in mathematics teaching talk can be overwhelming and detrimental for developing focused attention; hence, the guiding tools function to select some aspects over others. In line with Vygotskian views (Engeström, 2015), we take the tools as mediational and interactional, because they are expected to mediate the teachers' noticing practices, interacting between them and with other contextual circumstances. Our tools are a framework of mathematical naming and explaining practices in mathematics teaching talk (Planas et al., 2023), in articulation with a framework of students' mathematical learning challenges (Planas & Alfonso, 2023). These tools link teaching with learning by assuming connections between aspects of content teaching talk and the creation of opportunities for students' content learning.

In this report, we examine the potential of the tool-supported discussions with the seven teachers in one of the task-based workshops, specifically the extent to which they engaged in noticing practices of mathematical naming and explaining in relation to linear equations. Our research question is: *How can a framework of teaching practices of mathematical naming and explaining, in articulation with a framework of students' learning challenges, help teachers to notice collaboratively important and critical uses of mathematics teaching talk?* The emphasis is thus on the potential of the tools to support focused discussions rather than on, e.g., the function of the teacher educator or the tasks.

Literature review and theoretical framework

Mason (2002) refers to the discipline of noticing as a social practice and process whose development enriches the reading of professional and educational practices, and by doing this, it entails some potential for transformation of these practices. For teachers to develop a discipline of noticing aspects of teaching practice and further elucidating reasons for what they do and say in their teaching, they need to learn to give focused attention to these aspects and reasons so that they can reflect on them. The social stance in all this points to two central conditions of noticing development: participation in communities of practice, for shared work and learning in collaboration with the others and their experiences, and learning new specialized discourses, for interpretation of and decisions about aspects and reasons that can become visible with specific professional and educational meaning. Alongside the use of tools to focus attention, therefore, noticing development requires these tools to be placed in a social site of participation that allows the learning of a new specialized discourse.

In the thinking of the workshops with mathematics teachers as communities of practice for learning a new specialized discourse about the place of mathematics teaching talk in the enactment of students' understanding, we take noticing as a three-layered process. Parallel to Mason (2002), van Es and Sherin (2002) proposed a conceptualization of noticing that we adapt as consisting of: (1) identifying instances of mathematical naming and explaining in mathematics teaching talk; (2) interpreting the importance and criticality of these instances in the enactment of opportunities for students' mathematics learning; and (3) deciding how to improve or produce instances of mathematical naming and explaining to better support students in their content learning. In our study, the development in combination of a continuum of these noticing processes of identifying, interpreting and deciding, in the teacher education work, relies on the tools briefly described in the next two paragraphs.

We want teachers to engage in professional talk about mathematical naming and explaining in mathematics teaching talk that contributes to communicating meanings that are basic to the understanding of contents of secondary-school mathematics, such as linear equations. Naming here consists of giving word names or phrases from the mathematical register (Halliday, 1978), which for linear equations in two or more variables include, e.g., 'unknown', 'equal sign', 'equivalence' or 'algebraic expression', but also 'variable' and 'value variation', for distinctions with linear functions. Naming in the form of noun phrases includes the description with letters and words of mathematical objects symbolically or visually represented, e.g., 'y is triple the size of x' for $y=3x$. Explaining is a step further (Adler et al., 2023). It includes word names and phrases and consists of sentences that communicate explanations of mathematical meanings and relationships amongst them within a distinguished mathematical register. An example of explaining would be 'the equation $y=3x$ represents how the variables x and y relate to each other, while the function $y=3x$ represents how the value of y varies at specific values of x '. This sentence contributes to advancing some understanding of the distinction between linear equations and linear functions without necessarily adding new meanings, but relating those that may have been taught and introducing ways of reinterpreting. Our focus on and interpretation of the mathematical-linguistic practices of naming and explaining are rooted in the use of explicitness in language (Halliday, 1978) and in teaching that is sufficiently explicit in the communication of important content meaning (Planas et al., 2023).

In the workshops with secondary-school teachers, we aim to promote noticing that makes sense in mathematics teaching talk that is linguistically responsive to what students need to know and what they struggle to understand around specific content. For this, alongside a framework of mathematical naming and explaining in teaching talk, we adopt a framework of students' content learning challenges. Whereas many challenges originate in the epistemic complexity of the objects of learning and are tied to existing preconceptions, research also suggests connections between the persistence, reduction and prevention of students' learning challenges and the role of the classroom practice and the learning experience (e.g., Hiebert & Grouws, 2007). We thus connect the teachers' pedagogic knowledge of these challenges –and knowledge of learning and learners– with noticing work on mathematics teaching talk and the challenges embedded in mathematical naming and explaining that can support content learning. In this way, a framework of students' learning challenges is thought of as a practical tool to facilitate the teachers' noticing of what and how to name, and what and how to explain, that is mathematically and pedagogically important in the enactment of content teaching and learning, regardless of individual students and particular learning situations.

The mathematics education research literature reports various challenges in the students' thinking of linear equations across classroom contexts. These are often visible when students become confused or wrongly apply rules for transforming expressions and solving equations, with reasoning based on notation manipulation and arithmetical properties. Some learning challenges around linear equations are conceptually based, in the sense of indicating insufficient seeing of mathematical structures and relationships, including the algebraic meanings of equality and equivalence compared to the arithmetic meanings. In the design of the teacher education workshop, a framework of students' challenges in learning linear equations was elaborated around two largely documented challenges in the literature (Kieran, 1992; Rojano, 2022): linear equations reduced to manipulation of literal-symbolic notation, mostly linked to the mechanization of the procedures of algebra; and linear equations thought of as answering situations of finding the solution, mostly linked to the lack of distinction between calculation and structure. We assume that manipulating notation and finding the solution are important but do not reflect the totality of what needs to be communicated in classroom teaching talk and practice. For the workshop, we wanted the teachers to reflect on mathematical naming and explaining in teaching talk that could serve to revise the idea that manipulating notation and finding the solution is the kind of knowledge that linear equations are all about.

Research design and analytical methods

The empirical site of this study was a one-day workshop on linear equations with seven mathematics teachers. The workshop took place in the secondary school of three of them, the other four teachers being in the same educational district. All these teachers had responded to a call sent by the research team, in collaboration with the educational district, with the invitation to participate in one or more workshops on the teaching of a selection of contents of the mathematics curriculum in secondary-school grades. It was the first workshop for the seven teachers (none of them had chosen the early workshop on the teaching of angles reported in Planas et al., 2023). They all had mathematics or science university degrees and their experience of secondary-school mathematics teaching ranged from eight to twenty years. The first author was the teacher educator of the first round of one-day workshops in 2021 and 2022, including the workshop in this report. At the time of writing, a second

round of workshops has been initiated with the second author as the teacher educator and the enrollment of new mathematics teachers. Both authors have experience teaching mathematics in the secondary school and of conducting teacher education work with mathematics teachers.

We view the workshops in terms of participation in and learning social and focused practices of noticing. The use of professional tasks that enhance tool-supported discussions is thus important in the developmental dynamics. The two tools applied to linear equations were, on the one hand, for the information on students' understandings of linear equations as manipulating notation and finding the solution, and, on the other, for the consideration of the mathematical naming and explaining of algebraic aspects of sense structure in teaching. The workshop had three parts. In the first part, the framework of students' learning challenges was shared with the teachers. The teacher educator spent about 30 minutes to introduce data from the literature or prior projects of the team, and simulated interactions where students struggled to capture structure sense and apparently paid attention only to the numbers in the equations and the operational rules, considering each side or even each rule separately, with no evidence of strategies before manipulation. She asked the teachers if they also had examples of students in their classrooms experiencing similar challenges in the learning of linear equations. During this time, the emphasis was on documenting difficulties with structure sense, or with seeing equations as entities that could be represented into substructures in reciprocal relation. The second part consisted of about 120 minutes distributed equally for engagement in two professional tasks, PT1 and PT2, of identification, interpretation and proposal of mathematical naming and explaining for learning linear equations. After a break, the third part consisted of 90 minutes for discussion of the place of teaching talk in mathematics teaching and learning.

The teachers were given paper sheets and time to read instances of mathematical naming and explaining in teaching talk that could be seen as critical or improvable from the perspective of helping to challenge students' learning. The examples of naming and explaining were taken from three audio recorded lessons in which the three classroom teachers had proposed the same two mathematical tasks (MT1, MT2) –as part of the experimental design in a former project–. For PT1, MT1 was to discuss whether $x=5$ is a linear equation. For PT2, MT2 was to discuss whether $x=5$ and $3x=15$ are the same equation. Table 1 illustrates instances of naming and explaining of the three teachers indistinctly, translated from Catalan. The first author reviewed the recordings of the teaching talk in the lessons to search for naming and explaining oriented to the appreciation of structure and algebraic properties in the equations. A variably “relaxed” language of mathematics in the lessons was interpreted as reasonable attempts to avoid the continuous use of a too dense register. In this respect, an idea shared with the teachers in the workshop was that mathematical-linguistic “relaxed” instances of teaching talk do not necessarily reflect a lack of mathematical knowledge or lack of experience in teaching equations. Moreover, it was important not to present mathematical-linguistic correctness as the only way or determinant to support mathematical meaning making and reasoning.

During the day of the workshop, collaborative discussions started with the teacher educator asking the teachers to report what they had noticed in the teaching talk exemplified so that different reflections could be shared and examined. The teachers were encouraged to conduct the group discussions on their own by addressing the prompts written on the sheets. The workshop was audio recorded, and the written sheets, with the initial notes, were collected to double-check the proposals

of mathematical naming and explaining on which the teachers agreed in their responses to PT1.3 and PT2.3. A research officer transcribed the recordings, and the first author listened to them several times, organized the data and summarized the findings. She looked for discourse moves in which the naming and explaining in PT1 and PT2 had been raised as important or critical, and in which some reasons for this had been noticed, agreed on and shaped in the group. This process led to segments of data in the form of episodes illustrating discourse moves to identify, interpret or create teaching talk and challenges around this. The beginning of an episode was the move or statement in which a teacher suggested an instance of naming and explaining –taken from the task or created collaboratively– as challenging, in the sense of inhibiting or facilitating students’ understanding of linear equations. The end of an episode was marked based on content shifts in the discussion. The second author, a mathematics teacher educator and a secondary-school mathematics teacher in formal training as a doctoral student, examined the findings and moves identified, for discussion and clarification of what was not apparent to him or comprehensible from his reading of the summaries and from his knowledge of the workshop dynamics. This collaboration allowed the first author to go back to the transcription and examine how some findings could be more easily characterized and better argued.

Table 1. Summaries of the professional tasks in the workshop

First workshop task (PT1)		
Classroom task	Naming $x=5$	Explaining $x=5$
(MT1). Discuss whether $x=5$ is a linear equation.	<i>The unknown is five. // x is five. // x equals five. // The value of x is five. // An equation that is true for five and only five. // A linear, or one-degree equation with number five. // An equation that is five.</i>	<i>It is linear because it is already solved and we can check the one solution, five. // The power of x is one, so it is an equation of a linear type. // It does not look like an equation, but it is and linear, because we do not read a power of two or more. // It is an equation, and it is linear, but it is not written in standard form as usual. // Check whether you can represent it like a times x plus b equals zero and how this gives you the quick answer.</i>
PT1.1. How do these instances of teaching talk support the learning of linear equations? PT1.2. Which instances support algebra structure sense? Which ones rather promote unreasoned ways of manipulating notation and finding the solution? PT1.3. Propose your ways of naming and of explaining $x=5$ to support algebra structure sense.		
Second workshop task (PT2)		
Classroom task	Naming <i>same equation</i>	Explaining $x=5$ in relation to $3x=15$
(MT2). Discuss whether $x=5$ and $3x=15$ are the same equation.	<i>Equal equation. // Equivalent equation. // Equation with everything identical or almost. // Comparable in solution. // Equation after applying some rules of transposition. // Same number or expression adding or multiplying both sides. // Same line graph.</i>	<i>The value of x is not necessarily five in any equation, but it is for these two, so same equation. // They are identical to each other in the numerical solution. // Like five equals x same as x equals five. Different representation, same equation. // Let us think of three times five, and three times x, and how the three is important here. // Fifteen divided by three is five, that is the key to start.</i>

PT2.1. How do these instances of teaching talk support the learning of linear equations?
PT2.2. Which instances support algebra structure sense? Which ones rather promote unreasoned ways of manipulating notation and finding the solution?
PT2.3. Propose your ways of naming *same equation* and of explaining $x=5$ in relation to $3x=15$ to support algebra structure sense.

Findings and discussion

Consensus was reached on three broad discourse moves, i.e., three types of linguistic communication that helped to move the discourse forward on being responded to and accepted in the discussions. These moves characterize the teachers' noticing of important and critical uses of mathematics teaching talk in their tool-supported discussions. The moves and examples of statements that initiated them are: (1) challenges in the identification of mathematical naming in teaching talk (e.g., "I was surprised that none of the sentences on the sheet used the word structure. Did you see this?"); (2) mathematical explaining in teaching talk that voices the students' learning (e.g., "The teacher who says that it doesn't look like an equation, but it is ... To me, this issue of the equation looking is important, and what the teacher says here is a bit confusing"); and (3) classroom practice in relation to the mathematical naming and explaining of the teacher (e.g., "We can tell them that same here means same solution. We don't want them to run with the idea that same is exactly same when written"). Below, we briefly discuss the third discourse move by means of one episode.

Classroom practice in relation to the mathematical naming and explaining of the teacher

While the teachers spent much of the workshop time discussing the instances given, and anticipating challenges that students might face in making meaning for linear equations from them, they also produced teaching talk in forms that connected descriptions of *same equation* and explanations of $x=5$ in relation to $3x=15$. In this subsection, we reproduce an episode initiated by Teacher Quer in the discussion around MT2 and the reasons for $x=5$ and $3x=15$ being the same equation. In interaction with Teachers Isa and Vera, they talked about issues of communication in teaching related to engaging students in explanations with reasons that situated their thinking in the algebra domain.

- Quer: We can tell them that same here means same solution. We don't want them to run with the idea that same is exactly same when written.
- Isa: Yes, and I think it's a perfect moment for seeing the triple on both sides of the second one [$3x=15$].
- Quer: Yes, we cannot let them run with the idea that same equation is exactly same when written. So, it's a good task for making sure that they learn different ways of representing the same equation.
- Vera: Listen to what I wrote. We tell them that this one [$3x=15$] is this one [$x=5$] but now supporting the triple weight on each side. So, if you make the equation support the same weight on each side, you don't change the relationship between the two sides. Is that a good explanation?
- Quer: Yes, talking about keeping weights. Good idea.
- Isa: But we don't want to make them think of measuring by naming weights.
- Vera: Yes, that's true. Anyway, they can still think of numbers as having a weight value, and five something weighing more than fifteen something.
- Isa: And we can then talk about number of weights.
- Quer: Not too bad to think of a model of weights, so that they see the structure and what is kept in the relationship. If we show them the weighing balance with equal weights on both sides, what's wrong with talking about weights?

- Isa: It's funny but I use visuals with the algebra tiles and the weighing balance in one lesson. I don't use this kind of talk later, when all the rules, tests, and manipulation stuff come in, whether we want it or not.
- Vera: That's a pity, isn't it? We are not really getting to the content.

Two important decisions regarding talk in teaching are raised in this episode. First, the decision to tell students that 'same' in 'same equation' means 'same solution'. Second, the decision to talk about 'the triple weight on each side' and the relationship between the two sides that is common to $x=5$ and $3x=15$. This episode is unique in that the value of talk is interpreted in relation to forms of communication in mathematics teaching other than talk, particularly the use of visuals with the algebra tiles and the weighing balance. Isa does not only emphasize parallels between talk and non-linguistic communication, but she also raises some mismatch between her choices in the introductory teaching of equations –including talk to present the resources of the balance and the tiles– and her choices in later teaching concerning 'all the rules, tests, and manipulation stuff'. Teaching talk for literal-symbolic work with equations reduced to mentions of rules and manipulating procedures is then challenged. Furthermore, an underlying theme is the responsibility to use talk in teaching that is getting to the mathematical content and connected with students' challenges and needs. The teachers draw on their teaching practices and teaching talk to exemplify how difficult this can be. The point that the teaching talk of one teacher in different classroom lessons can be differently attentive to the students' learning challenges, because of some constraints, was taken up by the teacher educator so that the teachers could discuss possibilities of improvement and change in the final part of the workshop. They noticed teaching talk functioning differently –with respect to students' learning challenges– and, at the same time, that they were themselves using talk differently, sometimes promoting unreasoned ways to work with linear equations. By sharing insights to propose instances of talk, they brought up their teaching, and, by so doing, they noticed that changes in their teaching talk were about being more regular with talk that they already used in some of their lessons.

Conclusions

The tasks and discussions supported by our combined frameworks in the workshop guided the group of teachers on noticing that by examining and slightly changing teaching talk, they can support students' understanding of equations in their classrooms. A first contribution of this study to existing knowledge is that it has provided evidence of processes of learning to notice mathematical naming and explaining in teaching talk. A second contribution of this study is that it has shown that practices of mathematical naming and explaining can be reinforced by gaining clarity on what needs to be named and explained because of its general difficulty for secondary-school students. Although in the analyses we primarily searched for statements in which a teacher suggested an instance of mathematical naming and explaining as challenging and hence emphasis was on one framework, this search was considered from the perspective of how such an instance could be thought of as inhibiting or facilitating the students' understanding of linear equations. The framework of students' learning challenges was thus present in the episodes, for example, when the teachers discussed explanations so as not to let the students "run with the idea that same equation is exactly same when written."

Not all the three noticing processes intended were afforded in the same way. The interpretation of some instances of naming and explaining linear equations as critical with respect to student learning

was mediated by the recognition of talk of teachers who could have not developed further certain practices because of a diversity of reasons such as time constraints, students' cognition or epistemic complexity in mathematics. The teachers' noticing of critical teaching talk was particularly mediated by their seeing of their own teaching, students and classrooms. While it is not surprising that the teachers often referred to their classroom teaching, more so if they were confronted in the task prompts with talk of secondary-school mathematics teachers like them, the practices of mathematical naming and explaining, and their potential function of enacting students' content learning, remained in focus. This focus was recurrent and explicit in group discussions in which the teachers referred to the learning challenges of their students and to their mathematical naming and explaining in teaching.

Acknowledgments

To the Research Ministry of Spain (MICINN), Grant PID2019-104964GB-I00, to the Catalan Research Agency (AGAUR), Grant 2021SGR-159, to the teachers in the workshops and their schools.

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