



HAL
open science

Explorative study on language means used to explain the meaning of multiplying fractions

Jessica Mähnert, Kirstin Erath

► **To cite this version:**

Jessica Mähnert, Kirstin Erath. Explorative study on language means used to explain the meaning of multiplying fractions. Thirteenth Congress of the European Society for Research in Mathematics Education (CERME13), Alfréd Rényi Institute of Mathematics; Eötvös Loránd University of Budapest, Jul 2023, Budapest, Hungary. hal-04406739

HAL Id: hal-04406739

<https://hal.science/hal-04406739>

Submitted on 19 Jan 2024

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

Explorative study on language means used to explain the meaning of multiplying fractions

Jessica Mähnert and Kirstin Erath

Martin Luther University Halle-Wittenberg, Germany; jessica.maehnert@mathematik.uni-halle.de

In this paper, we give attention to how students engage in the exploration of mathematical concepts and in particular which language means students use during this exploration. The presented study is situated in the context of fraction operation, specifically focusing on fraction multiplication. Multiplying fractions is much more than learning and teaching the mathematical algorithm. It is about links and relationships and knowing why multiplying two fractions works as part of a part relationship. This qualitative study identifies (topic-specific) language means that students use to talk about their ideas of multiplying two fractions. The analyses show how important meaning-related language means are for building a conceptual understanding of multiplying two fractions.

Keywords: Language means, academic language, fraction multiplication, design research.

Introduction

The multiplication of fractions was chosen as the specific mathematical topic for the presented study since fractions are one of the most difficult topics in school, especially if the aim is developing a conceptual understanding. Many students know the algorithm for multiplying fractions and this algorithm can be effective in getting correct answers; however, often students cannot explain the rule explicitly (Padberg, 1986). It is therefore important to develop a conceptual understanding of the multiplication of two fractions. One central way of thinking about fraction multiplication is as *part of a part* (e.g., $\frac{1}{3} \cdot \frac{3}{5}$ is interpreted as taking $\frac{1}{3}$ of $\frac{3}{5}$), a conceptual understanding of the operation where the first fraction functions as an operator and the second fraction as an operand (Webel & DeLeeuw, 2016). Language plays an important role wherever conceptual understanding is to be developed (Erath et al., 2021). In mathematics education research, connecting language varieties (e.g., everyday, academic, and technical language) and multimodal representations (e.g., graphical or concrete representations) is one central design principle for learning arrangements that enhance language for mathematics learning (Prediger & Wessel, 2013). However, before designing instruction that enhances language or to improve existing approaches, there is a need to empirically identify (topic specific) language demands and means which are crucial for the particular learning processes in view in order to facilitate and design respective learning opportunities, particularly by engaging students in phases of explaining meanings (Prediger & Zindel, 2017). Thus, this paper examines students' language means that are used to explain the meaning of *part of a part*.

Theoretical background: Meaning-related language and conceptual learning

Following the idea of language and subject-integrated teaching and learning of mathematics (Prediger & Wessel, 2013) the greater project of the presented study aims to develop a language-responsive teaching-learning arrangement for the multiplication of fractions. We thus follow approaches emphasising the strong interplay of facilitating students' conceptual learning of mathematics and enhancing students' language (Erath et al., 2021). In particular, we follow a sociocultural perspective

that conceptualizes learning mathematics as a discursive activity, highlighting participation in (mathematical) practices, mathematical discourse, and conceptual understanding (Moschkovich, 2015). In addition, we emphasize the idea of language as “a tool for thinking and constructing knowledge via meanings” (Prediger & Wessel, 2013, p. 436) following Schleppegrell (2010). Therefore, and since mathematical objects are abstract and often relational entities, the construction of meaning as mental models involves the construction of new mental objects and relations and at the same time of “new linguistic, graphical, and symbolic means for expressing them” (Prediger & Wessel, 2013, p. 436).

Thus, to understand the multiplication of two fractions, learners need to build the mental model of the *part of a part* relationship; at the same time, they need to develop language means for explaining the meaning of this relationship. These processes of understanding can be supported by language-responsive teaching and rich discursive meaning-making processes (for an overview, see Erath et al., 2021). However, in the first step, language means for explaining the meaning of multiplying fractions (in the sense of taking *part of a part*) need to be identified. These language means for explaining meanings are mostly not part of the formal language vocabulary, which is classically regarded as technical language, but belong to the so-called meaning-related language means (Wessel, 2015), which have proven to be particularly important for building conceptual understanding. Meaning-related language means are linked to mathematical concepts in their meaning and differ from formal-related language means as they have a content-related, illustrative reference (Wessel, 2015). Many meaning-related language means emerge from everyday situations that are used to develop mathematical concepts. Such words may also appear in everyday language but become topic-specific formal language when they play a crucial role in mathematical understanding (Wessel, 2015). There are already some content- and language-integrated concepts for specific mathematical topics that systematically combine the development of conceptual understanding with language learning (e.g. Pöhler & Prediger, 2015, on percentages; Wessel, 2015, on fractions) but not yet on multiplying fractions.

Prepositions play an important role in teaching and learning mathematics as they are often used to express mathematical relationships (Niederhaus et al., 2016). When analysing comprehension processes of constructing meaning, prepositions present a particular challenge for understanding and solving mathematical problems (Niederhaus et al., 2016). A lack of understanding of prepositions can thus lead to incorrect interpretations and thus impair mathematical understanding (Niederhaus et al., 2016). In the phrase *part of a part*, the preposition plays a vital role in conveying its meaning since it indicates that a part of an already existing part is being considered. Although some aspects of language means for talking about mathematical meaning might be generic for each mathematical topic, lexical and syntactical phrases need to be identified by empirical investigation of topic-specific learning processes (Prediger & Zindel, 2017). This paper approaches the mathematical topic of multiplying two fractions. As a first step, students’ language means are empirically investigated.

Research question

Based on the theoretical background and the research gap in the context of empirically identified and topic-specific lexical and syntactical phrases for a conceptual understanding of fraction

multiplication, analyses of the learning processes presented in this paper were guided by the following research questions: Which language means do students employ to explain the meaning of *part of a part*? To what extent can language-related challenges also be identified?

Methods: Design research on language-responsive teaching and learning

The presented research is part of the first author's PhD project that has been conducted in the tradition of design research (Gravemeijer & Cobb, 2006). The presented data stems from the first cycle of design experiments. Design experiments are the methodological core of design research studies as they allow in-depth investigations of teaching-learning processes.

Methods of data collection

The data set used for this paper consists of the learning processes of five small groups of two to three 6th-grade learners aged 11 to 13 in an integrated comprehensive school; they were part of the data collection in the first cycle of design experiments. All students come from the same school in a large city in central Germany. The students had previous experience with fractions (fractions as part of a whole, expanding and reducing fractions) and were beginning with the addition and subtraction of fractions in regular classes. The first author or a colleague from mathematics education supported students' use of the material but rarely intervened with regard to meaning-making processes.

Methods of data analysis

The purpose of this qualitative study is to analyse the individual and collective processes by which students construct meanings for mathematical concepts and relations. In the first step, we collected all sequences in which students were working on the first guided practice problem (Figure 1). Then, each of these sequences was transcribed by the first author, starting at the beginning of the first guided practice problem and ending after this task. The analysis is based on the transcripts and students' written products. We came back to the video data to enrich the transcripts whenever there were uncertainties in students' utterances or actions as displayed in the transcripts. The presented analysis follows the process of inductive category formation (Mayring, 2015). For this, the critical moments of the learners' meaning-making processes are analysed to see which language means were central to the further development of conceptual understanding and which challenges might be caused by linguistic aspects. Thus, the categories (entries in the first column in Table 1) capture students' language means for explaining the meaning of *part of a part*.

Empirical insights

On the level of initiated learning processes, the research questions ask for students' language means and which challenges might be caused by linguistic aspects. We focus on students' discussions related to the first guided practice problem (Figure 1). Students had previously worked on the introductory task (Figure 1). After discussing this task, the students were introduced to a graphical and concrete representation of fraction multiplication that was new to them, and worked with it as a way to find a *part of a part*. With this knowledge, students were asked about the first guided practice problem (lower part of Figure 1). The activity aimed to stimulate students' intuitive strategies rather than memorized procedures. Students were allowed to use the (newly) introduced representations since we expected that students would seek to make meaning explicit while using them.

Introductory task – Hot Chocolate

Paul has a whole bar of chocolate. Paul shares this bar of chocolate with his sister. From his part, Paul would like to make himself a hot chocolate. For this, he needs half of his part. How much chocolate does Paul need for his hot chocolate?

First guided practice problem

How does the taste of the hot chocolate change when Paul has a larger part of the chocolate bar?

How does the taste of the hot chocolate change when Paul has a smaller part of the chocolate bar? Use the red and yellow transparent pages. Record your results in the table.

the whole	the part of a whole	the part of a part

Figure 1: Introductory task and first guided practice problem

First, we present extracts from Eva and Daria’s conversation on the task to reveal to which extent *part of a part* is part of their meaning-related language. However, the students lack any illustrative or content-specific reference to fully comprehend its significance. Afterwards, we refer to the analysis of all groups’ processes and present identified language means to explain in detail the meaning of *part of a part* that students use for talking about meaning, and show why linguistic challenges might also be important to consider for task design.

Insight 1: Part of a part as meaning-related language

The first excerpt of the transcript (S1-G2; translated from German by the first author) starts at minute 30:34 when the students begin to work on the first guided practice problem. In the first step, the students chose the task *half of a quarter*. Together with the teacher they visualised the task with the representation; they were then asked to fill in the table (Figure 1). The following excerpt shows how the students try to construct the meaning of fraction multiplication while working on the task.

- 52 Eva And. The part of a part, because he only takes half of it. If he takes half of a quarter again. A part of a part, a half of a wait
- 53 Daria Well, it is half of a quarter.

Eva uses meaning-related language at the beginning: “a part of a part” (line 52). The phrase *part of a part* is meaning-related language since *part of a part* refers to the meaning of the mathematical concept of multiplication of two fractions. It is not formal language because the phrase *part of a part* has a content-related and illustrative reference. Eva first tries to establish a relationship between the symbolic-numeric representation and the phrase *part of a part*. Daria supports Eva and successfully establishes a connection by saying, “Well, it is half of a quarter” (line 53) since she equates a quarter with the part and a half with the *part of the part*. After entering their results into the table, the students try to find a solution for the self-selected task *half of a quarter*.

- 73 Eva Half of a quarter, doesn’t half come out? No. That’s even less.
- 74 Daria Well, it is half of a quarter.
- 75 Eva Eh? Of a quarter. That would be a decimal. Oh God am I stupid. I was about to say something, but that would be a decimal.
- 76 Daria Yes, but in quarters.
- 77 Eva Of a, eh, a half of a quarter. Are?
- 78 Daria Yes.
- 79 Eva A half?
- 80 Daria A half. A quarter. A quarter?
- 81 Eva Eh? But that does not make any sense, does it?

82 Daria Two quarters?
 83 Eva Two quarters? That would be even more.
 84 Daria One quarter.
 85 Eva Two. We will leave it like that and move on.

In line 77 “Of a, eh, a half of a quarter. Are?” it becomes clear that Eva is trying to find an answer to the question. The students repeat the phrase “half of a quarter” (line 73, line 74, line 77) several times. She attempts to establish a relationship between the fractions, she struggles to determine how to approach and solve the problem. Daria names various fractions as possible solutions, e.g. “A quarter” (line 80) and “Two quarters?” (line 82). None of the named fractions seems to be a solution for Eva. They finish the task without a solution. Eva may find it difficult to connect the phrase *part of a part* with mathematical knowledge because the phrase is also used in the chocolate context and gives no indication of how to interpret it in the mathematical context of multiplying fractions. The meaning-related language mean *part of a part* is not sufficient for the two students to verbalise conceptual understanding and thus the meaning of *part of a part*. It can be assumed that other meaning-related language means are missing that relate to the meaning of the concept *part of a part*. Certain verbs can help to support the content-related, illustrative reference by conveying an action that clarifies the concept (see insight 2). At the same time, a special challenge of mathematical concepts becomes apparent because the phrase *part of a part* is abstract and relational terminology. The mental object, *part of a part*, records a specific relationship between the *part* and the *part of a part*. The specific meaning of the result of the task, one eighth in this case, lies precisely in the *part of a part* relationship between *a half* and *a quarter*. In German, the word *of* is a preposition that is used before the reference word and that indicates a relationship between it and another word. It is only through the preposition *of* that the meaning of the *part of a part* relationship becomes clear, which the students have to identify and recognise. Additional meaning-related language means might have been helpful for the two students to develop linguistic and conceptual clarity on the meaning of *part of a part*. The following section will provide insights into how students attempt to create such an illustrative reference to the concept *part of a part*.

Insight 2: Explaining the meaning of part of a part

We now present and refer to the collection of language means (the categories identified in the third step of our analysis) used by the students that contribute to the construction of the meaning of *part of a part* (Table 1). As in some cases, the nuances in the German language are of importance, we also included students’ original utterances and added our translation to English in italics.

Table 1: Categories of students’ language means

Language mean	Examples from students’ utterances
<i>students’ language means for the construction of the meaning of multiplication of fractions</i>	
Take a part of a part	Wenn er nur die Hälfte von ein Viertel nimmt. Der Anteil vom Anteil, die Hälfte von ein Viertel. <i>If he only takes half of a quarter. The part of a part, half of a quarter.</i>

$\text{of } \frac{a}{b} \text{ is } \frac{c}{d} \frac{e}{f}$	Von zwei Vierteln ist die Hälfte ein Halb. <i>Two quarters of one half is a half.</i>
$\frac{a}{b} \text{ of } \frac{c}{d} \text{ are } \frac{e}{f}$	Ja, die Hälfte von drei Viertel sind ein Viertel. <i>Yes, half of three quarters is one quarter.</i> Drei Viertel ja und wie machen wir das dann also ist das dann die Hälfte von drei Viertel oder nicht? Ja das ist 1,5. <i>Three quarters, yes, and how do we do that then so that it's half of three quarters or not?</i> <i>Yes, that's 1.5</i>
of that	Ein Ganzes und davon kriegt er, warte wie viele sind das. Davon kriegt er sechs, nein sieben, nein fünf. Genau. Also fünf Achtel. Davon braucht er dann wieder die Hälfte. <i>A whole and of that he gets, wait, how many are there. He gets six, no seven, no five. Exactly. So, five eighths. Then he needs half of that again.</i> Das sind acht Achtel und die Hälfte davon sind vier Achtel. <i>That is eight eighths and half of that is four eighths.</i>

The first line of the table gives an interesting example: “If he only takes half of a quarter. The part of a part, half of a quarter.” This example shows how students attempt to create an illustrative reference. In order to establish a connection between two fractions, namely a half and a quarter, the students utilise the preposition *of*. Additionally, they use the verb “to take” to imbue meaning into the sentence. By doing this, the students emphasize the action, as they are taking half of a quarter. It is important to note that not all verbs in the German language denote a specific action, e.g., the verb “*sein* – to be” does not describe a particular action. Consequently, the example in the second line would be less illustrative. However, by using the verb “take” the students link the mathematical concept of *part of a part* with a concrete mathematical action, which reinforces the illustrative reference.

We identified numerous other student utterances using the preposition *of* (Table 1, lines 2 to 4). In German, the preposition *of* is used to establish a link between a reference word and another word. Specifically, it is placed before the part from which the *part of a part* should be taken. The examples from lines 2 to 3 vary solely in terms of word order. German is recognized for its highly flexible word order, allowing for the phrase “*Von zwei Viertel ist die Hälfte ein Halb*” (*Two quarters of one half is a half.*) or “*Ja, die Hälfte von drei Viertel sind ein Viertel.*” (*Yes, half of three quarters is a quarter.*) to be expressed without grammatical error. In the first sentence, the preposition *of* is located at the beginning of the sentence, whereas in the second sentence, it is positioned between two fractions. These examples reveal that while the word order in German may be flexible, the preposition *of* always precedes the reference word. Through sentence restructuring, students attempt to create meaning by establishing how two fractions are related. These examples also highlight the complexity of the German sentence structure. Moreover, a misunderstanding of the sentence structure can affect students’ interpretation of taking a *part of a part*. The next step is to imbue this relationship with meaning, which proved challenging for the students. Therefore, it is crucial to use language that is based on meaning to develop a clear concept of what *part of a part* relationship signifies.

The use of the phrase *of that* is also interesting: “That is eight eighths and half of that is four eighths” (Table 1, line 4). This indicates that the students are trying to create meaning, particularly a relationship. In the German language, they replace the preposition *of* with the phrase *of that* and thus

establish a relationship between *the part* and *the part of a part* in a linguistically and mathematically appropriate way. This phrase is ideal for expressing the relational structure. In this example, the phrase *of that* serves as shorthand for eight eighths and refers to the preceding part of the sentence. This indicates that the students are attempting to find the solution of half of four eighths. The linguistic challenge in using the phrase *of that* in the sentence is to ensure that it is clear what it refers to. To understand the sentence, students need to identify what the phrase *of that* refers to, which requires them to look at what was said previously.

Prepositions and syntactical structures have a central role in the expression of relations. Students who have not yet mentally constructed this relationship cannot fill the linguistic phrase *part of a part* with mathematical meaning. This demonstrates that language is an important “tool for thinking and constructing knowledge via constructing meanings” (Prediger & Wessel, 2013, p. 436).

Conclusion

In the context of the current study, the focus is on the language means used by the students. Results from insight 1 show that the development of conceptual understanding and language means is necessary to further clarify the mathematical concept *part of a part*. In answering the research question, insight 1 demonstrates that additional language means with an illustrative reference are necessary to articulate the precise meaning of *part of a part*. Meanwhile, a particular challenge of mathematical concepts becomes clear because the mental object, *part of a part*, captures a specific relationship between *the part* and *the part of a part*, which students need to recognise and understand. For a conceptual understanding of *part of a part*, two fractions must relate to one another. This provides evidence that the relationship between language and meaning is of fundamental importance in any context where conceptual understanding is to be built. Meaning-related language means thus represent a central part of the subject-specific language and are crucial for understanding and, as has been shown here, have to be acquired or addressed by the students. For this, students need meaningful language means, verbs and relational linguistic structures with prepositions and prepositional adverbs. Insight 2 shows that students used verbs, the preposition *of* and *of that* when talking about *part of a part*. A conceptual understanding of the *part of a part* notion requires the establishment of a relationship. Especially the verb “take” demonstrates an illustrative reference for the students. This is also encouraged by the graphic and concrete representation we introduced to the students. Other meaning-related vocabularies that underlie the *part of a part* relationship will be elaborated in further detail in the next cycle so that they can be explicitly offered to the students, as these belong to academic language rather than everyday language and thus might not be familiar to the learners.

References

- Erath, K., Ingram, J., Moschkovich, J., & Prediger, S. (2021). Designing and enacting instruction that enhances language for mathematics learning: A review of the state of development and research. *ZDM – Mathematics Education*, 53(2), 245–262. <https://doi.org/10.1007/s11858-020-01213-2>
- Gravemeijer, K., & Cobb, P. (2006). Design research from a learning design perspective. In J. van den Akker, K. Gravemeijer, S. McKenney, & N. Nieveen (Eds.), *Educational design research* (pp. 17–51). Routledge.

- Mayring, P. (2015). Qualitative content analysis. Theoretical background and procedures. In A. Bikner-Ahsbals, C. Knipping, & N. Presmeg (Eds.), *Approaches to qualitative research in mathematics education. Examples of methodology and methods* (pp. 365–380). Springer https://doi.org/10.1007/978-94-017-9181-6_13
- Moschkovich, J. N. (2015). Academic literacy in mathematics for English Learners. *The Journal of Mathematical Behavior*, 40(A), 43–62. <https://doi.org/https://doi.org/10.1016/j.jmathb.2015.01.005>
- Niederhaus, C., Pöhler, B., & Prediger, S. (2016). Relevante Sprachmittel für mathematische Textaufgaben. Korpuslinguistische Annäherung am Beispiel Prozentrechnung [corpus linguistic approach using the example of percentages]. In E. Tschirner, O. Bärenfänger, & J. Möhring (Eds.), *Deutsch als fremde Bildungssprache: Das Spannungsfeld von Fachwissen, sprachlicher Kompetenz, Diagnostik und Didaktik* (pp. 135–162). Stauffenburg.
- Padberg, F. (1986). Über typische Schülerschwierigkeiten in der Bruchrechnung. Bestandsaufnahme und Konsequenzen [Typical student difficulties in fractions. Review and consequences.]. *Der Mathematikunterricht*, 32(3), 58–77.
- Pöhler, B., & Prediger, S. (2015). Intertwining lexical and conceptual learning trajectories – a design research study on dual macro-scaffolding towards percentages. *Eurasia Journal of Mathematics, Science & Technology Education*, 11(6), 1697–1722. <https://doi.org/10.12973/eurasia.2015.1497a>
- Prediger, S., & Wessel, L. (2013). Fostering German-language learners' constructions of meanings for fractions. Design and effects of a language- and mathematics-integrated intervention. *Mathematics Education Research Journal*, 25(3), 435–456. <https://doi.org/10.1007/s13394-013-0079-2>
- Prediger, S., & Zindel, C. (2017). School academic language demands for understanding functional relationships: A design research project on the role of language in reading and learning. *Eurasia Journal of Mathematics Science and Technology Education*, 13(7b), 4157–4188. <https://doi.org/10.12973/eurasia.2017.00804a>
- Schleppegrell, M. J. (2010). Language in mathematics teaching and learning: A research review. In J. Moschkovich (Ed.), *Language and mathematics education. Multiple perspectives and directions for research* (pp. 73–112). Information Age Publishing.
- Webel, C., & DeLeeuw, W. W. (2016). Meaning for fraction multiplication: Thematic analysis of mathematical talk in three fifth grade classes. *The Journal of Mathematical Behavior*, 41, 123–140. <https://doi.org/https://doi.org/10.1016/j.jmathb.2015.12.003>
- Wessel, L. (2015). *Fach- und sprachintegrierte Förderung durch Darstellungsvernetzung und Scaffolding. Ein Entwicklungsforschungsprojekt zum Anteilbegriff* [Content- and language integrated teaching by relating registers and scaffolding. A design research study on understanding fractions]. Springer Spektrum. <https://doi.org/10.1007/978-3-658-07063-2>