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Students' use of symbolical mathematical language and their development in their first year of study

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According to research, one of the major problems for students beginning with their mathematics studies is the mathematical language. To investigate the process of language development, we interviewed a group of 15 mathematics students from a German university at seven different times during their first year of study. In this paper we focus on the aspect of using symbolical mathematical language when formulating theorem-like statements. Our results imply three types of students with respect to their development of mathematical language. Students of the first type seem to be able to use mathematical symbols correctly from the beginning, whereas students of the second and third type show severe problems and difficulties in their use of language. But while students of the third type show no real development and the majority of them dropped out of their studies, improvements in using symbolical mathematical language can be observed for those of the second type.

Keywords: Language, symbols (mathematics), transition, university students.

Introduction

Students' difficulties with the transition from school to university mathematics have been studied from different perspectives in mathematics education research and therefore multiple interventions as for example bridging courses were developed and established (Guedet, 2008). Nevertheless, at least in Germany, the drop-out rates in mathematics studies are still high (Heublein, 2014). One of the reasons for that can be the transition from school-mathematical to university-mathematical language, as research showed that students struggle with mathematical language especially at the beginning of their mathematics studies (Guedet, 2008) as they have to deal with differences and changes regarding language use and language requirements.

In school mathematics, everyday language and colloquial paraphrases are often used and accepted to describe and discuss results and mathematical objects, whereas at university, formalism and a correct and precise use of mathematical language is required (Reiss & Nagel, 2017). Clark and Lovric (2008, p. 29) therefore characterise the secondary-tertiary transition as "a shock of passage from informal to formal language" for mathematics students and Epp (2003, p. 886) states: "My students seem to live in a different logical and linguistic world from the one I inhabited". Following Guedet (2008), the mathematical language acts as a gatekeeper at universities to get access to the mathematical community and this deep dependency of mathematical learning and language is emphasised in many publications (Schleppegrell, 2007; Tabach & Nachlieli, 2015). Thus, mathematical language can be seen not only as a learning medium, but also as a learning prerequisite and a learning object that seem to be an obstacle for many students.

However, research concerning university students' development of mathematical language is sparse, which is why we contribute to this line of research with a project that has the aim to investigate this

process and is conducted within the Competence Center for Higher Education in Mathematics ([khdm, www.khdm.de](http://www.khdm.de)). At CERME 12, we already presented parts of this project, focusing on definitions as a specific part of mathematical language (Körtling & Eichler, 2022). In this paper, we focus on students' use of mathematical language when having to formulate theorem-like statements symbolically. In particular when developing mathematical proofs, students need to be able to understand and use formal symbolic language, but according to research, this is an obstacle for many students (Moore, 1994). For example, Trigueros & Ursini (2003) showed that first-year students struggle often to differentiate between variables that are used as a specific number and variables that are used as a specific unknown. Begg & Pierce (2021) found out that especially the use of subscripts and symbols that are used with different meanings depending on the context can lead to misunderstandings for students. To conclude the main research question for this paper is the following:

What development do students show in their use of mathematical language over the course of their first year of study when having to formulate mathematical theorem-like statements symbolically?

Construct and content of mathematical language

For a characterisation of the mathematical language and its key features, usually a subdivision into word, sentence and text level is used (Prediger et al., 2019). The mathematical vocabulary (technical terms) is to be classified on the word level. In comparison to everyday language, a division into three categories can be made (Maier & Schweiger, 1999; Monaghan, 1999): Firstly, there are words that are only used in mathematical language (e.g. “homomorphism”). Words as “completeness”, which are used in everyday language with nearly similar meaning than in mathematical language, can be assigned to a second category. The third category includes words that are used in both registers but with different meanings (e.g. “series”).

In addition to technical terms, mathematical symbols can be assigned on the word level too, as they can be understood as specific linguistic entities (Meyer & Tiedemann, 2017). In general, a differentiation between symbols with a fixed meaning (constants) and symbols without an independent meaning (variables) can be made (Maier & Schweiger, 1999) and the use of symbols plays an important role in mathematics which is emphasised in many papers (Arcavi, 2005). They are used as abbreviations for technical terms or mathematical objects and therefore lead to abstraction and conciseness of mathematical texts and in many cases it is only possible to describe and grasp objects, constructs and therefore their meanings with the help of symbols (Schleppegrell, 2008).

On the sentence level, special patterns of grammar and syntactical structures like using long and dense noun phrases, passive voice and the verbs “be” and “have” are typical for mathematical language (e.g. Schleppegrell, 2008; Maier & Schweiger, 1999). Regarding to the use of symbols, there are certain norms and rules about how mathematical symbols may be linked with each other and may be linked to words. For this, it is possible to establish references between different concepts and objects and to express mathematical relationships (Maier & Schweiger, 1999). Apart from such general syntactical characteristics, mathematical definitions are to be located on the sentence level. Typically their grammatical structure is the following: The condition or premise is presented in a (conditional)

subordinate clause and the consequence that results from the premise is described in the main clause (Meyer & Tiedemann, 2017).

Besides definitions, mathematical theorems are another important element of mathematical language. As they often consist of more than one sentence, they can be assigned to the text level, as well as of course mathematical proofs (Meyer & Tiedemann, 2017). In general, properties of technical jargon such as completeness or precision are also characteristically for mathematical language.

Methods

The sample for this research is a group of 15 students, aged between 18 and 23 years (ten female, five male), that we obtained through convenience sampling (willingness to participate in the research). The group consists of prospective mathematics teachers (upper secondary school) and mathematics students, who were all beginning with their mathematics studies and most of them had recently graduated from high-school. To analyse their language development we chose a longitudinal setting and interviewed them at seven different times during their first year of study (four interviews during their first semester, three during their second semester). Each interview lasted about 30 to 60 minutes. For those interviews, they were paid in the same way as student workers are paid at university.

Among other aspects, the students were asked in the interviews to give definitions of mathematical terms (e.g. the term “function”), to explain a definition using an example, to read out specific mathematical sentences, to explain the meaning of a mathematical object in their own words and to formulate a given theorem-like statement symbolically. Regarding the latter aspect, one of the tasks was to formulate the following statement symbolically: “If you add two odd natural numbers, the result is always an even natural number” (The statement was changed slightly for later interviews, e.g. to “If you add an odd and an even natural number, the result is always an odd natural number”). The participants were asked to write down that statement using formal symbolic language. In case the interviewer noticed that there occurred difficulties, he asked the participants to try to explain what exactly makes it difficult for them to write down that statement.

The recorded audio of the interviews was transcribed fully and analysed through applying the procedure of inductive coding following Mayring (2014). To analyse the students written answers, a coding scheme was developed to identify and describe differences between the students and their language use and their development (Kuckartz, 2016). For that, the technical terms and symbols the students used in their answers were coded as follows:

- the number of technical terms; the number of symbols used overall; the number of variables, quantifiers, logical connections, arithmetic operators, sets and sets operations

Regarding this, it was also coded whether a quantifier or logical connection was expressed by its symbol or written out and to record the failures that occurred the codes listed below were used:

- the number of undefined variables; erroneous terms; missing parts/aspects of a statement in the answer; syntactically complete sentence; errors in the syntax e.g. in the connection of symbols

In Figure 1, an example of a student’s answer on the task described above is given (original expression). This student uses 16 symbols overall in his answer, of which two are variables (n, m)

that are defined, three arithmetic operators (+, (, =) and two symbols for sets or set operations (\in , \mathbb{N}). He expresses the sum of two odd natural numbers completely symbolically and does not use a technical term or any word in general. The symbols are used and connected in a syntactical correct way and therefore no erroneous terms were coded. Nevertheless, his answer is not a syntactically complete sentence and the universal aspect in the given statement (“is always”) is not mentioned in his answer (missing parts).

$$n, m \in \mathbb{N}$$

$$(2n + 1) + (2m + 1) = 2n + 2m + 2$$

Figure 1: Example of a student’s symbolical expression of the mathematical statement “If you add two odd natural numbers, the result is always an even natural number” (First Interview)

Comparing the coding of the students written answers based on the procedure of type-building content analysis (Kuckartz, 2016), students with similar patterns regarding their development of using symbolical mathematical language were summarised to one type.

Results

Referring to the task to formulate the given mathematical statement symbolically, it can be noted that the student’s formulations become more elaborate in general during their first year of study. This can be seen in particular in the overall number of symbols used by the students. As shown in Figure 2, a large increase in the number of symbols used can be observed from the first to the second interview, followed by a decrease until interview 5 and an increase after that again.

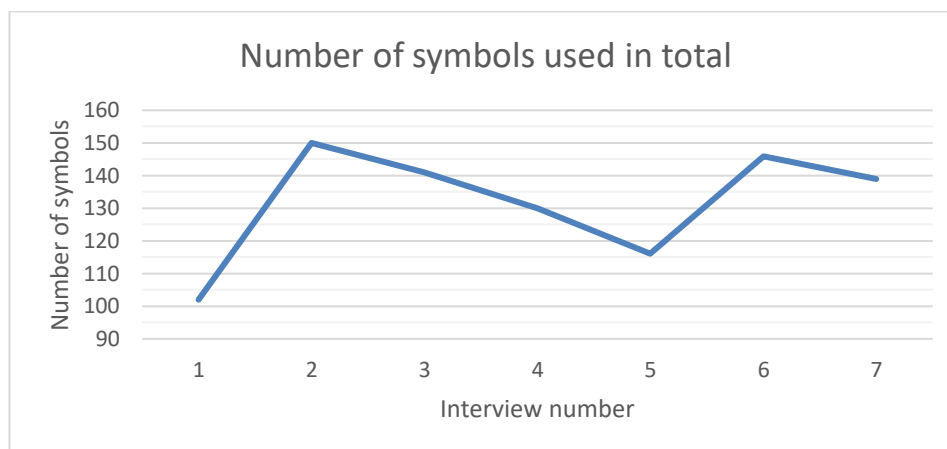


Figure 2: Number of symbols used in total (by the students who took part in all interviews)

Based on the evaluation of the students written answers as described above, three types of students could be identified. On the one hand there are students (type one), who make nearly no errors in their use of symbolical mathematical language from the first interview on. They use a correct symbolic representation for even and odd natural numbers, use at least two variables that are defined in their formulation of the given statement and make no errors in connecting symbols with each other. A development can be noticed in the way that their formulations become a bit more elaborate after the first interview and thus sometimes longer than necessary, as the example in Figure 3 shows (all used

variables could have been defined in one line and it was not asked to prove the statement), but they switch back to shorter formulations towards the end of their first year of study.

$n_1, n_2 \in \mathbb{N}$ $m_1, m_2 \in \mathbb{N}$ $m_1 = 2n_1 - 1 \quad m_2 = 2n_2 - 1$ $m_1 + m_2 = 2n_1 - 1 + 2n_2 - 1 = 2(n_1 + n_2) - 2 = 2(n_1 + n_2 - 1)$ $2(n_1 + n_2 - 1) \text{ ist immer gerade.}$	<p>“ist immer gerade” can be translated to “is always even”</p>
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Figure 3: Example of the symbolical formulation of the mathematical statement “If you add two odd natural numbers, the result is always an even natural number” of a student of the first type

On the other hand, there are students who struggle to formulate that mathematical statement symbolically or make many errors in using mathematical language (type three) and show no real improvement in their language use during the interviews. Figure 4 shows examples of the formulations of a student of the third type (who dropped out of his studies after the third interview). He formulates the statement using symbols only from the beginning on, but makes multiple mistakes. In the first interview, he uses capital letters for the variables, which is more common for sets than for elements of a set and uses the symbol for divisibility in the equation. In the third interview, he does not use capital letters for the variables and adds the definition of the variables as elements of the natural numbers, but still stays with representing an odd number symbolically by the expression “ $2 \nmid a$ ” and connects all that information in one equation.

<p>Interview 1:</p> $2 \nmid A + 2 \nmid B = 2 \nmid (A+B)$	<p>Interview 3:</p> $2 \nmid a \in \mathbb{N} + 2 \nmid b \in \mathbb{N} = \nexists 2 \nmid c \in \mathbb{N}$
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Figure 4: Examples of the symbolical formulation of the mathematical statement “If you add two odd natural numbers, the result is always an even natural number” of a student of the third type

Students that we summarized to a second type, have difficulties with using mathematical language correctly at the beginning too, but in contrast to students of the third type, a positive development can be observed over the course of their first year of study. An example for that is given in Figure 5. In his version from the first interview, the student mixes set notation with arithmetic operators, representing an odd natural number by using the set symbol for the natural numbers and adding the information “odd” (“ungerade” on German) in subscript. From the second to the fifth interview, he changes to using variables (that he does not define) instead of set symbols for the numbers that are added, but stays with representing whether a number is odd or even by putting that in subscript. In the seventh interview, his formulation becomes more elaborate as he uses “ $2x + 1$ ” and “ $2x$ ” to represent an odd and an even number and explicitly expresses the universal statement by using the quantifier “ \forall ”. However, due to using only the variable “ x ” in his defining of “ m ” and “ n ”, only the case of the sum of two consecutive numbers is considered. Additionally he uses variables with subscripts in his following equation that are therefore not explicitly defined as even or odd numbers.

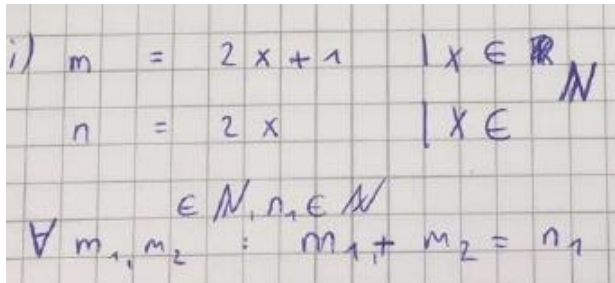
Interview 1: $N_{\text{ungerade}} + N_{\text{ungerade}} = N_{\text{gerade}}$
Interview 3: $N_{\text{ungerade}} + m_{\text{ungerade}} = k_{\text{gerade}}$
Interview 7: 

Figure 5: Examples of the symbolical formulation of the mathematical statement “If you add two odd natural numbers, the result is always an even natural number” of a student of the second type

Another difficulty that occurred in most of the formulations of students from the second type was that they did not seem to be aware of needing to use two different variables for two different natural numbers and that they mixed up certain symbols, for example the symbols for the logical connections “and” and “or”.

Discussion

In this paper we focused on student’s ability to formulate mathematical statements symbolically and their development of using symbolical mathematical language. We observed a large variety in the formulations written down by the students and that their use of symbols got more elaborate in general during their first year of study. Our results indicate a division into three types of students with respect to the development of mathematical language and the tendencies and difficulties that occurred described above regarding the symbolical formulation of the given statement could also be seen in other tasks where the participants are asked to give a written response using mathematical language.

The students that we assigned to the first type seem to be able to use mathematical language with regard to symbols correctly from the beginning and the only errors we coded in their answers were in some cases missing parts or linguistically incomplete sentences. In contrary to that, students of the second and third type were either not even able to write down something to the given prompt at the beginning or showed many difficulties with the use of mathematical language. They try to use new symbols quite quickly but therefore one major issue for them is choosing the correct symbols for the given context and connecting them in the right way. In some cases, it seems that they are trying to imitate the mathematical language they have seen in lectures or textbooks, without having grasped the meaning of the symbols they used fully already at that point. Berger (2004) calls this imitation “functional use” and holds the hypothesis, that students can learn a correct and sophisticated use of mathematical language in this way. In our study, this definitely seems not to be the case for the students of the third type as they showed no real development and progress in using mathematical language correctly and most of them dropped out of their mathematics studies over the course of their first year of study or switched to studying another subject instead of maths. However, a positive

development especially in the last two interviews could be observed in the group of students of the second type, which could be seen as that their “functional use” of terms and symbols is successful at least to some extent. Nevertheless, they have not reached the same level of linguistic sophistication as the students of the first type in general, and the question whether they will succeed with that without specific support for their language development remains open. Taking into account the results from research about students’ difficulties with symbols and following Berger’s hypothesis, one possible option for language support could be to create more occasions where students have to use formal symbolic language but detached from new content and knowledge, e.g. tasks that they should be able to solve alone with the help of their knowledge from school mathematics. In this way, the focus could be more on the correct use of language and mathematical language as a learning object, as many students described in the interviews that the higher pace in lectures at university together with newly introduced content and new linguistic aspects is a major difficulty for them in the transition from school to university. Furthermore, failures of students in using mathematical language correctly could be discussed more often in lectures or tutorial classes to make use of them in a productive sense (Loibl & Rummel, 2014).

Although the results of our study are not representative because of the qualitative approach we chose, they imply that many students have severe problems formulating mathematical statements symbolically and using mathematical language accurately in general even at the end of their first year of study.

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