Research-infused teacher-led innovations in university mathematics education

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We introduce a model for knowledge mobilization between university mathematics teachers and researchers in didactics of mathematics. The model revolves around innovations that teachers lead in their instruction and researchers assist to study in a disciplined manner. The model is illustrated in action with an assignment, in which students in a large first-year course video-recorded and peer-reviewed solutions to problems from past exams. A collaborative teacher-researcher inquiry into the impact of the assignment generated insights that were of a practical and research value.

Keywords: Knowledge mobilization, research-practice gap, teacher-researcher partnerships, university mathematics teaching, video podcasts.

Introduction

The gap between research and practice is not exclusive to university mathematics education. Yet, there is something paradoxical about it in that context: being researchers themselves, university mathematics teachers (UMTs) could be expected to be open to engaging with and endorsing the findings of research into the didactics of mathematics. Many UMTs genuinely care about teaching, aspire to improve their students' mathematical experiences, are employed in institutions that encourage educational scholarship, and some UMTs even work with researchers of didactics of mathematics (RDMs) side-by-side (e.g., Artigue, 2021). Still, these promising foundations are often insufficient to address the research-practice divide without deliberate initiatives.

Artigue (2021) suggests that a change is unlikely to come from research dissemination in the traditional sense. Instead, she advocates for thinking

> in terms of collaborative projects, building and negotiating, jointly with mathematicians and other university teachers, *problématiques* that make sense for all those involved, and meet their respective interests and needs. And then we should combine our respective knowledge and expertise in these projects through appropriate praxeologies (p. 14, italics in the original).

This approach aligns well with the longstanding interest of TSG14 in the rapprochement between different mathematics stakeholders, with special attention to the UMT and the RDM communities (e.g., Winsløw et al., 2018). Artigue (2021) highlights that some members of these communities have been establishing successful collaborative projects since the inception of research into university mathematics education (UME). But such projects remain relatively rare, while many of them have been spontaneous and unsustainable. Hence, it is important to consider what kinds of projects may lie at the heart of UMT-RDM collaborations, how the respective interests and needs can be combined with available knowledge and expertise, and what the parties can "gain" from partnering with each

other. Given the acknowledged complexity of UME (e.g., Artigue 2021; Nardi, 2016), we maintain that considerations of this sort should pragmatically account for the affordances of this context.

In this paper, we introduce a model for teacher-researcher collaboration that is centered around innovations that UMTs lead in their instruction and RDMs assist to study in a disciplined manner. We present the model and illustrate it in action after laying out our theoretical underpinnings.

Theoretical underpinnings

Mathematics education research has come a long way from viewing teachers solely as research subjects to bearers of valuable professional knowledge. A growing line of research argues for the potential of teachers and researchers establishing mutually beneficial *co-learning partnerships* (e.g., Jaworski, 2006). Wagner (1997) explains that in such partnerships, both teachers and researchers are

> engaged in action and reflection. By working together, each might learn something more about the world of the other. […] However, each may learn something more about [their] own world (p. 16).

Matthews et al. (2021) elaborate on the infrastructure that is needed to support partnerships of this sort. The conceptual pillar of the infrastructure is the Knowledge Mobilization framework (KMf), which stems from an alternative conceptualization of the relationship between teachers and researchers. Instead of framing it in terms of a two-cultures problem, a divide, or bifurcation, the communities are conceived with an emphasis on crosscutting identities and commitments.

> Many researchers are quite committed to teaching practice, and many practitioners value the insights to be gained from research. Moreover, ample evidence suggests that members of each community are genuinely interested in exchanging with each other (Matthews et al., 2021, p. 494).

Then, KMf posits that genuine knowledge mobilization is inherently bidirectional, meaning that both researchers and teachers are viewed as *producers of knowledge* that is specific to their own practice.

Researchers rarely doubt the relevance of knowledge they produce to teachers. KMf argues that knowledge that teachers produce may be of value to researchers as well. Indeed, by working in real classrooms, teachers collect the so-called data of practice. These data are not as formal and systematic as researchers are used to, but it can be paramount to getting a sense of students' common challenges, preferred modes of work, and responses to certain instruction. Accordingly, from KMf's viewpoint, researchers are natural consumers of knowledge that teachers produce (Matthews et al., 2021).

Within KMf, knowledge mobilization is conceived as a proactive and an intentional process unfolding between teachers and researchers so that research and teaching inform each other in a reciprocal manner. This approach is consistent with many frameworks of research-practice partnership that aim to establish sustained collaborations, strive for mutual understandings and shared commitments, while the partners work towards joint research goals (e.g., Wagner, 1997). Jaworski (2006) highlights a key role of *inquiry* in such partnership: students' inquiry into mathematics, teachers' inquiry into their teaching and students' learning, and inquiry as a form of research that surrounds these activities.

In the university context, the road to knowledge mobilization between UMTs and RDMs contains formidable obstacles—institutional, epistemological, historical, stereotypes, to name a few (e.g., Artigue, 2021). These obstacles can be expected to feature in communication. Thus, a special

communication space is needed where "alternative and competing discourses and positionings transform conflict and differences into rich zones of collaboration and learning" (Gutiérrez et al., 1999, p. 286–287). Nardi (2016) demonstrates how such spaces can be created through storytelling research findings in a form of a fictional dialogue between a UMT and an RDM. She argues that the communication between the two can be *non-deficit, non-prescriptive, context-specific, example-centered and mathematically focused*. We contend that UMTs and RDMs can commit their actual communication to similar principles as co-learning partners.

RITLI model

We introduce a model for knowledge mobilization between UMTs and RDMs, a model targeted at producing *Research-Infused Teacher-Led Innovations* (RITLI). A distinctive feature of the model is that in the initial stages, UMTs have a significant degree of agency over some didactical innovation and they assume a leading role regarding its implementation in their instruction. The innovation can be embedded in the UMTs' teaching already, or it can still be in the design stages. By "research infusion", we refer to collaborative disciplined inquiry into the innovation (cf. Shulman, 1981). In this way, the familiarity with the innovation and the expertise in disciplined inquiry a priori position UMTs and RDMs as producers of knowledge that is of mutual value.

RITLI constitutes a sharable outcome of knowledge mobilization between UMTs and RDMs, when the initiative may come from either partner. This process is fueled by the partners' *mutual inquiry*, that is "a willingness to wonder, to ask questions, and to seek to understand by collaborating with others in attempt to make answers to them" (Wells, 1999, p. 122). The partners' communication is subject to Nardi's (2016) principles of non-deficit and non-prescriptive interaction that is centered around the innovation and its affordances for students' mathematics learning. In the case of UMTs, the aspiration for knowledge mobilization can be broadly motivated by the interest in how their innovation is experienced by the students and why, how it can be reshaped to better align with UMTs' needs and interests of practice, with an eye to contemporary research. In turn, RDMs can engage in the process to support educational efforts of their colleagues as well as seize an authentic opportunity to pursue their own research agenda (we elaborate on this point shortly).

In an RITLI model, the partners oscillate between three types of innovation-centered inquiry:

- *Inquiry into the innovation's design*. In the beginning stage, UMTs communicate the innovation to RMDs, and together the partners trace the story of its development, how it came about, and what objectives it is meant to achieve. By brokering the contextual details, UMTs are provided with opportunities to unpack and reflect on the decisions of design. UMTs can also share their data of practice (e.g., anecdotal feedback from the students) or intuitions if the innovation is yet to be implemented. However, it is important that these data stimulate UMTs' appetite for disciplined inquiry rather than being interpreted as conclusive evidence.
- *Inquiry into a joint research ground*, and specifically, into potential foci, aims, and questions to pursue. UMTs may enter the partnership with concrete ideas to explore but they may also present their inquiry interests broadly (e.g., "To understand how the innovation is working for students"). As part of this inquiry, RMDs link the innovation to the research literature and draw UMTs' attention to a range of innovation-related aspects and possible methods of study.

Many of these can initially appear tacit and become visible through certain theoretical lenses. The role of non-deficit and non-prescriptive communication is critical at this point since this kind of inquiry is targeted at figuring out whether there exists a joint research ground, standing on which may satisfy professional needs and interests of both UMTs and RDMs.

We note that eventually agreed foci, aims, and questions may not necessarily be equally appealing to the partners. Yet, they can decide to pursue them jointly since these research aspects will determine only a sub-set of all the activities that will unfold as part of the partnership. For example, while UMTs may be primarily interested in students' views on the innovation, RDMs could be keen to study the impact of these views on UMTs' teaching. In other words, a RITLI model legitimizes a scenario where UMTs and RDMs collaborate without necessarily reaching a consensus about the end goals of their collaboration.

- *Disciplined inquiry* or an implementation of the previously agreed course of research action. This includes systematic data collection, its analysis with rigorous educational methods, and discussion of the findings. RDMs can be expected to lead this activity, but UMTs can still contribute meaningfully in a range of capacities (see the next section for examples). The findings of disciplined inquiry constitute a co-produced outcome, a sharable result of knowledge mobilization that the partners are unlikely to generate on their own.

The findings of disciplined inquiry may give an impulse to another knowledge-mobilization round. Now that the UMTs have obtained systematically constructed feedback on their innovation, it only seems reasonable to re-interpret its alignment with their original needs and interests. This process is likely to lead to changes and new ideas that summon a new inquiry cycle. In a similar fashion, a posteriori, researchers often realize how a study could or should have been carried out differently, delineating a new course of research activity. In other words, the outcomes of disciplined inquiry may endow the partners with new professional interests for further collaborative work. In this sense, a RITLI model bears resemblance to design research that "attempts to support arguments constructed around the results of active innovation and intervention in classrooms" (Kelly, 2003, p. 3); attempts that are innovation transformative, research iterative, and knowledge generative.

Illustration: The impact of video assignment on students' exam performance

Teacher-led innovation

"Communication and engagement" are a new capability that the University of Auckland (UoA) recently introduced to a list of attributes it aspires its graduates to develop. The capability is expected to be demonstratable through students being able to "receive and interpret information, express ideas and share knowledge with diverse audiences in a range of media and formats" (UoA, n.d.). However, it is challenging to develop this capability in first-year mathematics courses that have often been content-dense, lecture-based, and taken by hundreds of students.

To address this challenge, UMTs introduced a *video assignment*. The assignment includes a bank of 30 problems from past exams with especially low success rates. The students' task is to select one problem and produce a 1–5 minute-long video clip with a solution. The guidelines explain:

> You want to not just solve the problem, but explain what you are doing and why […]. Talk like you would to another student in the class who was having a hard time with the concepts.

The assignment has been implemented in a large course for non-mathematics majors, covering standard topics in first-year Calculus and Linear Algebra. The final exam, and accordingly the problems in the assignment bank, are multiple-choice.

The students are typically given 10–14 days to submit their presentations in the learning management system. Each presentation is assigned to four random peers, who independently review the mathematical validity of the solution and the quality of the explanation. Final answers to the bank problems are released before peer review. The assignment is given in the last week of the semester, and the peer-review component is followed by a final exam within a few days.

Since the introduction of the assignment, the course UMTs honed its guidelines, produced video-recorded examples of "good" and "bad" presentations, and resolved myriad technical issues. Turning the assignment into a regular part of the course enabled rendering it as affording students to advance "communication and engagement". But as the UMTs noted, "we don't really know what the assignment does to students' mathematics learning."

Research infusion

A RITLI model was employed to explore the opportunities for mathematics learning that the video assignment provides. In this paper, we present an abbreviation of our ongoing disciplined inquiry into *the impact of the assignment on students' performance on similar problems in final exams*. The inquiry entails many methodological challenges since we pursue it with original presentations and peer reviews that the students submitted as part of the course. The decision to operate with the authentic assignment is consistent with the UMTs' professional interests and growing attention to didactics of mathematics as it unfolds within institutional affordances (e.g., Winsløw et al., 2018).

Two questions underpin our inquiry: (i) *Do students, who submit presentations to and/or peer review problems that are similar to the ones featuring in the exam, perform better, compared to students who engaged with different problems in the assignment?*; and (ii) *Did exam performance improve after the introduction of the assignment?* In both questions, we explore the exam performance on specific problems that are similar to those in the assignment's bank.

We operate with data from three pre-pandemic semesters, when students sat the exams in closed-book invigilated conditions. To pursue the analysis, we identified *matching dyads* – pairs of problems that appeared sufficiently similar in terms of the involved mathematical concepts, questions, and options for answers (see the Supplementary materials). In the first phase, the second author nominated potentially matching dyads. We independently assessed each one of them to decide whether it can be considered as matching or not. Disagreements were discussed until consensus was reached.

To pursue Question (i), we drew on 16 matching dyads (P_A, P_E), where P_A came from the assignment's bank and P_E from the final exam that followed (Supplementary materials, Table 1). For each P_E, the cohort was divided into two sub-groups: (a) students who engaged with a matching P_A as problem solvers, peer reviewers, or both; and (b) those who worked on other problems from the bank as it has been documented in the learning management system. We used Pearson's Chi-squared test with Yates' continuity correction to compare the performance between the two groups.

For Question (ii), we used data on exam performance in the last decade. Indeed, the course syllabus, resources, and instructional methods remained relatively stable over the years. This enabled us to compare performance before and after the introduction of the assignment. We scrutinized past exams in the search for problems that were sufficiently similar to each P_E in the focal semesters. Eventually, we found three matching dyads (dyads 1–3 in Supplementary materials, Table 2) for which the relative exam performance of the past and focal cohorts on the relevant topics in the exam (e.g., derivatives) was statistically *insignificant*. Indeed, a difference in performance on a matching dyad could be rationalized in different ways. But if the cohorts performed similarly on a certain topic, the difference in the dyad performance that had a matching problem in the assignment bank can be associated with the impact of the assignment. Concurrently, similar performance on problems that did not have a match in the bank may be viewed as a complementary marker of the impact. We found 16 dyads of this sort (dyads 4–19 in Supplementary materials, Table 2).

Results of disciplined inquiry and further knowledge mobilization

The analysis suggested engagement with the assignment improved exam performance. In all 16 problems, the sub-group that produced video-solutions or peer reviewed the matching problems outperformed the rest of the cohort; this difference was statistically significant in 5 problems. The cohort that submitted video-presentations to matching problems outperformed their peers on 11 problems, 3 of which were significant. The peer reviewers of matching problems outperformed other students in 7 cases, 4 of which were significant.

On two problems, the focal cohort significantly outperformed the cohort who took the course before the introduction of the assignment. This result held for students who submitted a presentation to a matching problem and those who submitted a presentation to another problem. There was no significant difference in the performance on the third problem: not for the focal cohort nor its two sub-groups. For problems without a similar problem in the video-assignment's bank, there was no significant difference in the performance of the past and focal cohorts in ten out of 17 cases. These findings offer complementary evidence about the impact of the assignment: focusing on topics where students' performance was statistically similar, an inclusion of a problem in the assignment's bank was followed by a more successful performance on a similar problem. In turn, when an exam problem did not have a match in the bank, there mostly was no significant difference in the exam performance.

The discussion of these results engendered multiple rounds of knowledge mobilization in our partnership. For example, the RDMs highlighted that in many cases, the bank problems were selected for presentation by a small group of students, which did not allow for statistical analysis. This made UMTs realize that the decision to include 30 problems in the bank was rather arbitrary, and that explicit criteria are needed to include and regularly revise the bank problems. This is an example of a research consideration that yield a didactical reflection.

Some results informed the debates within our partnership. One of them pertained to publishing final answers to the bank problems before the peer review. When inquiring into this feature, the UMTs explained that having access to the answers allows students to check their work and makes a peer-review process organizationally viable in a large course. The researchers agreed, but raised concerns

about the possibility of mechanical answer-driven reviewing. The frequent outperformance of the reviewers, supported the UMTs' viewpoint, suggesting that many students peer review meaningfully.

The discussion of the results led to further questions about the assignment. Being limited to the outcomes of students' work, we knew little about the processes and decision-making that led to it. For instance, how do students select problems for their presentations? If they seize the assignment as an opportunity to learn about topics that they find challenging, then it seems legitimate to interpret the performance on similar problems in the exam as evidence of the assignment's impact on learning. But if students choose problems that they feel confident about, then there is little surprise in them performing better on similar problems in the exam a few days later. Overall, the results opened the door for another round of inquiry regarding questions that are of value for research and practice.

Concluding remarks

Two attributes of a RITLI model are worth highlighting. First, the gap between research and practice has been often discussed in terms of the latter not building on the former, i.e. "without considering the possibility that researchers may have a substantial amount to learn from teachers" (Matthews et al., 2021, p. 499). Consistently, RITLI is centered around innovations that UMTs already implement or consider implementing in their instruction. By establishing knowledge mobilization partnerships around teacher-led initiatives, RITLI recognizes university teachers as professional innovators and producers of valuable knowledge about didactics of mathematics; knowledge that is worthy of disciplined inquiry and dissemination within both professional and research communities.

Second, three types of inquiry embedded in RITLI provide RDMs with opportunities to learn about, support, inform, and shape practice (cf. Artigue, 2021). Systematic study of these inquiry processes can lead to better understanding of teacher-led innovations and teacher-researcher partnerships. These understandings are paramount if research into UME wishes to be relevant and useful for practice.

Until now, we used RITLI in four projects, with the total of three RDMs and four UMTs from the same department of mathematics (Kontorovich & Bartlett, 2021; Kontorovich & Greenwood, 2022). Accordingly, RITLI's capability to act as a modus operandi for other faculty and institutions remains to be shown. Nevertheless, we believe that this model may have a pragmatic appeal to a wide range of RDMs and UMTs. For the former, the model opens the door to study ground-led innovations and processes that unfold in teacher-researcher partnerships—topics of an enhanced research interest (e.g., Artigue, 2021). RITLI findings are of added value as they attest to how an innovation operates "on the ground" rather than in laboratory conditions. The obtained results may be seen as "proofs of concept", informing future studies. In turn, in many tertiary settings, UMTs are expected to teach in a research-informed manner, often without being provided with necessary resources to do so (e.g., time, training). By joining forces with RDMs and locating their own instructional innovations at the center of disciplined inquiry, UMTs will fulfill these institutional expectations.

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