



HAL
open science

Identifying static- and emergent-shape thinking in students' language about graphs of functions

Elizabeth Kimber, Cathy Smith

► **To cite this version:**

Elizabeth Kimber, Cathy Smith. Identifying static- and emergent-shape thinking in students' language about graphs of functions. Thirteenth Congress of the European Society for Research in Mathematics Education (CERME13), Alfréd Rényi Institute of Mathematics; Eötvös Loránd University of Budapest, Jul 2023, Budapest, Hungary. hal-04406720

HAL Id: hal-04406720

<https://hal.science/hal-04406720>

Submitted on 8 Feb 2024

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

Identifying static- and emergent-shape thinking in students' language about graphs of functions

Elizabeth Kimber and Cathy Smith

The Open University, UK; elizabeth.kimber@open.ac.uk

Perspectives on graphs of functions can involve static-shape thinking, which focuses on perceptual shapes, or emergent-shape thinking, which views a graph as a trace of covarying quantities. These, and more formal covariation perspectives, are used flexibly and together to solve graphical problems. Researchers have grappled with establishing empirical-theoretical relationships between the language students use and the meanings made available. Systemic Functional Linguistics (SFL) offers tools to analyse how linguistic choices contribute to the creation of mathematical texts and can thus be used to investigate how features of students' language communicate forms of shape-thinking or covariation. In this paper we present an SFL analysis of students' short written descriptions of a graph. We uncover similarities to the language used in their lessons and how shape thinking or covariation may be constructed by linguistic features of students' discourse.

Keywords: Graphs, functions, covariation, language, systemic functional linguistics (SFL).

Theoretical background

Functions are a core area of the secondary mathematics curriculum and are important for progression to calculus and for participation in a wider discourse on change. Research has suggested different interpretations of graphs, e.g., as picture or icons resembling objects (Leinhardt et al., 1990); as an inherent shape that is independent of scales and orientation of axes (static-shape thinking); or as a trace of covarying quantities (emergent-shape thinking) (Moore & Thompson, 2015). These are not mutually exclusive types of reasoning. We have inherited the names 'static- and emergent-shape thinking' but see them as forms of reasoning constructed in discourse, not as states of mind.

Understanding covariation involves coordinating how the values of two varying quantities change in relation to each other. It is fundamental to a conceptual development of functions and may occur at different levels of sophistication (Thompson & Carlson, 2017). Teachers are necessarily constrained by the curriculum and we are interested in whether the richness of covariation is made available in lesson topics that focus on graphs and functions, for example, here, lessons on transforming graphs.

Students' learning and discourse on graphs

Learning mathematics can be seen as developing and participating in mathematical discourse. This includes learning how to use technical terms and grammatical structures that differ from everyday language, with teachers' language playing a crucial role in this development (Schleppegrell, 2007; Sfard, 2007). Researchers have called for more analysis of topic-specific discourse (Erath et al., 2021; Prediger & Zindel, 2017) and Thompson and Carlson (2017) have asked researchers to articulate in more detail the basis of claims that someone is reasoning covariationally. Temporality and fictive motion (where movement verbs are used for objects that do not move) are important for developing concepts of smooth, continuous covariation (Thompson and Carlson, 2017). Antonioni et al. (2020) further found that, after working with functions in dynamic environments designed to highlight

motion, students' written texts involved dynamic language expressing variation around points. While not formal, this 'transitional discourse' can be related to formal discourse, and its expression of variation could be beneficial for learning. In linguistic terms, dynamism and temporality would be evident in choices that foreground actions (in SFL terms, material processes with explicit Actors), but these are not typical of formal mathematics discourse, which tends to concern timeless relations between objects rather than material processes (Antonini et al., 2020; Morgan, 1998; Sfard, 2007).

In this paper, we use tools from Systemic Functional Linguistics (SFL) to analyse students' written descriptions of graphs for linguistic features that may correspond to static- or emergent-shape thinking or covariation. This analysis is part of a larger study that focuses on teachers' classroom discourse about functions and graphs and contributes to answering our research question: 'What features of language are evident in classroom discourse on graphs?' And in particular for this paper, 'How do linguistic features of students' written descriptions of graphs construct or interweave shape thinking or covariation perspectives?'

Systemic Functional Linguistics

SFL is used to analyse written or spoken texts, and offers a theoretical basis for distinguishing the characteristics of formal, scientific language-in-use (Halliday & Matthiessen, 2004). Within SFL, texts are seen as the product of ongoing conscious or unconscious choices. These affect how a text functions for the audience via its interrelated ideational, interpersonal and textual metafunctions.

SFL analysis of mathematics texts has identified characteristic features of the mathematics register, such as technical vocabulary, nominalisation (through which processes become objects) and the prevalence of relational processes (such as *being* or *having*) which identify objects or assign attributes (Schleppegrell, 2007). Through the ideational function, choices of process types and participants contribute to the construal of mathematical activity in a text. In particular, in the mathematics register relational processes are typically used to set up static, timeless relationships between mathematical objects (Morgan, 1998). In contrast, material processes of *doing* and *happening* are common in everyday use; they may have mathematical objects or humans as Actors and provide an action-oriented narrative that unfolds over time (Halliday & Matthiessen, 2004). For example, *the x-intercept is at (1,0)* is a timeless, identifying description, whereas *the graph crosses the x-axis at (1,0)* allows an interpretation of unfolding – emerging – movement with the graph as Actor.

Related to choice of process, the choice of active or passive voice contributes to the ideational and interpersonal functions (Halliday & Matthiessen, 2004). The interpersonal function constructs relationships between author, audience and mathematics. For example, the active clause *I translated the graph* has a human Actor, whereas a passive form *the graph has been translated* omits the Actor. Mathematicians and mathematics teachers commonly use passive voice to indicate generality but, by obscuring human agency, this diminishes the perception of mathematics as a classroom activity (Morgan, 2016). Finally, the textual function organises the text to facilitate the other two functions.

It is these different portrayals of mathematics, mathematical participants and mathematical activity that can be brought into focus using SFL and may have the potential to suggest different forms of shape thinking or covariation.

Method and data

We are investigating classroom discourse in lessons on functions, graphs and change in secondary mathematics classrooms in England. Our main focus is on teachers' classroom discourse, but we also analyse students' language, to compare it with their teacher's language and language-focused practices in the lesson. The schools, teachers and student groups are an opportunity sample, and the data presented here was selected because its analysis allowed comparisons between two student groups taught the same topic by the same teacher. The topic of graph transformations is relevant as it offers opportunities to discuss the behaviour of individual graphs and the relationships between them.

The students were from two groups at a state school in England, taught by the same experienced teacher. The lessons involved similar activities using software to plot and compare graphs. Lesson 1 was for 30 students aged 15-16 who were due to take national examinations within eight months. Lesson 2 was for 28 students aged 14-15, much earlier in the same course. Ten lesson 1 students and 23 lesson 2 students gave consent for their responses to be analysed.

The data analysed includes lesson videos, teaching materials, a teacher interview and, centrally, students' written responses to an exit pass, i.e., a short task used at the end of a lesson to inform future lessons (William, 2018). We wrote the task (Figure 1) for the lessons in consultation with the teacher.

Describe the blue graph.
(Please write 1-2 full sentences.)

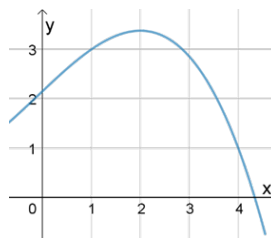


Figure 1: Exit pass given to students

Language-focused teaching in the lesson

We now describe features of the teacher's language, to illustrate our use of SFL tools and to allow later comparison with students' language. In lesson 1, the teacher emphasised that examinations required students to use words such as *translation* and *vector* (verbal and symbolic nominalisations) rather than everyday words such as *move*. To demonstrate this, he displayed the following model text, which students were asked to copy and complete. He used similar text three times in the lesson.

Given the graph of $y = f(x)$, the graph of $y = f(x) + c$ represents a translation.

The translation is given by the vector $\begin{pmatrix} 0 \\ c \end{pmatrix}$.

This model text aligns with several features of the mathematics register: it uses a relational process *represents* to describe a relationship between graphs; *translation* occurs as a noun, i.e. the material process *translate* has been nominalised; *is given by* is a passive construction which, textually, achieves a shift in attention between mathematical objects, from *translation* towards *vector*.

The teacher also discussed two other forms of words with students:

Teacher: you can say it is being translated that's ok or it is a translation either is ok

The second of these proposes the simpler relational process *is* instead of *represents*. The first introduces the material process *translated*, but in passive voice with an unspecified participant *it*. The teacher appears to be endorsing this material process+passive form as a more familiar but acceptable wording and the same form was used in a worksheet *The graph ... is transformed by the vector ...* .

Lesson 2 did not involve model text. Instead, the teacher encouraged students to develop their own articulations. He displayed descriptions of transformations in note form rather than sentences; most used the material process+active form but lacked Actors, e.g. *translates the graph up by c units*.

In the interview the teacher explained that he included the model text in lesson 1 “to make sure all their books had accurate notes that were easy to read”. He also explained that students are expected to support each other to use technical language in discussions.

Analysis

In the first stage of analysis we considered each exit pass response holistically in terms of mathematical content, informed by the literature on shape thinking and covariation (Moore & Thompson, 2015; Thompson & Carlson, 2017).

The main SFL analysis considered the form of words in individual clauses based on the definitions and examples in Halliday and Matthiessen (2004) and using the local mathematical context. We focused on features of text that contribute to the portrayal of mathematics and thus we identified relational and material processes and their participants. Finding few human Actors, we also identified whether material processes were used in active or passive voice.

The final stages were to make detailed comparisons of responses from each lesson and between the lessons, and to consider the alignment of students’ language with their teacher’s language. By combining linguistic analysis with our initial holistic analysis, we considered whether certain linguistic features of students’ responses suggest static- or emergent-shape thinking or covariation.

Findings

Holistically, most exit pass responses fall into two clear categories. Some describe the graph in terms of transformations (9 responses), echoing the lesson language. Some describe the shape or graphical features such as intercepts (22 responses). Finally, four responses, all from lesson 1, could not be categorised as the meanings are unclear: three refer to a *transition* and one may refer to intercepts.

Describing transformations

All six categorised lesson 1 responses refer to translations and offer no description of the graph’s covariational behaviour. For example

Response 1A: *the curve has been translated by the vector* $\begin{pmatrix} 2 \\ 3.5 \end{pmatrix}$

These responses from lesson 1 show a uniformity and clear alignment with the lesson language. They use *translation* or *translated* and give the vector symbolically. Two use the teacher’s nominalisation, but five use the teacher’s alternative form of material process+passive, as in 1A. Whether they use a relational process or material process+passive, all the responses start with a mathematical object as participant, either *the curve* or *it*. In most cases *it* is likely to refer to the graph students were asked to

describe, but in *It is a translation*, the *It* is more ambiguous and could equally refer to the relationship between the given curve and a perceived ‘parent’ curve. We note that none of the transformation responses explicitly refer to the idea of two curves from the model text.

In contrast with lesson 1, only three responses in lesson 2 describe transformations at all and do so without using *translation* or *translated*. Use of processes is similar to the lesson 1 responses as they include a mix of relational processes and material processes+passive. An example is

Response 2B: *It is flipped in the x-axis and placed in a weird angle. It's moved up in the y-axis and left in the X axis and it is a blue line.*

As with the lesson 1 responses, 2B does not describe the shape of the curve, though the two other responses do include this using a relational process, e.g. *It is a blue parabola but it has been stretched*.

Overall, most of the responses (7/9) that describe transformations use material processes (*translated, moved, flipped, placed, stretched*), and all in passive voice, so these are processes that students considered are being done ‘to’ a graph rather than ‘by’ it.

Describing the graph’s shape, features or activity

The 22 responses in this category are all from lesson 2. They are highly varied, as the examples below show, but refer to a named graph (12), points on the graph (15) and/or the graph’s activity (13).

Response 2C: *It is a parabola that goes through the point (2,3.5). The y-intercept is (0,2.1).*

Response 2D: *the blue graph is a parabola that peaks at (2,3.2) and intercepts the y-axis at 2.1*

Response 2E: *The graph heads as a linear line in a positive x-axis direction. Continuing along the x-axis, it begins to curve at a plateau, and gradually curves in a negative direction.*

Responses 2C and 2D are typical of responses in this category: they talk about mathematical objects (*parabola, y-intercept*) and use a relational process *is* to name the graph. But they also use material processes (*goes through, peaks*) that construct the vertex as part of the graph’s activity. They use different process types to describe intercepts: 2D continues the description of activity by using the material process *intercepts*, while 2C locates the object *y-intercept* using a relational process. The responses that described the graph via points on an unnamed curve were similarly split between using material or relational processes, or both. Response 2E shares only some features with other responses in this category: it refers to a *linear line* but does so using a material process *heads*; it describes the graph’s activity, but in relation to the x-axis rather than locating particular points.

Unlike the material clauses in descriptions of transformations, these responses all use active voice. There are also patterns in the types of material processes chosen: as responses 2C-2E show, students chose material processes that invoke visual features of the graph (*peaks, curves*) or movement and direction (*goes through, intercepts, heads, crosses*). These portray the graph as participating in processes through which its shape emerges or which relate to motion.

Discussion

We only have a small number of responses, especially from lesson 1, so we can only make tentative claims about students’ language.

Uniformity and alignment with the lesson language

Lesson 1 responses align closely with the lesson language including the use of technical vocabulary and stating the vector, which were emphasised by the teacher as examination requirements. A difference is that the model text used the nominalisation *translation* in a relational process, whereas most students adopted the teacher's alternative of using the material process+passive.

One reason for common features of responses in the same lesson is that students were allowed to discuss their responses to the exit pass. From the seating plans we know that only some of the lesson 1 students who described translations in a similar way were seated together, so this does not account for all the similarity. We suggest that a stronger reason could be the language-focused practices in lesson 1, providing model text and emphasising the use of technical language.

In contrast, only three of the 23 responses from lesson 2 describe the graph as a transformation and the variety between these responses corresponds to the variety in lesson language. Lesson 2 had lower emphasis on formal language, which may have contributed to the overall variety of responses, as the students did not have immediate access to an authorised way to describe the graph. As with lesson 1, students' responses to the task will have been filtered by their perceptions of what was expected.

Passive voice is a distinctive feature of all the transformation responses. In English, the start of a clause is usually what the clause is about (Halliday & Matthiessen, 2004). Since most students started with *the curve* (sometimes referred to as *it*), we suggest the curve was their focus of attention in this task. However, from the transformations work they were also concerned with describing this curve as the result of activity, thus used a material process form familiar from the everyday register. The material process+passive construction, which was made available in the lesson, achieves a description of activity but with the curve as the starting point. Previous research compares using passive rather than active voice and concludes that the passive retains a focus on activity but obscures human agency (Morgan, 1998, 2016). However, here we consider students' adoption of material process+passive, e.g. *the curve has been translated*, instead of a more formal relational process with nominalisation, e.g. *the curve is/represents a translation*. In this context we agree that the material process+passive construction introduces activity and temporality, which are less apparent in the relational form. However, rather than obscuring human agency, we see it as allowing for human activity, as it portrays the given graph as a product of activity of an unspecified but potentially human actor.

Shape thinking and covariation in students' responses

We now examine how students' linguistic choices may communicate shape thinking or covariational reasoning. Our analysis is based on a small dataset generated in the context of graph transformations, so these are tentative ideas. Furthermore, we do not claim that features of students' language mean they are or are not thinking in particular ways.

In the transformations responses, the material process+passive form, combined with choices of material process that involve deformation or repositioning (*translated, moved, flipped, placed, stretched*), suggest that the given graph results from physically moving a material object around the plane. This invokes the material 'graph as wire' metaphor within static-shape thinking rather than

more sophisticated emergent-shape thinking (Moore & Thompson, 2015). We note that this portrayal involves only one curve, rather than a relationship between two curves afforded by the model text.

Within the graphical responses, we see many material processes in active voice with the curve as Actor (as in 2C-2E). When material processes are connected with shape, such as *peaks* and *curves*, this may allow for static- and emergent-shape perspectives to be intertwined or co-exist. These word choices describe perceptual features, suggesting or allowing static-shape thinking. But the students describe features of the graph using a material process, which portrays the visual features as arising through unfolding change, and therefore could simultaneously afford emergent-shape thinking.

Other material processes relate to movement as travel (*goes through, intercepts, heads, continuing along, crosses*). This is fictive motion, as the graph does not actually move. Notions of continuous covariation may require this conception of travel, but also require coordination of x-and y-values (Thompson & Carlson, 2017). Language that combines movement processes with coordination of variables, such as 2C *a parabola that goes through the point (2,3.5)* or 2D *the blue graph ... peaks at (2,3.2)*, could therefore express covariational reasoning. In contrast, the use of material processes in 2E suggests a different form of covariational reasoning. This relates the behavior of the graph to movement along the x-axis (*continuing along the x-axis it begins to curve*) but does not state values of variables, which could be interpreted as emergent-shape thinking with gross coordination of values, in which “the person envisions a loose, nonmultiplicative link between the overall changes in two quantities’ values.” (Thompson & Carlson, 2017, p. 441).

We also see relational processes that classify and identify. Many responses use relational clauses to name the curve e.g. *it is a parabola*. Given students’ probable prior work with functions and that the curve is actually part of a cubic, this is likely to have been based on the graph’s perceived shape. A simple interpretation of this could be that the students are foregrounding static-shape thinking. However, many expand their responses to include material process descriptions, offering richer interpretations. For example, the relational form in 2C *The y-intercept is (0,2.1)*, treats intercepts as objects and expresses a timeless relationship, suggestive of static-shape thinking. In contrast, the material form in 2D *the blue graph ... intercepts the y-axis at 2.1*, does not require the x- or y-intercept to be objects: instead they are part of the graph’s unfolding behaviour.

These contrasts between relational and material processes are largely due to the detemporalising effect of the relational form, in which activity becomes static (Morgan, 1998). Relational processes and passive voice are features of formal mathematics texts. However, in this context, we note that our argument above suggests that they may both indicate less-sophisticated static-shape thinking, or at least do not challenge that, while certain material processes in active voice have potential to communicate the richness of emergent-shape thinking.

Conclusion

Within the limitations described, and although it is not surprising, we have provided tentative evidence that teachers’ language and choices about language-focused practice in lessons affects students’ language. We have also demonstrated how tools from SFL can help to identify how students’ choices of words or grammatical structures indicate static- or emergent-shape thinking or covariation. Encouraging certain choices of words or phrases may help students both express and

understand covariation, but further work would be required to establish such a link. Our work here adds to detail of why forms of shape thinking or covariational reasoning may be attributed to students.

References

- Antonini, S., Baccaglini-Frank, A., & Lisarelli, G. (2020). From experiences in a dynamic environment to written narratives on functions. *Digital Experiences in Mathematics Education*, 6(1), 1–29. <https://doi.org/10.1007/s40751-019-00054-3>
- Erath, K., Ingram, J., Moschkovich, J., & Prediger, S. (2021). Designing and enacting instruction that enhances language for mathematics learning: A review of the state of development and research. *ZDM – Mathematics Education*, 53(2), 245–262. <https://doi.org/10.1007/s11858-020-01213-2>
- Halliday, M. A. K., & Matthiessen, C. (2004). *An introduction to functional grammar* (3rd ed.). Hodder Arnold. <https://doi.org/10.4324/9780203783771>
- Leinhardt, G., Zaslavsky, O., & Stein, M. K. (1990). Functions, graphs, and graphing: Tasks, learning, and teaching. *Review of Educational Research*, 60(1), 1–64. <https://doi.org/10.2307/1170224>
- Moore, K. C., & Thompson, P. W. (2015). Shape thinking and students' graphing activity. In T. Fukawa-Connelly, N. Infante, K. Keene, & M. Zandieh (Eds.), *Proceedings of the 18th Annual Conference on Research in Undergraduate Mathematics Education* (pp. 782–789). <http://sigma.maa.org/rume/RUME18v2.pdf>
- Morgan, C. (1998). *Writing mathematically: The discourse of investigation*. Routledge. <https://doi.org/10.4324/9780203017715>
- Morgan, C. (2016). Studying the role of human agency in school mathematics. *Research in Mathematics Education*, 18(2), 120–141. <https://doi.org/10.1080/14794802.2016.1176595>
- Prediger, S., & Zindel, C. (2017). School academic language demands for understanding functional relationships: A design research project on the role of language in reading and learning. *EURASIA Journal of Mathematics, Science and Technology Education*, 13(7b), 4157–4188. <https://doi.org/10.12973/eurasia.2017.00804a>
- Schleppegrell, M. J. (2007). The linguistic challenges of mathematics teaching and learning: A research review. *Reading & Writing Quarterly*, 23(2), 139–159. <https://doi.org/bqmcwq>
- Sfard, A. (2007). When the rules of discourse change, but nobody tells you: Making sense of mathematics learning from a commognitive standpoint. *Journal of the Learning Sciences*, 16(4), 565–613. <https://doi.org/10.1080/10508400701525253>
- Thompson, P. W., & Carlson, M. P. (2017). Variation, covariation, and functions: Foundational ways of thinking mathematically. In J. Cai (Ed.), *Compendium for research in mathematics education* (pp. 421–456). National Council of Teachers of Mathematics.
- William, D. (2018). *Embedded formative assessment* (2nd ed.). Solution Tree Press.