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► To cite this version:

Thomais Karavi, Angeliki Mali. Exploring a lecturer's identification of students: a case study of teaching in university analysis lectures. Thirteenth Congress of the European Society for Research in Mathematics Education (CERME13), Alfréd Rényi Institute of Mathematics; Eötvös Loránd University of Budapest, Jul 2023, Budapest, Hungary. hal-04406712

HAL Id: hal-04406712

<https://hal.science/hal-04406712>

Submitted on 19 Jan 2024

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Exploring a lecturer's identification of students: a case study of teaching in university analysis lectures

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This study explores a lecturer's identification of students for their intended participation in the mathematical discourse. The case is set around the teaching of a lecturer in analysis lectures. The study is framed within commognition and identity from a commognitive perspective. The 17 lectures of the course were analysed through discourse analysis and constant comparative analysis. Through the identification of the 2nd Person and collective stories, we consider the intended participation as authored in the identified metarules. The identified metarules related to students' transition from calculus to analysis, potential obstacles, and self-study. The identification of the students by the lecturer provides valuable insights into the affective aspects of lecturing that influence students' participation in the discourse, suggesting ways for engaging students in the advanced learning processes.

Keywords: Metarules, lecturing, identity, participation, mathematical discourse.

Introduction

University mathematics teaching has been investigated in relation to students' efforts to participate in the new teaching-learning settings (Sfard, 2014). Specifically, in the setting of university mathematics lectures, the role of the lecturers is essential because they introduce students to advanced mathematical practices and set specific expectations for the students who attend their courses (Sfard, 2014). In the courses, both cognitive and non-cognitive aspects of mathematics teaching and learning emerge. Thus, in recent years in order to connect the cognitive and non-cognitive aspects of learning and teaching mathematics, researchers have turned their attention to the construct of identity (Heyd-Metzuyanım et al., 2016; Gardee, 2021).

The focus of the literature on the cognitive aspect of a learner's identity is prominent (Gardee, 2021). However, the students also shape their learning identities in relation to the valued actions for mathematical learning that are presented in the teaching setting they attend (Heyd-Metzuyanım & Shabtay, 2019). In that way, lecturers' role in the development of the non-cognitive aspect of a learner's identity is essential, influencing students' participation in the mathematical discourse.

Tait-McCutcheon and Loveridge (2016) analysed teachers' positioning when they teach lowest-ability students. Teachers' positioning of themselves and their students led to non-equal opportunities for participation in the mathematical classrooms the students attended. The findings suggested that the positioning and pedagogy were influenced more by the teacher and less by the teaching material, suggesting the importance of the issue. Earlier, in a case explored by Heyd-Metzuyanım (2013), the teacher identified the student as "clueless", limiting in that way the opportunities for autonomous participation in the mathematical discourse and restricting the student to passive engagement with the teacher. The identification of the teacher did not align with the identification of the student about

herself, who was motivated and interested at the beginning of the course. However, at the end of the course, the student endorsed the identification of the teacher, thus limiting her participation in the course. Similarly, Gardee (2021) focused on the relationships between two teachers' practices, the narratives of their practices, and learners' mathematical identities. The analysis provided insights into the opportunities offered by the teachers for students' development of their mathematical identities. Teachers' narratives were about the regulation of participation in the classroom community and the specification of who can learn mathematics, informing in that way students' learner identity development. This study offered valuable insights into the influence of teachers' practices on the development of students' learner identity. The researcher suggested further examination of changes in teachers' identifications offered to the learners. In our study, we initiate an exploration in that direction, in university mathematics lectures. Thus, using the construct of identity and commognition, we focus on lecturers' identification of students highlighting the intended participation in the university mathematical discourse in analysis lectures.

Theoretical considerations

This study is framed within the commognitive framework (Sfard, 2008) and identity from a commognitive standpoint (Heyd-Metzuyanim, 2013; Sfard & Prusak, 2005). Learning mathematics in commognition relates to becoming a participant in the mathematical discourse that is a particular form of communication (Sfard, 2008). Each discourse is governed by rules: object-level or meta-level (metarules). The object-level rules reflect a growth in the number of endorsed narratives (series of communicational acts, accepted by the mathematical community), whereas the metarules reflect the patterns in the activity of the participants of the discourse (Sfard, 2008). In the discourse, metarules appear either endorsed (recognised as a person's own) or enacted (when they are identified by external observers). The transition between discourses requires the development of a new set of metarules (Sfard, 2008), regulating in a way the participation in a certain mathematical discourse (Viirman, 2021). Teaching aims to facilitate students' participation assisting them during the shifts in metarules. In the case of university mathematics, endorsed metarules reflect the conscious choices of the lecturer while teaching (Viirman, 2021).

Identity is defined as "collections of stories about persons or, more specifically, as those narratives about individuals that are reifying, endorsable, and significant" (Sfard & Prusak, 2005, p. 16). Identity can be current, reflecting the present identity of the person or designated identity referring to the expectation of one's identity (Sfard & Prusak, 2005). Each of the identity stories can be represented using the triple ${}_B A C$ that exemplifies "*who identifies whom and to which audience*" (Heyd-Metzuyanim, 2013, p. 345, emphasis in original). In this triple, B is the author, A is the subject – the person that is being identified – and C is the audience. Three types of stories occur, 1st Person (1st P) stories are ${}_A A C$ (A is identifying A to C), 2nd P stories are ${}_B A A$ (B is identifying A to A), and 3rd P stories are ${}_A B C$ (A is identifying B to C). Identity supports the exploration of "how collective discourses shape personal worlds and how individual voices combine into the voice of a community" (Sfard & Prusak, 2005, p. 15). In this study, the focus is on 2nd P stories, authored by the lecturer about the students to the students (${}_{\text{Lecturer}} \text{Students} \text{Students}$) and the collective stories authored by the lecturer.

The purpose of this study is to explore a lecturer's expectations for students' participation in the learning processes in university analysis lectures. The research question of this study is "*How does the intended participation of students in the mathematical discourse emerge in the lecturer's identification of students?*" Through the identification of the 2nd P and collective stories, we look at the intended participation as authored in the identified metarules. Thus, we characterise the intended participation through an exploration of students' designated identity and collective identity. 2nd P stories were identified in the Heyd-Metzuyanin et al. (2016) study where students' designated identities were teachers-to-be or mathematicians and emerged directly and indirectly in the lecturer's talk. The identification of 2nd P stories appeared valuable for the discussion of the assumptions of the lecturer about students' learning and was connected with the offered opportunities to students to participate in the discourse.

Methods

Design, context, and data collection

This single case study explored a lecturer's teaching (he/him) in analysis lectures. The lecturer is a mathematician whose mathematical research focuses on mathematical analysis as well. He had already taught the introductory analysis course six times before the time of the data collection. In the course 103 (general or applied) mathematics bachelor's students were registered. Around 20-50 students attended live lectures while they had access to the recording of the lectures after the live lectures. In total 17, online, 90-minute lectures took place within the teaching period (eight weeks). The 17 lectures were observed while field notes were kept and their live recordings were transcribed.

Data Analysis

For the data analysis, first, the transcripts were divided into teaching episodes around the presentation of a theorem (or lemma), its proof, and the follow-up commentary where available. The episodes of the lectures were analysed, using discourse analysis, aiming at the identification of endorsed metarules concerning the expectations of the lecturer for students' participation in the mathematical discourse through the exploration of his discursive actions. In the episodes, themes of metarules occurred through the search for excerpts that relate to normative aspects of the lecturer's mathematical discourse for teaching. For example, the excerpts were related to the views of the lecturer on mathematics, views on students' study of mathematics, references to calculus and analysis, and the use of textbooks. Through constant comparative analysis (Charmaz, 2006) of the excerpts, themes that suggested endorsed metarules occurred and were complemented with enacted metarules. Last, a close examination of the themes aimed at a micro-analysis to address the nature of the endorsed and enacted metarules. The themes were added into tables where the following appeared: the articulation of the metarules, their potential influence on students' participation, the identification of possible tensions that prohibit students' participation, and the direct or indirect identification of the students from the lecturer (2nd P). The goal of the last step was to connect the emerged metarules to the identification of the students by the lecturer, addressing the research question of this study.

Findings

In this section, the lecturer's discursive actions are related to the 2nd P identifications (authored by the lecturer to the students for the students) and to the collective stories authored by the lecturer about himself and the students who attend the online course. The following excerpts are representative of the lecturer's introductory lectures where the move from calculus to analysis was discussed:

In calculus, you learn to compute stuff. Analysis, on the contrary, is not about computing, it's about proving things. And that's a quite different thing. So, we're not interested in computing the derivative of a function, but instead, we are interested, for instance, in proving that you can differentiate a function. [Lecture 01, turn 12]

So, it takes a little bit of practice to get used to this kind of reasoning. So, this is completely different from calculus. So, here we are not computing, we are reasoning. [Lecture 01, turn 214]

The metarule in these excerpts is *we are not computing, we are reasoning* in an effort of the lecturer to comment on the differences between calculus and analysis, promoting a collective identity of the participants of the analysis discourse. In the first excerpt, the lecturer identified the students based on his assumptions of their prior knowledge, saying that "you learn to compute stuff" in the calculus course. This 2nd P identification addressed the assumption of students' activity as participants in the calculus discourse. The current identity of the students contradicts the designated and collective learning identity the lecturer tried to establish in his talk ("we're not interested in computing [...], but instead, we are interested, for instance, in proving"). In the second excerpt, he continued with the 2nd P identification of students' current identity, implicitly this time, saying that "it takes a little bit of practice to get used to this kind of reasoning". Then, he moved to the designated identity by endorsing the metarule that underlines these two excerpts. In the excerpts, the lecturer used "we" to discuss the collective and designated identity, and the shift the students need to attend in order to participate in the analysis discourse establishing a feeling of collective struggle for the transition from calculus to analysis.

Next, the metarules that appeared when the lecturer informed students how they should work on their own will be discussed.

As an exercise, I'd also recommend that you redo the proofs yourself. It's a great way of getting used to working with these abstract definitions and working with these abstract theorems. And it really takes some time to get used to this. This is different from calculus.... But here, it's much more subtle. It's about reasoning. It's about finding strategies to solve a problem. And sometimes the strategy is not clear at all. Sometimes you have to prove by contradiction. Sometimes you don't have to prove by contradiction. And it takes a little bit of time to get a feeling of this. [Lecture 02, turns 255-256]

The endorsed metarule in this excerpt is *it takes a little bit of time to get a feeling of this*, indirectly referring to students' designated identity as participants in the analysis discourse. Connecting with the previous excerpt and the need for a shift from computing to reasoning, the lecturer directly identified students' activity of computing, suggesting this time ways with which the students could attend the shift between the calculus and the analysis discourse, towards the activity of reasoning

(designated identity as learners of analysis). He recognised that students may have difficulties when proving on their own, emphasizing the need to take some time to *get a feeling of this*. With the word *feeling*, in this case, the talk of the lecturer was around the experience that needed to be developed for the selection of the appropriate proving strategies. In this excerpt, the lecturer suggested ways that would support the activity of the students for the development of the feeling, involving students redoing proofs in their own time to get used to abstract definitions, theorems, and the different proving strategies in analysis. The need for time to participate fully in the new discourse was also discussed in Viirman (2021) where metarules may have not been recognised in real-time. The metarule that is emphasized in this excerpt reflected an observed obstacle by the lecturer that may prohibit students' participation in the discourse from his perspective. The 2nd P stories were not based on an assumption of students' background but on a struggle that students may face during their effort to participate.

Similarly, a detailed list of what the lecturer thought students should do when they work on their own, is apparent in another excerpt:

So, this is a demanding course, and just attending the lectures and the tutorials will not be sufficient; you need to put in a decent amount of self-study. And what do I mean by a decent amount of self-study? I will say at least eight hours per week. That is, you read the book, and you read it carefully. You write down the questions that you have, you can also post your questions on the forum, and you can email me but you have to be active. [Lecture 01, turn 14a]

In this excerpt, the endorsed metarule is *you need to put in a decent amount of self-study* where the lecturer stated that just attending the lectures would not reassure students' participation in the analysis discourse. The lecturer directly addressed the students, giving them tasks that they should do while working on their own, thereby reporting the properties of their membership in the analysis discourse for the development of the designated collective identity. He proposed working "a decent amount" of time which for him meant at least eight hours per week for the course. At this time, he described that they need to read the textbook and "be active" by raising their questions in the learning platforms (forum, email) where he implicitly assigned himself as responsible for giving responses to their questions. Throughout the excerpt, the lecturer discussed the designated collective identity for participation in the mathematical discourse. This excerpt is parallel to the discussion described by Sfard and the lecturer (Sfard, 2014) where the lecturer said "I don't expect them to understand in real-time," and "Understanding is something a person can only achieve while working on her own in the quiet of her room." (p. 201). Thus, after the lectures where the lecturer introduced the metarules of the analysis discourse, in order for the students to participate they need to spend time self-studying. The lecturer in the case of this study recognised students' current identity ("just attending the lectures and the tutorials will not be sufficient") and set his expectation for their designated identity as learners of analysis by guiding the students on how to attend the self-studying process. The next expert continues with self-study, clarifying the expectations of the lecturer for students' work on their own.

There's no such thing as just reading a mathematics textbook. So please take this very, very seriously. So, if you open a textbook, you will see that the textbook is filled with definitions and theorems. And I recommend that you learn those by heart. So, this is old-fashioned studying. And there's a reason why I say this: definitions and theorems are tools. You need to use those tools in

order to solve problems. If you don't know definitions and theorems, you will not be able to solve problems. So, this is really important. There are definitions and theorems, and I really recommend that you learn those by heart. So, this is old-fashioned studying. So, this is completely different from what you might have done in calculus. [Lecture 01, turn 14b]

The lecturer in this excerpt brought attention to the way the students should read and use the textbook, identifying it as necessary to inform them about the learning processes that are taking place in the analysis course. Inferred in that case appeared the metarule *you need to put in a decent amount of self-study* as the lecturer implied that students need to spend time on their own on the learning material. He suggested that just reading the textbook is not enough for achieving participation whose goal is to be able to solve problems. Again, a distinction between calculus and analysis was mentioned where in analysis, tools such as definitions and theorems are needed to be available for problem-solving, whereas in calculus it is assumed that this is not the case. The lecturer suggested “old fashioned studying” which meant learning the definitions and theorems by heart that seemed as the necessary step towards the designated collective identity of analysis learners. The following excerpt concludes the discussion about self-study:

So, the point here is that whether or not you have uniform convergence also depends on the set A . Yeah? So, in this respect, it is a little bit similar to uniform continuity... Yeah? So, I really recommend you to do this exercise in a little bit more detail, to convince yourself that we do have uniform convergence on intervals from zero to b , where b is strictly less than one, but you don't have uniform convergence on the half-open interval. [Lecture 12, turn 198]

In this excerpt, the metarule *you need to put in a decent amount of self-study* is inferred and, in this case, self-study had the role of conviction. The excerpt highlights the collective identity they have been developing in the course in the talk of the lecturer where he assumed that the students had already a background of theorems and definitions and they could use them as a basis to build on. Indeed, he referred to the uniform continuity and reminded the students of the sets on which they considered the functions. He assumed that in uniform convergence, the situation is similar but they needed to *convince* themselves about it by adding the details he skipped in the lecture.

In the last three excerpts of this section, the highlighted metarule emphasised the lecturer's strategies that would enable students' participation in the analysis discourse towards the designated collective identity as analysis learners. The identification of students' current identities as not participants in the analysis discourse led to 2nd P stories that are based on the expectation of what students need to do in their personal time when they work for the course. This identification shifted towards the last excerpt, from Lecture 12, where the lecturer identified students as more experienced participants in the discourse than in the first lectures.

Discussion

This study explored the identification of the 2nd P and collective stories as authored by the lecturer in the identified metarules about students' participation in the mathematical discourse. So, the intended participation was characterised by students' designated identity and collective identity. The endorsed metarules emphasised the regulation of students' participation in the course and the lecturer's expectations for students' learning. The first metarule related to the transition from calculus

to analysis and was named *we are not computing, we are reasoning*. Within the excerpts, the lecturer identified students' current identity that contradicted the designated and collective identity of the participants in the mathematical discourse. He suggested a change in their activity based on the assumptions about their background from calculus. The second metarule *it takes a little bit of time to get a feeling of this* addressed a potential obstacle of the students in their effort to participate in the mathematical discourse during the development of the designated identity as learners of analysis. The lecturer again referred to the differences between calculus and analysis but this time the focus was on analysis and the strategies that would enable their participation. Last, commenting on self-study and self-conviction (*you need to put in a decent amount of self-study*), the lecturer emphasised his expectation about students' personal work that would enable them to expand in the mathematical discourse and develop the designated identity as participants in the discourse. The 2nd P stories of the lecturer about students' current and designated identity revealed a number of metarules that appear in the talk of the lecturer. Those three metarules underlined the process of participation in the new for students' mathematical discourse from the lecturer's perspective.

The lecturer in the course identified the students as non-participants in the mathematical discourse and suggested ways of developing a designated and collective identity that would enable participation in the discourse. In the Heyd-Metzuyanım et al. (2016) study, the lecturer recognised students as "teachers to be" and potentially engaged students in the otherwise stressful mathematical activity, bridging in a way the gap between the current and designated mathematical identities. In our case, it seems to us that the lecturer promoted a collective designated identity to facilitate the move towards the mathematical discourse in the analysis course, establishing a non-cognitive aspect of the communication in the lectures (i.e., the emergence of metarules for supporting students' participation in the mathematical discourse). In the excerpts, he attended to the differences between incommensurable discourses (calculus and analysis) for the participation of the students in the mathematical discourse. The talk of the lecturer brings to the forefront the social relationships of the lecturer and the students, despite the fact that in lectures the interaction of the lecturer and the audience is limited. Studies in that direction are limited (Gardee, 2021) but necessary as they discuss a level of interaction of the participants of the discourse beyond the cognitive level.

Students' participation from the teachers' perspective has been highlighted through the exploration of teachers' practices and opportunities for the development of students' learner identity, suggesting directions for future exploration (e.g., Gardee, 2021; Tait-McCutcheon & Loveridge, 2016). Especially in the case of university mathematics lectures, which is our case, we used commognition and identity from a commognitive standpoint to explore the instances of identification of students' participation in the mathematical discourse in the analysis lectures. Commognition enabled a micro-level analysis of the instances of the interaction of the non-cognitive and cognitive phenomena in the mathematical lectures (Heyd-Metzuyanım, 2013) (e.g., lecturer's suggestions of a decent amount of self-study of uniform convergence [Lecture 12, turn 198]). The identification of the lecturer's metarules emphasised the need for the shift the students need to make from the current to the designated identity for participation in the mathematical discourse as envisioned by the lecturer.

Exploration of such instances in observational data could give access to when and where exactly the pedagogical considerations of the lecturers happen and for which reasons, relating them to the

expectations of the lecturers about the students who attend their courses. Moreover, further exploration of the non-cognitive aspects of lecturing is needed, especially in such a context where the students are mostly silent but present. The findings of studies on the non-cognitive aspects of lecturing could benefit lecturers' reflection on their teaching by supporting them in making informed choices regarding the expected participation of the students in their courses. This knowledge can be valuable input for professional development programs for lecturers of advanced mathematical courses.

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