

# Designing bridging tasks to work on Klein's first discontinuity

Leslie Jiménez<sup>1</sup> and Macarena Flores González<sup>2</sup>

<sup>1</sup>Universidad de Chile, Chile; [jpleslie@uchile.cl](mailto:jpleslie@uchile.cl)

<sup>2</sup>CY Cergy Paris Université, LDAR, France

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## Introduction

This poster aims to introduce a methodology to shorten the gap of secondary-tertiary transition in calculus. For this, the poster will discuss the construction of two tasks created thanks to the advanced methodology and some theoretical tools of the didactics of calculus.

### Klein's first discontinuity in calculus

Klein's first discontinuity refers to teacher training and the problem that young university students face when they enter university by not using school mathematics (Klein, 1932). The study of this discontinuity is not easy, especially when one looks at the international situation. According to Viirman (2022), we can distinguish at least three training profiles: teacher training starts from the first year of university (1), university training starts with a bachelor's degree in mathematics followed by postgraduate teacher training (2), and teachers only take a bachelor's degree in mathematics (3). Our work focuses on the first profile, on the beginning of teacher training in Chile.

Thanks to F. Klein's legacy (Kilpatrick, 2019), one of the common domains between the end of secondary school and the beginning of teacher training is calculus, which makes it a domain particularly favorable to deal with the first discontinuity. Moreover, future teachers will teach calculus later on, making it important that future teachers should experience the didactics of calculus in their training (Viirman, 2022). Focusing on the first training profile described above, we wonder, how can the design of calculus tasks bridge the gap between secondary school and university courses for future teachers?

### Designing bridging tasks to work on the Klein's first discontinuity

To address this question, we constructed a *bridging task design methodology* that reviews both the high school curriculum and the early university curriculum in calculus; we analyze the tasks proposed there and modify the existing ones (if necessary) in order to link the tasks proposed in both institutions. Then, we focus on two main ideas: looking for tasks in the secondary curriculum or in the first year of university curriculum and, modify them to bridge both institutions. We may modify them to belong to the same institution but to be closer to the other or modify to be applied in the other institution. From a theoretical point of view, we identify the elements that allow us to enhance visualization according to Duval (2005). Especially understood as the possibility of matching the units of the drawing with the characteristics of the object described in a given task. On the other hand, we seek to decrease the level of mathematical abstraction using the definition of "paradigms of analysis" (Delgadillo & Vivier, 2016), which are defined as: *Arithmetic/geometric analysis (GA)*, that allows interpretations with implicit assumptions based on geometry, arithmetic calculations or

the real world; *Computational analysis* (CA), where computational rules are more or less explicitly defined and applied independently of reflection on the existence and nature of the objects introduced; *Real analysis* (RA), characterized by approximation and neighborhoods' work, including topological work; the definition and properties are established theoretically which allows an " $\varepsilon$ -work" characteristic of this paradigm; limits, inequality, "the negligible", which we will use for our discussion and study. This poster proposes the study of two bridging tasks that have already been modified according to the proposed methodology to address Klein's first discontinuity:

**Tasks that promote inductive thinking:** In our contexts recursive sequences are found at the end of secondary curriculum and also in the first year of university. We have reviewed a Fibonacci sequence task that is divided into three parts according to the three paradigms defined by Delgadillo and Vivier (2016). The main idea in this task is to promote a dialectic among the defined paradigms to bridge the nature of mathematical work in both institutions, using a content that lives in their intersection of contents.

**Real context task:** Modeling is an ability that is found in both institutions: secondary school and university. A task has been modified in the secondary curriculum with the idea that students work on the formation of a semiotic representation (in the sense of Duval, 2005), generating a geometric model of a given situation in a real context, as well as modeling and optimizing a function that determines the lowest cost of a construction.

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