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“Like finding the acceleration”: A praxeological analysis of a calculus/mechanics task with and without its physics context

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Although mechanics and calculus are strongly related disciplines and share common objects, like the derivative, students do not always seem capable of leveraging the knowledge they acquire in studying one discipline when solving tasks in the other. In this paper, we continue our study of pairs of ‘similar’ tasks, a task from physics and the same task set in a pure calculus context, to analyse students’ learning of the derivative in these two contexts. Our results indicate that students have difficulties identifying that, beneath the context, the two tasks are essentially the same. On the one hand, this hinders students from using simpler algebra-based techniques from physics to solve the calculus task, while on the other, it favours the mobilisation of richer rationales.

Keywords: Calculus, mechanics, anthropological theory of the didactics (ATD), derivative task, college mathematics.

Introduction and research problem

As Liu (2022) and others before him have pointed out, “Among all modern sciences, physics is the most mathematically intensive subject” (p. 1657). In particular, the derivative is not only at the core of differential calculus, but it is also central to kinematics (the study of motion in mechanics), where it appears as velocity and acceleration. Therefore, calculus is often a prerequisite or co-requisite to mechanics and other physics or engineering courses for post-secondary STEM (Science, Technology, Engineering, and Mathematics) students (e.g., Hitier & González-Martín, 2022a). In their recent literature review on “the diverse roles of calculus courses for students, professionals and teachers”, Biza et al. (2022) note a “filtering role” of calculus courses that may account for high dropout rates in STEM studies. While Marrongelle (2004) observes that some students can use physics to solve calculus problems, Biza et al. (2022) raise the issue “that content and approaches of calculus courses may not be transferred or may not be relevant for transfer to other disciplines” (p. 402).

The notion of “transfer” has been conceptualised in the research literature, initially using a purely cognitive approach. More recently, the perspective of the *Actor Oriented Transfer* (AOT) (e.g., Lobato, 2012; Roorda et al., 2015) defines transfer as “the influence of a learner’s prior activities on his activity in novel situations” (Lobato, 2012, p. 233). Using AOT, Roorda et al. (2015) analysed the transfer of procedures learned in mathematics to physics and vice versa, although it is “usually expected to occur from mathematics to physics” (Planinic et al., 2019, p. 235). While not always stated explicitly, their high-school participants were able to establish some relationships between procedures learned in mathematics and in physics. However, the encouraging results of the studies of Marrongelle (2004) and Roorda et al. (2015) are rather a minority in the international literature, and it is important to consider that these studies are primarily interested in the physics context, in contrast to other studies that use a physics context but where the focus is on the covariational reasoning. Our own literature review instead stresses the “conflicting views on how derivatives are best learned in a

physics context, as well as the lack of research in mathematics education focusing on actual practices in physics courses” (Hitier & González-Martín, 2022a, p. 389).

The above considerations are at the origin of the first author’s PhD project, which studies the practices around the notion of the derivative in a kinematics context, both in calculus and in mechanics courses. This paper presents some preliminary results from the third stage of this ongoing research, which investigates student practices through an online questionnaire and interviews with students. In previous stages, we (1) conducted a praxeological analysis of calculus and mechanics textbooks (Hitier & González-Martín, 2022a), and (2) analysed teaching practices through interviews with teachers (Hitier & González-Martín, 2022a) and classroom observations. We identified inconsistencies between the practices in the calculus course, which rely mainly on the limit definition of the derivative and differentiation formulas, and the mechanics course, where ready-to-use formulas were often at the heart of the techniques. This led us to focus on students’ learning as a consequence of practices in each course. Here, we expand on the analyses initiated in Hitier and González-Martín (2022b). We discuss a second pair of “similar tasks”, that is, “the same task [...] presented both with its physics context and without it” (Hitier & González-Martín, 2022a, p. 2422). This task is a familiar one for students in a mechanics context but becomes unfamiliar in a calculus context. In the previous paper, our analysis of students’ answers to two questions seemed to confirm the compartmentalisation of techniques, as only a marginal number of participants were able to use the approaches from mechanics to solve the decontextualised calculus problem. In this paper, we continue to explore how the praxeologies that students use when solving an unfamiliar calculus task relate to the praxeologies they use when solving a similar but familiar task in a mechanics context.

Theoretical Framework

The Anthropological Theory of the Didactic (ATD—Bosch et al., 2020) considers all activity as institutionally situated and provides useful tools to study practices in different institutions. For our context, the calculus and physics courses are considered as different institutions, as defined by ATD.

A key tool of ATD is the concept of *praxeology*, which consists of two blocks, each with two components. The *praxis* (or practical) block is formed by a *type of task*, T , and a *technique*, τ , to accomplish that task. The *logos* (or theoretical) block contains the *technology*, θ , that sustains the technique, and which is itself included in a larger *theory*, Θ . Praxeologies can only be considered in relationship to a given institution, as moving a praxeology across institutions could involve the transformation of one or more of its four components (e.g., González-Martín et al., 2022). Our previous praxeological analysis of textbooks (Hitier & González-Martín, 2022a) allowed us to identify how some praxeologies around derivatives may differ between calculus and mechanics courses, depending on the institution. In this paper, we investigate how praxeologies used by students in solving a familiar task in physics differ from those used to solve the exact same task that has been “translated” into a pure calculus context.

Methods

In the third stage of our research, we created an online questionnaire and conducted interviews with students in a large Canadian college (College A hereinafter) where our teacher participants are employed. In Quebec, colleges are post-secondary institutions that offer, among other programmes,

a four-term pre-university science diploma that is compulsory for students intending to pursue university-level studies in a scientific field. Most science students at College A take their differential calculus and mechanics classes in the fall, during the first term of their studies. Near the end of the Fall 2020 term, at a time when all courses were taught online, we sent a questionnaire to the approximately 1,200 science students at College A. The questionnaire ended with an invitation to students to participate in an online task-based interview scheduled for the study break of the following term (Winter 2021). Four students volunteered. At the time of the interview, all four were enrolled in their second calculus course (Integral Calculus). Two of them, S2 and S4, were following the *Pure and Applied Sciences* path and were also enrolled in their second physics course (Waves, Optics and Modern physics), while S1 and S3, who were following the *Health Sciences* path, were planning to take that course during the following Fall term.

The online interviews varied in length from approximately one hour (S1, S2, S4) to slightly over 90 minutes (S3); they were video recorded and transcribed. The participants were also asked to send us photographs of any written work they completed during the interview. After a few introductory questions, the participants were invited to answer six questions; the number of tasks in a purely mathematical context was balanced with the number of tasks in a kinematics context. The interviewer remained as neutral as possible as the students completed the tasks, but would answer participants if they asked for clarifications. Moreover, because several months had elapsed since the participants took their calculus and mechanics courses, and in order to smooth the process, formulas were offered to the participants if they struggled to remember them. In particular, we expected the students to have difficulties recalling the kinematics formulas, as these were provided to them during their mechanics examinations.

In this paper, we focus on a pair of questions: Question 3 (Q3) and Question 6 (Q6) (see Figure 1). Taken from Knight (2017, p. 62),¹ Q6 is a familiar kinematics problem, a type of task that appears in all the mechanics textbooks we analysed. Q3 is essentially the same question stripped of its kinematics context, with the function f being the equivalent of the position function in Q6.² We consider it an unfamiliar calculus task, as this type of task does not appear in the textbooks we analysed (Hitier & González-Martín, 2022b) nor in the calculus course we observed. We consider Q6 and Q3 as a pair of similar questions in the sense that they are mathematically equivalent (it is basically the context in Q6 that differs). In principle, they could be solved using the same technique. However, their context places them in two different institutions (mechanics and calculus). Based on our previous work (Hitier & González-Martín, 2022b), we expected that students would use different praxeologies: with a technique based on algebra and using kinematic equations (reflecting the mechanics textbooks) for Q6, and with a technique relying on the properties of the derivative for Q3. For both questions, we identified the praxeologies used by each student (mainly the technique and

¹ This textbook was used in the mechanics class of three (S2, S3, S4) of our four interviewees, and this type of task can also be found in the textbook used by S1's mechanics teacher.

² The continuity of the first derivative is implicit, but if students had asked about this, the information would have been provided.

rationale), before comparing the individual praxeologies for each question.

Question 3	Question 6
<p>Below is what we know about a certain continuous function f. Determine the value of the constant k.</p> <ul style="list-style-type: none"> • $f(0) = 0$ • $f'(x) = 20$ on $[0, 0.5]$ • $f(x_1) = 110$ • $f'(x_1) = 0$ • $f''(x) = k$ is constant on $(0.5, x_1)$ 	<p>You are driving to the grocery store at 20 m/s. You are 110 m from an intersection when the traffic light turns red. Assume that your reaction time is 0.50 s and that your car brakes with constant acceleration. What magnitude braking acceleration will bring you to a stop exactly at the intersection?</p>

Figure 1: Questions 3 and 6 of the task-based interviews

Data analysis

Question 6

The techniques proposed by all participants (even S1, who struggled the most) were the same as in the textbook. All identified Q6 as a two-stage problem with a constant velocity of 20 m/s during the first 0.50s leading to a travelled distance of 10 m, then a constant (negative) acceleration that can be found using the third kinematic equation $v_f^2 = v_i^2 + 2a\Delta s$ (see Figure 2). As exemplified by S2 below, the technology used was more algebraic, rather than relying on physics reasoning:

S2: And I know that my final velocity has to be 0. And my initial velocity is still 20. What I want to find is the acceleration. So I can use the equation of $\Delta s = (v_f^2 - v_i^2)/2a$ and solve for the acceleration.

As expected, all participants, except S2, needed help recalling the kinematics equations. In S3's worked solution (right-hand column of Figure 2), we can see the equations as provided by the interviewer when needed.

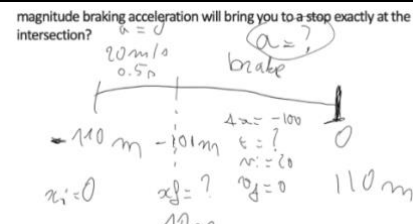
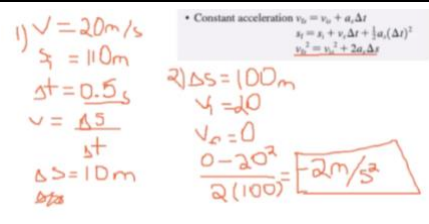
 <p>magnitude braking acceleration will bring you to a stop exactly at the intersection?</p> <p>$v = 20 \text{ m/s}$ $t = 0.5 \text{ s}$ brake $a = ?$</p> <p>$\Delta x = -100$ $v_i = 20$ $v_f = 0$ $x_i = 0$ $x_f = ?$ $t_i = 0$ $t_f = ?$</p> <p>110 m 10 m 100 m</p> <p>Identification of the two stages (S1)</p>	 <p>1) $v = 20 \text{ m/s}$ $s = 110 \text{ m}$ $\Delta t = 0.5 \text{ s}$ $v = \frac{\Delta s}{\Delta t}$ $\Delta s = 10 \text{ m}$</p> <p>2) $\Delta s = 100 \text{ m}$ $v_f = 0$ $v_i = 20$ $0 - 20^2 = 2a(100)$ $a = -2 \text{ m/s}^2$</p> <p>• Constant acceleration $v_f = v_i + a\Delta t$ $s_f = s_i + v_i\Delta t + \frac{1}{2}a(\Delta t)^2$ $v_f^2 = v_i^2 + 2a\Delta s$</p> <p>Worked solution to Q6 (S3)</p>
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Figure 2: Sample solutions to Q6

Question 3

As expected, the decontextualised version proved to be more difficult for the students, who identified the question as challenging (e.g., S1: “ah, it is hard...”) and unfamiliar (e.g., S2: “I don’t think I remember how to solve this.”). The easiest part for them was to find an equation for f on $[0, 0.5]$ and use it to determine $f(0.5) = 10$. Figure 3 provides an overview of the graphical representations and technology used by the four participants.

In what follows, we summarise the main points of each participant’s attempt to solve Q3.

S1 had difficulties conceiving that f is a piecewise-defined function:

S1: What I put is wrong [...] it's not $20x$ [...] 'cause k would be 0 if it was double derived [...], so that's bad.

Unable to reconcile this issue, he gave up.

	S1	S2	S3	S4
GRAPHICAL REPRESENTATION				
TECHNOLOGY	<ul style="list-style-type: none"> The derivative of $f(x) = mx$ is m, \Rightarrow if $f'(x) = m$, and $f(0) = 0$, then $f(x) = mx$ as $m \cdot 0 = 0$. If $f'(a) = 0$, then f has to be constant. 	<ul style="list-style-type: none"> $f'(x_1) = 0 \Rightarrow$ extremum $f'(x) = m \Rightarrow$ constant slope $\Rightarrow f(x)$ linear $f'(x)$ linear $\Rightarrow f''(x)$ constant $f''(x)$ constant \Rightarrow no change in concavity $f''(x) < 0 \Rightarrow$ concave down Properties of quadratics 	<ul style="list-style-type: none"> $f'(x) > 0 \Rightarrow$ value increases $f'(x) = 0 \Rightarrow f(x)$ constant $f''(x) = 0 \Rightarrow y = f(x)$ straight line $f''(x)$ constant $\Rightarrow y = f(x)$ "looks like a parabola" The sign of $f''(x)$ gives the concavity Properties of quadratics 	<ul style="list-style-type: none"> $f'(x) = 0 \Rightarrow f(x)$ constant $f''(x)$ constant $\Rightarrow f'(x)$ linear \Rightarrow "the function itself could be a curve, but I don't know what curve exactly" Integration

Figure 3: Technologies and graphical representations for Q3

S2 quickly identified that f has a maximum at x_1 , and was immediately able to sketch a possible graph for f , from which she obtained a graph of the first and second derivatives. She did so on separate coordinate systems, as is done in physics (see Figure 3, S2 column); however, the usual technique in mechanics textbooks would be to start from the constant second derivative (i.e., acceleration) and, from there, obtain the graph of the first derivative (i.e., velocity), before then using the first derivative to sketch an approximate graph of the (position) function. At first confused by the fact that " f' goes from being 20 to 0, but f goes from [10] to 110", S2 then identified that f is a quadratic on $(0.5, x_1)$, and has an equation of the form $f(x) = ax^2 + \dots$. Based on $f'(0.5) = 20$, she deduced that a has to be 10 (implicitly identifying the linear derivative with the tangent line to the quadratic at 0.5). She left it there, having forgotten all the formulas for the quadratic (reduced form, coordinate of the vertex, etc.) and discouraged that she did not know the coordinates of the vertex.

S3 started with a sketch. After finding the equation on $[0, 0.5]$, she identified k as negative since if f were concave up, it would lead to a cusp at x_1 (see Figure 3, S3 column). Then, reflecting upon the nonlinear part, she concluded that "this part maybe, I'm not sure but [...] we can say that it is a part of the quadratic." Writing a possible equation for the parabola, $a(x - x_1)^2 + 110$, she obtained a first equality by using the point $(0.5, 10)$: $0.25a - x_1a + x_1^2 + 100 = 0$, but noticed that she had

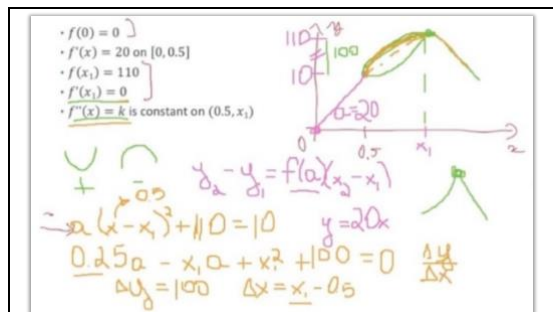


Figure 4: Detail of S3's work

two unknowns. She rejected integrating $f''(x) = k$, since this was not a part of her first calculus course (*Differential Calculus*). She then wrote “ $\Delta y = 100$ ” and stated that it is “like finding [...] the acceleration.” We note that, at the same time, her work seemed to indicate that she considered working with the average velocity (see the detail of her work in Figure 4 above). She concluded: “I tried to find x_1 and it didn’t work [pause] ah, it’s hard,” before moving on to the next question.

S4 started out sketching as well, clearly drawing the linear part and identifying that its slope is 20. But she was unable to recognise the parabolic part:

S4: Well, because if the first derivative is linear, that would, I mean, the function itself could be a curve. But I don’t know what curve exactly. I mean, I know the initial and final point of that curve.

Leaving the graphical approach aside, she resorted to integration:

S4: Yeah, now I’m thinking about integration. [...] maybe I could integrate that [...] and then from that, with the two points, I could go on to find like the actual equation [...] between those two points and then just go back to f' and f'' .

Interviewer: That would work. That definitely would work.

S4: Yeah. So, while we did learn a bit of integration in Cal 1, so I guess I could do that.

After this, she moved on to the next question.

Overall, the techniques used by the participants were as follows: finding the equation of a linear function given the slope and a point, and using it to find the value of the function at 0.5 for the first part; and attempting to find a quadratic equation using techniques from algebra or integral calculus for the second part. However, as we can see in Figure 3, their attempts rely on a variety of technologies that the participants were not able to articulate in order to clarify their reasoning and solve the task. We note that with the unfamiliar (calculus) task, they tried a variety of approaches that had them considering properties of the functions at play and their derivative; this contrasts with their techniques for the familiar (mechanics) task, where they plugged values into a formula, without necessarily thinking about the relationships between distance, velocity, and acceleration.

Concluding Remarks

In this paper, we analysed two ‘similar’ questions: one familiar question lifted from a mechanics textbook used at College A, and the same task presented in a pure mathematical context. Although all the numerical values were kept the same, none of the students, not even S3 who mentioned “acceleration”, identified the link between the two questions, which would have allowed them to solve the unfamiliar calculus task using the simpler technique from mechanics. As a result, in contrast with some of the participants in Marrongelle’s (2004) study and two of the students who answered our questionnaire (Hitier & González-Martín, 2022b), none of our interviewees showed the capacity to transfer procedures from one discipline to the other. Physics education research has already highlighted students’ failure to transfer knowledge from mathematics to physics (e.g., Planinic et al., 2019), and our study tends to indicate that students also have difficulty transferring praxeologies from physics to mathematics. It may be that they are unable to identify the similarity between tasks, so they automatically associate them with different institutions and, therefore, different techniques. As hypothesised in our previous work (Hitier & González-Martín, 2022a), this may be due to “different epistemological approaches in both disciplines, with calculus favouring the use of a motion function

(which can be then differentiated [...]) and mechanics using specific values as data” (p. 307). In line with the ruptures identified in our previous work (Hitier & González-Martín, 2022a), we ascribe this to the “institutional disconnection” between mathematics and the “client disciplines”, as pointed out by Romo-Vázquez and Artigue (2022), among others.

If we focus now on both tasks more specifically, we observe some uniformity in the techniques used to solve the familiar mechanics task, as opposed to the unfamiliar calculus task. The solving of the familiar task relied on pure algebraic reasoning, consistent with our textbook analysis findings (Hitier & González-Martín, 2022a), which identified that once kinematics equations were introduced, the derivative disappeared from the praxeologies used in kinematics. This may facilitate students’ recollection of the technique, since they simply need to use one equation and plug in values to obtain an answer. This situation does not seem to be specific to college physics courses. For instance, González-Martín et al. (2022) note that “only vestigial traces of calculus are visible in the technique and in the final result, confirming that these phenomena also occur in engineering courses” (p. 567). However, according to Planinic et al. (2019), this “almost exclusive reliance on formulas [...] presents [...] an important obstacle for the development of students’ deeper reasoning in physics and sometimes even an obstacle for the application of their already existing knowledge and reasoning developed in other domains” (p. 243). This “reliance on formulas” may be observed in the way participants attempted to recall techniques for both tasks, instead of trying to give sense to the task (e.g., S2: “I don’t think I remember how to solve this”).

Finally, we believe that our results concerning ‘paired’ questions (in this paper and in Hitier & González-Martín, 2022b) seem to confirm expectations regarding students’ difficulty using praxeologies from one course in the other. In both cases, the praxeologies used to solve the familiar mechanics question were rather homogeneous and students were able to provide answers without needing to understand the context. In contrast to the familiar mechanics task, our participants used a wider range of approaches in solving the unfamiliar task, due to the lack of a ready-to-use technique. This task also seemed to encourage greater use of the graphic register by the participants. Additionally, we note that the rationales used included mentions of calculus notions like concavity or variation, although these were not really exploited by the students, who ultimately decided to use the properties of parabolas or integration techniques. We therefore believe that discussing these kinds of unfamiliar tasks in calculus classes, and drawing links to similar tasks in mechanics courses, would help students deepen their understanding of the concepts involved. However, this would require calculus teachers to be familiar enough with the mechanics content to make the necessary connections themselves. This last point resonates with Biza et al. (2022), who advocate for “a balance[d] input from mathematics educators and educators of other disciplines at practice and research levels” (p. 409).

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